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Estimating the Performance of Series System's Production Process

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Abstract

In former Lifetime performance index C_L studies, it is usually assume that quality characteristic is the lifetime of an electronic component, engine, camera or in special case lifetime of business. In this paper we suppose that the quality characteristic is the lifetime of a series system and under the assumption of exponential distribution for component lifetime, we provide a maximum likelihood estimator of C_L and then this estimate used to develop testing procedure of C_L. Finally, we give an example to illustrate the use of the testing procedure.

Keywords: Lifetime performance index , Series system, Capability analysis.

1 Introduction

Process capability indice (PCI) is an effective means for measure the ratio of the spread between the process specifications to the spread of the natural variation. Montgomery [1] proposed PCIs such as, C_p, C_{pk}, C_{pm} and C_{pmk} that measure the target-the-better type quality characteristics. Beside above PCIs, they also proposed the indices C_{PL} or C_L (lifetime performance index) for measure the larger-the-better type quality characteristics, where L is the lower specification limit. Clearly, a longer lifetime implies a better product quality, so the lifetime is a larger-the-better type quality characteristic. Recently, C_L have been an interesting subject for many researchers in capability analysis field, for example, Tong et al. [2], Hong et al. [3] and Ahmadi et al. [4] worked on statistical inference of C_L. In former C_L studies, it is usually assume that quality characteristic is the lifetime of an electronic component, engine, camera or in special case lifetime of business. For

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example, Tong et al. [2] assume that the quality characteristic is the lifetime of electronic components or Hong et al. [3] suppose that the quality characteristic is the lifetime of businesses.
A series system is a system that work if and only if all of it components work. Let $X_1, X_2, ..., X_n$ be the lifetime of the system components. Since the failure time of the series system depends on the failure time of the weakest component, so the failure time of a series system can be modeled by $X_{1:n}$ (first order statistic).
In this paper we want to do a statistical inference about the performance of series system's production process. In Section 2, we introduce some properties of C_L for lifetime of series system. In Section 3, we discusses about the conforming rate. Sections 4 and 5 presents the ML estimator of C_L and a new hypothesis testing procedure for C_L based on lower confidence bound, respectively. Finally, in Section 6 we present a numerical example to illustrate proposed testing procedure.

2 The lifetime performance index

Lifetime performance indice C_L is defined as follows:

$$C_L = \frac{\mu - L}{\sigma}, \tag{1}$$

which μ denotes the process mean, σ represents the process standard deviation, and L is the known lower specication limit. Suppose that m series systems are placed independently in test at time zero. Also suppose that the components of each system are independent and identically distributed and comes from the one-parameter exponential distribution with below probability density function (p.d.f.):

$$f_{(X)}(x, \lambda) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0, \tag{2}$$

where λ is the scale parameter. Let X_{ij} $\forall j = 1, 2, ..., n$ and $\forall i = 1, 2, ..., m$ be the lifetime of the j-th component of the i-th system and $T_i = X^i_{1:n}$ $\forall i = 1, 2, ..., m$ be the lifetime of the i-th series system, respectively. By a simple computation, it can be seen that the lifetime of systems (T) follow one-parameter exponential distribution with p.d.f (2) with scale parameter $n\lambda$. By substituting the mean and standard deviation of T in (1), the lifetime performance index can be obtained as follows:

$$C_L = \frac{\frac{1}{n\lambda} - L}{\frac{1}{n\lambda}} = 1 - nL\lambda, \quad -\infty < C_L < 1, \tag{3}$$

where $\frac{1}{n\lambda} = E(T) = \sigma(T)$. Obviously, when the mean lifetime of system exceed L (i.e $\frac{1}{n\lambda} > L$), then the lifetime performance index $C_L > 0$.

3 The conforming rate

If the lifetime of a system, T, exceed the lower specification limit (i.e $T > L$) then the system defined as a conforming product. The ratio of conforming products is known as the conforming rate P_r which is defined as follows:

$$P_r = P(T > L) = e^{-nL\lambda} = e^{C_L - 1}.$$

Obviously, a strictly relationship exist between conforming rate P_r and lifetime performance index C_L.

4 MLE of lifetime performance index

The likelihood function corresponding observed sample $(t_1, t_2, ..., t_m)$ is given as follows:

$$L(\lambda) = \prod_{i=1}^{m} n\lambda e^{-n\lambda t_i} = (n\lambda)^m e^{-n\lambda \sum_{i=1}^{m} t_i}. \tag{4}$$

By setting the first partial derivatives of the natural logarithm of the likelihood function (Eq (4)) equal to zero with respect to λ, the MLE of λ obtained as $\widehat{\lambda} = \frac{m}{nW}$, where $W = \sum_{i=1}^{m} T_i$. From (4), it can be seen that $W = \sum_{i=1}^{m} T_i$ is a complete and sufficient statistic for λ and also $W \sim Gamma(m, n\lambda)$ therefore, $2n\lambda W \sim \chi^2_{(2m)}$. According to the invariance property of the MLE, the MLE of C_L can be written as:

$$\widehat{C_L} = 1 - \frac{mL}{W} \tag{5}$$

By taking expectation from $\widehat{C_L}$, it can be seen that when $m \longrightarrow \infty$ the MLE of C_L is a unbiased estimator for C_L.

5 Testing procedure for the lifetime performance index

At first suppose that the lifetime performance index target value is shown by c^*. Given the specified significance level α and the pivotal quantity $2n\lambda W$, which is distributed as $\chi^2_{(2m)}$, the level $100(1-\alpha)\%$ one-sided confidence interval for C_L can be derived as follows:

$$\begin{aligned} P\Big(2n\lambda W < CHIIN(1-\alpha, 2m))\Big) &= 1-\alpha, \\ \Rightarrow P\Big(C_L > 1 - \frac{(1-\widetilde{C_L})CHIIN(1-\alpha, 2m)}{2m}\Big) &= 1-\alpha. \end{aligned} \tag{6}$$

where CHIIN$(1-\alpha, 2m)$ function represents the lower $100(1-\alpha)$ percentile of $\chi^2_{(2m)}$. From (6), the level $100(1-\alpha)\%$ lower confidence bound for C_L can be derived as:

$$\underline{LB} = 1 - \frac{(1-\widetilde{C_L})CHIIN(1-\alpha, 2m)}{2m}, \tag{7}$$

where $\widetilde{C_L}$, α and m denote the MLE of C_L, the specified significance level and the observed number, respectively. So the testing procedure can be constructed with the one-sided confidence interval as follows:

step1: Determine the lower lifetime limit L for products and performance index target value c^*.

step2: Specify a significance level α.

step3: Calculate the value of lower confidence bound $\underline{LB}$ from (7).

step4: The decision rule of statistical test is "If performance index target value $c^* \notin [\underline{LB}, \infty)$, it is concluded that the lifetime performance index of products meets the required level".

6 Numerical example

Example 1. *Simulated data set*

A simulated data set of the failure times of $m=$ 20 series systems with n=5 components from exponential distribution with p.d.f (2) and parameter $\lambda=$ 0.2 ($n\lambda = 1$) are: 1.69, 0.98, 0.54, 0.16, 1.23, 3.92, 0.39, 5.11, 0.01, 0.08, 2.42, 0.42, 0.80, 1.18, 0.56, 0.18, 0.29, 0.41, 0.95, 2.29. **step1** the lower lifetime limit L and the lifetime performance target value c^* are assumed to be 0.1 and 0.8, respectively. In **step2** Specify a significance level α=0.05. Calculate the value of $\underline{LB} = 1-\frac{((\frac{20*0.1}{23.61})*55.76)}{2*20} = 0.88$ in **step3**. In **step4**, because of $0.8 \notin [0.88, \infty)$, so it is concluded that the lifetime performance index of products meets the required level.

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