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Goodness-of-Fit Test Based on Kullback-Leibler Information for Progressively First-Failure Censored Data

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Abstract

In this article, We constructed a goodness-of-fit test statistic based on Kullback-Leibler information for exponential distribution by using maximum likelihood estimate of the model parameter. A Monte Carlo simulation is performed to evaluate the power of the proposed test for several alternatives under different sample sizes and progressive first-failure censoring schemes.

Keywords: Entropy, Goodness-of-fit test, Kullback-Leibler information, Monte Carlo simulation, Progressively frist-failure censored data.

1 Introduction

Censoring is very important in determining the distribution of life-time products and where as units test are often censored based on cost and time. Although progressively Type- II shortens the test duration, but it is still too long for products having a high reliability that made Johnson [1] proposed a new censoring scheme known as the first-failure. Wu & Kus [6] combined the concepts of fist-failure and progressively censoring to introduce a new concept called progressively first-failure censoring scheme.

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1.1 Progressively First-Failure Censored Data

Suppose n independent groups with k items in each group are placed on a life-testing experiment that their life-times are identically distributed with probability density (p.d.f), $f(x;\theta)$, and cumulative distribution function (c.d.f), $F(x;\theta)$. Where θ is the unkown the vector of parameters and $m(n)$ is fixed prior to the exprement. When the first failure (X_1) occurs, R_1 groups and the group with observed failure are randomly withdrawn from the experiment. When second failure (X_2) observed, R_2 groups and the group with observed failure are randomly withdrawn from the experiment, and so on. Finally, when the failure m is observed, the remaining l_m groups and the group with observed failure are all withdrawn from the experiment. This censoring is called a progressive first-failure censoring scheme. The joint p.d.f of all progressively first-failure censored order statistics $(X_{1:m:n:k}, X_{2:m:n:k}, \ldots, X_{m:m:n:k})$ with progressive censoring scheme proposed by Wu & Ku? [?] that is given by

$$f_{X_{1:m:n:k},\ldots,X_{m:m:n:k}}(x_1,\ldots,x_m) = ck^m \prod_{i=1}^{m} f(x_i;\theta)\left(1-F(x_i;\theta)\right)^{k(R_i+1)-1}$$

$$,0 < x_1, \ldots, < x_m$$

where $c = n(n-R_1-1), \ldots, (n-\sum_{i=1}^{m-1} R_i - m + 1)$.

1.2 Nonparametric Entropy Estimate of Progressively First-Failure Censored Data

Balakrishnan et al. [1] has been simplified the joint entropy of progressively Type-II censored order statistics in terms of an integral involving the hazard function $h(x)$. Since the joint p.d.f Progressively first-failure censored is similar to the joint p.d.f progressively Type-II censored, the nonparametric estimate of the joint entropy $H_{1\cdots:m:n:k}$ is given by

$$H_{1\cdots:m:n:k} = -\log c + nkH(w,n,m,k)$$

where

$$\begin{aligned} H(w,n,m) &= \frac{1}{nk}\sum_{i=1}^{m} log\left(\frac{x_{i+w:m:n:k} - x_{i-w:m:n:k}}{E(U_{i+w:m:n:k}) - E(U_{i-w:m:n:k})}\right) \\ &+ \frac{m}{nk} - \frac{1}{nk}\sum_{i=1}^{m}\sum_{j=1}^{i}\frac{D_i}{\gamma_j^2} \end{aligned}$$

where $D_i = \prod_{j=1}^{i}$, $\gamma_i = m - i + 1 + \sum_{j=i}^{m} R_i$ for $1 \le i \le m$.

1.3 Kullback-Leibler Information

For a null density function $f^0(x_i;\theta)$, the KL information from progressively first-failure censored data can be estimated by

$$\begin{aligned} T &= -H(w,n,m,k) - \frac{1}{nk}\sum_{i=1}^{m}\log f^0(x_i;\widehat{\theta}) \\ &- \frac{1}{nk}\sum_{i=1}^{m}\left(k\left(R_i+1\right)-1\right)\log\left(1-F^0(x_i;\widehat{\theta})\right) \end{aligned}$$

where $\widehat{\theta}$ is a MLE estimator of θ.

1.4 Goodness-of-Fit Test for Exponential

Suppose we are interested in a goodness-of-fit test for

$$H_0 : f^0 = \left(\frac{1}{\theta}\right) exp\left(-\frac{x}{\theta}\right) \quad VS. \quad H_A : f^0 \neq \left(\frac{1}{\theta}\right) exp\left(-\frac{x}{\theta}\right)$$

where θ is unknown. If we replace the maximum likelihood estimate in place of the unknown parameter θ, then the KL information for progressively first-failure censored data can be estimated by

$$T = -H(w, n, m, k) + \frac{m}{nk}\left[\log\left(\frac{1}{m}\sum_{i=1}^{m} k\left(R_i + 1\right)X_{i:m:n:k}\right) + 1\right].$$

If $T(w, n, m, k)$ is close to 0, H_0 will be acceptable, and therefore large values of $T(w, n, m, k)$ will lead to the rejection of H_0.

Table 1: VALUE OF THE WINDOWS SIZE m WHICH MINIMUM CRITICAL VALUES OFα FOR 0.1

nk	k	m	w
20	(2,2)	(5,7)	(3,4)
30	(2,2,3)	(5,7,10)	(3,4,6)
40	(2,2,2,4,4)	(5,10,15,5,7)	(3,6,8,3,4)
50	(2,2,2,2,5,5)	(5,10,15,20,5,7)	(3,6,8,11,3,4)

2 Implementation of Test

Because the sampling distribution of $T(w, n, m, k)$ is intractable, we determine the percentage points using 10,000 Monte Carlo simulations from an exponential distribution. In determining the window size w which depends on n, m, k and α, we consider the optimal window size to be one which gives minimum critical points. However, we understood from the simulated percentage points that the optimal window size w varies much according to m rather than n, k and does not vary much according to α, if $\alpha \leq 0.1$. In view of these observations, our recommended values of w for different m are presented in Table 1.

3 Main results

Since the suggested test statistic is related to the hazard function of the distribution, we consider the following alternatives according to the type of hazard function as

(a) **Monotone increasing hazard including Gamma and Gexp (shape parameter 2) and Chi-square (degree of freedom 3),**

(b) **Monotone decreasing hazard including Gamma and Gexp (shape parameter 0.5) and Chi-square (degree of freedom 1), and**

(c) **Non-monotone hazard including Beta and Log-logistic (shape parameter 0.5) and Burr (shape1 and shape2 1).**

To estimated the power of proposed test statistic, We used 10,000 Monte Carlo simulations for $nk = 20(10)40$, each with own different $k's$, and some m under null hypothese. However, we understood following results when the alternative is either monotone decreasing hazard or monotone increasing hazard functions:

(a) **Censoring scheme** $R = (m, \ldots, 0)$ **and** $R = (0, m, \ldots, 0)$ **show better power than other censoring schemes when the alternative is a monotone increasing hazard function.**

(b) **It is observed that for fixed n and k, as m increases the power is improved but when k increases the power is decreased .**

(c) **for** $nk = (20, 30, 40)$**, the best power is shown at** $k = 2$ **and** $m = (7, 10, 15, 20)$ **respectively.**

References

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