First Seminar on
Reliability Theory and its Applications
27-28, May 2015

A Representation of the Residual Lifetime of a Repairable System

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Abstract

In this paper, the residual lifetime of a repairable system is studied when the failure status of the system is known. A mixture representation of the reliability function of the conditional residual lifetime of a repairable system in terms of the reliability function of residual records is provided. Some stochastic properties of the conditional probabilities and the residual lifetimes also are given.

Keywords: Aging properties, Minimal repair, Residual lifetime, Stochastic ordering.

1 Introduction

For a repairable system, carrying out minimal repairs is a natural approach, because it can keep the system working at a minimal cost. That is, minimal repair restores the system to its functioning condition just prior to failure with the failure rate of the system remaining undisturbed. Many authors have been followed this model, see e.g., for example [1], [2], [3], [4] and [5]. The present paper explores some applications of the residual life of record values in analysis of a repairable system. For this purpose we consider a repairable system with minimal repairs, whose number of repairs is a positive random variable with a given probability vector. We obtain some mixture representations for residual lifetime of a repairable system and compare two systems. For briefness, we just mention the following orders for comparison of arithmetic random variables.

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Definition 1. *If X and Y are discrete random variables taking on values in $\mathbb{Z}^+$, with distributions $\mathbf{p} = (p_1, p_2, \ldots)$ and $\mathbf{q} = (q_1, q_2, \ldots)$, where $p_i = Pr(X = i)$ and $q_i = Pr(Y = i)$, $i \in \mathbb{Z}^+$. Then*

1. *$X \leq_{st} Y$ if and only if $\sum_{j=i}^{\infty} p_j \leq \sum_{j=i}^{\infty} q_j$, for all $i \in \mathbb{Z}^+$;*
2. *$X \leq_{hr} Y$ if $\sum_{j=i}^{\infty} p_j / \sum_{j=i}^{\infty} q_j$ is decreasing in i, for all $i \in \mathbb{Z}^+$;*
3. *$X \leq_{lr} Y$ if p_i/q_i is decreasing in i, for all $i \in \mathbb{Z}^+$ when $p_i, q_i > 0, \forall i \in \mathbb{Z}^+$.*

We refer the reader to [6] and [7] for more details on stochastic orderings and their applications.

2 Model description

Consider a repairable coherent system under the condition that, at time t, some information about the status of the system lifetime is available. Suppose the system can be repaired $N-1$ times and the Nth failure is fatal to the system with probability vector $\mathbf{p}$, where

$$\mathbf{p} = (0, \ldots, 0, p_{\ell_1}, p_{\ell_1+1}, \ldots, p_{\ell_2-1}, p_{\ell_2}),\ \ell_1 = 1, 2, \ldots, \ell_2,\quad \ell_2 = 1, 2, \ldots, n. \tag{1}$$

Chahkandi et al. [8] investigated some reliability properties of this model. Sometimes, an operator may know that, at time $t > 0$, the system is still operating, i.e. $T > t$, which is related to the residual lifetime of the system. Thus, we are interested in the probability that the system can be repaired $(i-1)$ times by assuming that $T > t$. Let us denote $\Pr(T = T(i)|T > t)$ by $b_i(t)$, then we have

$$b_i(t) = \frac{p_i \bar{F}_{T(i)}(t)}{\sum_{j=\ell_1}^{\ell_2} p_j \bar{F}_{T(j)}(t)} \quad i = \ell_1, \ldots, \ell_2. \tag{2}$$

3 Mixture representations

Navarro et al. [9] considered the residual lifetime of a coherent system and obtained a mixture representation for the system's residual lifetimes in terms of the order statistics of its components. Here, we consider the situation that one may have some partial information about the system lifetime and interested in finding the dynamic probability of system failure. We derive a mixture representation for the survival function of used but working repairable system, i.e. for distribution function of the system lifetime T given that its lifetime is greater than t, $(T - t|T > t)$. In the next result, we obtain a mixture representation for the residual lifetime of a repairable system in terms of the record values of its original distribution.

Theorem 1. *If T is the lifetime of a repairable system that can be repaired $(N-1)$ times and the Nth failure is fatal to the system with probability vector $\mathbf{p}$. Then, for all $x \geq 0$ and $t > 0$, such that $\bar{F}_T(t) > 0$, we have*

$$\Pr(T - t > x|T > t) = \sum_{i=\ell_1}^{\ell_2} b_i(t) \Pr(T(i) - t > x|T(i) > t), \tag{3}$$

where the coefficient $b_i(t)$ *is given in* (2) *such that* $\sum_{i=\ell_1}^{\ell_2} b_i(t) = 1$.

Consider two repairable systems with different probability vectors for their repairable numbers. For example two manufactures may guaranty their products with different number of repairs:

$$\mathbf{p} = (0, \ldots, 0, p_{\ell_1}, p_{\ell_1+1}, \ldots, p_{\ell_2-1}, p_{\ell_2})$$

and

$$\mathbf{q} = (0, \ldots, 0, q_{\ell_1}, q_{\ell_1+1}, \ldots, q_{\ell_2-1}, q_{\ell_2}),$$

for $\ell_1 = 1, 2, \ldots, \ell_2$, $\ell_2 = 1, 2, \ldots, n$, respectively. Suppose two systems are repairable $(N-1)$ and $(M-1)$ times, and the Nth and Mth failures are fatal to the systems with probability vectors $\mathbf{p}$ and $\mathbf{q}$, respectively. Take

$$\mathbf{b}(t) = (0, \ldots, 0, b_{\ell_1}(t), \ldots, b_{\ell_2}(t)), \tag{4}$$

where $b_i(t)$ is given as in (2). In this case, if $\mathbf{p} \leq_{lr} \mathbf{q}$, then $\mathbf{b}_{\mathbf{p}}(t) \leq_{st} \mathbf{b}_{\mathbf{q}}(t)$, where $\mathbf{b}_{\mathbf{p}}(t)$ is the dynamic probability vectors corresponding to $\mathbf{p}$.

Theorem 1 shows that the reliability function of the residual lifetime of a repairable system can be expressed in terms of a weighted summation of record values' residual lifetimes. Here, we consider a situation in which the system is alive after a known number of repairs and investigate its residual. For this purpose, we study the residual lifetime of a repairable system when the system is working, and at least $k-1$ repairs are done on the system at time t; namely the conditional random variable $[T - t | T > t, T(k) \leq t]$, $k = \ell_1, \ldots, \ell_2 - 1$. The next theorem presents a mixture representation for the conditional residual lifetime of $[T - t | T > t, T(k) \leq t]$, $k = \ell_1, \ldots, \ell_2 - 1$.

Theorem 2. *Consider a repairable system with probability vector* $\mathbf{p}$*, as in* (1)*, for the random maximum number of minimal repairs that can be performed. If* $\Pr(T > t, T(k) \leq t) > 0$*, then*

$$\Pr(T - t > x | T > t,\, T(k) \leq t) = \sum_{i=k+1}^{\ell_2} b_i(t,k) \Pr(T(i) - t > x | T(i) > t, T(k) \leq t), \tag{5}$$

where $\mathbf{b}(t,k) = (0, \ldots, 0, b_{k+1}(t,k), \ldots, b_{\ell_2}(t,k))$*, with*

$$b_i(t,k) = \frac{p_i \Pr(T(k) \leq t < T(i))}{\sum_{j=k+1}^{\ell_2} p_j \Pr(T(k) \leq t < T(j))}, \tag{6}$$

such that $\sum_{j=k+1}^{\ell_2} b_j(t,k) = 1$.

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