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A Switch Model in Redundant Systems

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Abstract

Redundancy is a technique that has been widely applied to improve the system reliability and its availability. In this paper, a new switching model is proposed to increase the reliability of a unit (system) with a cold standby backup. It is assumed that the switch over to the standby unit is not failure-free, contrary to what we have in standby redundancy. The optimal time to switch between the key unit and its cold standby backup is find such that the mean lifetime of the system to be maximized. Finally, an example is presented to compare the mean lifetime of the proposed switching model and a system with parallel redundancy.

Keywords: Parallel system, Redundancy, Survival function, Switching.

1 Introduction

Redundancy is a common method to increase system reliability. There are various methods, techniques, and terminologies for implementing the redundancy. Standby redundancy is one of the main methods. In general, there are three types of standby, i.e. cold, hot and warm standby. In cold standby, the secondary unit is powered off, thus preserving the reliability of the unit. In hot standby, an inactive unit undergoes the same operational environment as when it is in active state. Warm standby is an intermediate case. In this case an inactive unit undergoes operational environment that is milder than the environment of the same component in active state. The performance of the standby system was studied by some of researchers such as [1], [2], [3], [4] and [5]. For the simplicity of the standby redundancy models, we assume that the switch over to the standby unit is perfect, i.e. instantaneous and failure-free. But there are some real situations that we

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haven't any time to switch the failed unit to its backup. Because after the unit failure, the system would be failed. Here, we focus on this case, i. e. the case that the switch over to the standby unit is not failure-free. In these situations, we can use the dual modular redundancy (DMR) or parallel redundancy, where the key component (system) and its backup begins to operate together, to increase the system reliability. But, parallel redundancy increases the cost and complexity of the system. Because, the lifetime of the key unit and its backup decrease simultaneously. In this paper we present a new model for a system with a cold standby component such that it is not required to work the key component and its backup continuously. In the next section, we present the new model that allows to switch between the key unit and its backup before the units failures. The optimal switching time for increasing (decreasing) failure rate distributions is obtained in Section 3. A parametric example is given to compare the mean lifetime of our new model and a system with parallel redundancy.

2 A new switching model

In this section we present a new switching model for a unit (system) with a cold standby backup. Consider a system consisting of two units A (the key unit) and B (the cold standby unit) connected in parallel branches, and a switcher (S) as shown in Figure 1.

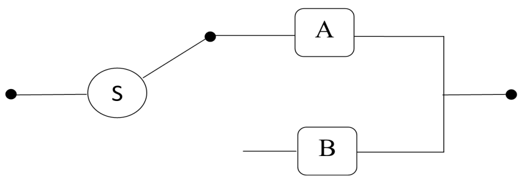

Figure 1: Standby system with two units.

At the beginning, the units are new with a good reliability. Thus, it may not be required to operate both of them. We consider a switcher key before the units to allows us to switch between the ones in specified times. First, the key components begins to operate and its back up is in an 'off' state. After a specified time t_1, a switch turns on the 'standby' backup (while the key component goes on 'off' state), and the system continues to operate. After specified time t_2 a switch turns on the key components and its backup together. This arrangement implies that the system may be failed before the first switch, when the key unit is working, between the first and second switch, when the backup unit is operating, or after the second switch, when both of the units are operating. For simplicity, suppose that $t_1 = t_2 = c$, and the units' lifetimes are the same, with distribution function F. After some manipulations, the survival function of the system can be expressed as

$$\bar{F}_{T_1}(t) = \begin{cases} \bar{F}(t) & t \le c \\ \bar{F}(c)\,\bar{F}(t-c) & c < t \le 2c \\ 2\bar{F}(c)\bar{F}(t-c) - \bar{F}^2(t-c) & t > 2c. \end{cases} \tag{1}$$

The failure rate function of T_1 also is given as

$$r_{T_1}(t) = \begin{cases} r(t) & t < c \\ r(t-c) & c < t < 2c \\ 2r(t-c)\left[\dfrac{\bar{F}(c) - \bar{F}(t-c)}{2\bar{F}(c) - \bar{F}(t-c)}\right] & t > 2c, \end{cases} \tag{2}$$

Table 1. The values of mean lifetimes

(λ, β)	c_0 (the optimal switching time)	$E(T_1)$	$E(T_2)$
(1, 0.5)	0.000	3.500	3.500
(1, 1)	0.000	1.500	1.500
(1, 1.5)	0.164	1.286	1.236
(1, 2)	0.347	1.303	1.145
(1, 3)	0.507	1.359	1.077
(1, 4)	0.586	1.472	1.037
(1, 5)	0.637	1.530	1.028
(1, 7)	0.703	1.616	1.023

where $r(t)$ is the units' failure rate function. Utilizing (1), the expectation lifetime of the system can be found as

$$E(T_1) \;=\; \int_0^c \bar{F}(t)dt + \bar{F}(c)[E(X) + \int_c^\infty \bar{F}(t)dt] - \int_c^\infty \bar{F}^2(t)dt = g(c). \qquad (3)$$

We are interested in finding a value of c that maximize the mean lifetime of the system.

3 Main results

In this section, we find the optimal time of switching to maximize the mean lifetime of the system. In the next results the optimal time of switch is found in DFR and IFR distributions.

Theorem 1. *Let F be a DFR distribution, then the function $g(c)$ in (3) would be maximized at $c = 0$.*

Theorem 2. *Let F be an IFR distribution and $f(0) < \frac{1}{2\mu}$, where f is the density function of F and μ is its mean, then the function $g(c)$ in (3) takes its maximum value at a point on its domain (on the interval $[0, \infty)$) not on the boundary points.*

Now, we compare the mean lifetime of the system in switch model, $E(T_1)$, and the mean life of a parallel system with two units, $E(T_2)$. $E(T_1)$ is obtained in equation (3) and $E(T_2)$ can be obtained as

$$E(T_2) = 2\mu - \int_0^\infty \bar{F}^2(t)dt,$$

where X_1, X_2 are the units' lifetime with distribution F and mean μ. The failure rate function of T_2 also is given by

$$r_{T_2}(t) = 2r(t)a(t), \qquad (4)$$

where $a(t) = \frac{F(t)}{1+F(t)}$. Note that $0 \leq a(t) \leq 0.5$, and is an increasing function of t. By comparing the equations (3) and (4), it would be found that $E(T_2) = g(0)$. In the next example we find the optimal time for our switching model when the units' distribution lifetimes are Weibull. Consider the following distribution for the components' lifetimes of the system given in Figure 1

$$\bar{F}(t; \lambda, \beta) = e^{-(\lambda t)^\beta}.$$

Table 1 confirms the results of Theorems 1 and 2.

The failure rate functions of T_1 and T_2 are also plotted in Figure 2 for $\lambda = 1$, $\beta = 3$ and $c_0 = 0.507$. Note that the failure rate of the new switching model is less than the failure rate of a parallel system after c_0.

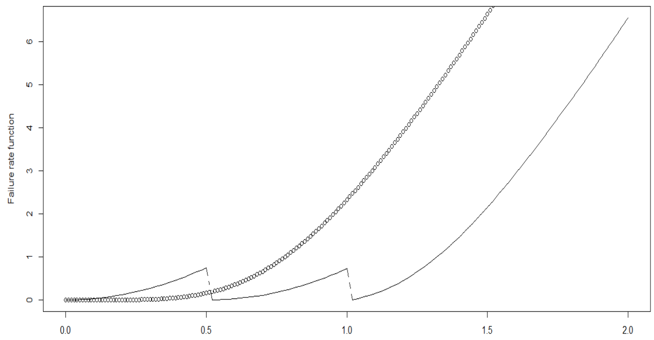

Figure 2: The failure rate functions of T_1 (thick line) and T_2 (dotted line) for the Weibull distribution.

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