First Seminar on
Reliability Theory and its Applications
27-28, May 2015

A Non-Parametric Test Against Renewal Increasing Mean Residual Life Distributions

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Abstract

In this paper we introduce a new aging class of life distributions when a device is operating in a realistic environment. We study the behavior of such life distributions through the mean residual life notion, when a device is experiencing number of shocks. Due to these shocks the lifetime of such device has become shortened or prolonged. These tempered events are governed by a homogenous Poisson process. A moment inequality which characterizes this new aging class, namely renewal increasing mean residual life, is derived. We propose a new U-statistic test procedure to address the problem of testing exponentiality against such class of life distributions. It is shown that the proposed test enjoys a superior power for some commonly used alternative.

Keywords: Poisson Shock model, Increasing mean residual life, Exponential distribution, Moment inequalities, U-statistics.

1 Introduction

The mean residual lifetime, MRL, is the remaining lifetime of a component alive at time t. If X denotes a nonnegative random variable with a continuous life distribution function F and finite mean $\mu = \int_0^\infty \bar{F}(x)dx$, then the MRL function at time t is defined as

$$m(t) \ = E\left(X - t | \, X > t\right) = \frac{1}{\bar{F}(t)} \int_t^\infty \bar{F}\left(u\right) du. \tag{1}$$

The properties of mean residual life (MRL) of a component (subjected to no shocks) have been widely used for deriving maintenance and replacement policies.

The goal of this paper is to study the age replacement models through the remaining life time of a device in a more real life environment in which unit fail by physical deterioration

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suffered from some damage. In the latter case, units fail when the total damage due to shocks has exceeded a critical level. Such damage models may apply to the actual units that are working in industry, service, information, computers etc. In the past decade, various properties of failure distributions when shocks occur in a Poisson process were extensively investigated. We refer our reader to Li and Xu (2008), Ahmad and Mugdadi (2010), and Izadkhah and Kayid (2013), Sepehrifar et al. (2014), among others. We study the cumulative damage model for an operating unit through the increasing mean residual life function, which follows by the related definitions. We use the innovative features of this function to introduce the related moment inequality function and to derive a new U-statistic testing exponentiality against renewal increasing mean residual life under shock (RIMRl_{shock}) alternatives. Finally, the power simulation ane some numerical results are presented.

2 Basic definitions and properties

Consider a unit which is subjected to successive shocks and each shock causes some damage to this unit in some amount. Let random variables $\{T_j$: j=1, 2,...$\}$ denote the sequence of time intervals between successive shocks, and random variables $\{W_i$: i=1, 2,...$\}$ denote the amount of life-damage produced by the i-th shock, where $W_0 \equiv\ 0$. It is assumed that the sequence of $\{W_i\}$ is nonnegative, independent and identically distributed. We also assume that W_i is independent of T_j. Let random variable $N(t)$ denote the total number of shocks up to time t. Then, the total cumulative life-damage up to time t is defined as $Z\left(t\right)$where $Z\left(t\right)=\sum_{i=0}^{N(t)} W_i$.

It is assumed that the unit fails when the total damage exceeds a pre-specified level x(>0). Usually, the failure level x is statistically estimated and is already known. Let X be the life variable of a device with survival function $\bar{F}\left(t\right)=P\{X\geq t\}$ which is subjected to *N(t)* shocks with$P\left\{N\left(t\right)=j\right\}=F^{(j)}\left(t\right)-F^{(j+1)}\left(t\right),\ j=0,\ 1,\ 2,\ .$ Let the random variable W_i be the amount of hidden lifetime absorbed by the i^{th} shock, with common distribution $G\left(x\right)=P\{W_j\leq x\}$. Then the distribution of$Z\left(t\right)$ is $Q\left(x\right)=P\{Z\left(t\right)\leq x\}=\sum_{j=0}^{\infty} G^{(j)}\left(x\right)\left[F^{(j)}\left(t\right)-F^{(j+1)}\left(t\right)\right]$. Glynn and Witt (1993), studied the distribution of$Q\left(x\right)$.

Let $X^*=X-Z\left(t\right)$ be the lifetime of an item (with lifetime X, and survival function $\bar{F}\left(t\right)$) in a service with total life-damaged $Z\left(t\right)$. Set $m^*\left(t\right)=E\left[X_t^*\right]=E\left[X^*-t|\geq t\right]$, which is the mean residual lifetime of such item in the age replacement model subjected to N(t) shock, given that the item is in operating situation hours after installation or the total life-damaged is not exceeding the threshold level x, whichever comes last. We assume that X and Z(t) are independent. First, we present definitions and basic properties that will be used in the sequel.

Definition 1. The mean residual life of a device under shock model (MRL_{shock}) at time t, is defined as

$$m^*\left(t\right)\ =\frac{\int_t^{\infty}\bar{v}\left(z\right)dz}{\bar{v}(t)}, \tag{2}$$

where $\bar{v}\left(z\right)=\int_0^x\bar{F}\left(z+w\right)dQ\left(w\right).$

Definition 2. The distribution function F is said to be a renewal increasing mean residual life under shock models (RIMRL_{shock}) if $m^*(t)$ is an increasing function in $t\geq 0$.

Corollary 1. The distribution F belongs to RIMRL$_{shoch}$ if

$$(\bar{v}(t))^2 \leq \int_0^x f(t+w)\,dQ(w) \int_t^\infty \bar{v}(z)\,dz\ .$$

Corollary 2. The distribution function F belongs to RIMRL$_{shock}$ if

$$E_{f^*}\left[Min(X_1^*, X_2^*)\right] \leq \frac{1}{2}\left(E_{f^*}(X_1^*)\right)$$

where $X_i^* = X_i - W_i$.

3 Testing exponentiality against RIMRL$_{shock}$ alternatives

Consider the problem of testing H$_0$: F is exponential with mean,$\mu < \infty$ versus H$_1$: F is RIMRL$_{shock}$ and not exponential. We consider corollary 2 for RIMRL$_{shock}$ as the measure of departure from the null hypothesis H$_0$:

$$\delta = \frac{1}{\mu^*}\left\{E_{f^*}[Min(X_1^*, X_2^*) - \frac{1}{2}X_1^*]\right\}$$

where $\mu^* = E(X_i^*)$.
This measure may be estimated by the following statistics:

$$\hat{\delta} = \frac{1}{\bar{X^*}} \times \frac{2}{n(n-1)}\left\{\sum_{\substack{i=1\\ i<j}}^{n}\sum_{j=1}^{n}\left\{Min(X_i^*, X_j^*) - \frac{1}{2}X_i^*\right\}\right\} \qquad (3)$$

where $\bar{X^*} = \frac{1}{n}\sum_{i=1}^{n} X_i^*$ is the sample mean based on a random sample from distribution F. Note that $\hat{\delta}$ is derived based on the standard U-statistic theory.Let

$$\phi(X_1^*, X_2^*) = \frac{1}{\mu^*}\left[Min\,(X_1^*, X_2^*) - \frac{X_1^*}{2}\right]$$

$$\phi(X_2^*, X_1^*) = \frac{1}{\mu^*}\left[Min\,(X_2^*, X_1^*) - \frac{X_2^*}{2}\right]$$

and define the symmetric kernel

$$\psi(X_1^*, X_2^*) = \frac{1}{2!}\sum \phi(X_{i1}^*, X_{i2}^*)$$

where the sum is an overall arrangement of X_1^* and X_2^*. It can be shown that $\hat{\delta}$ in equation (3.1) is equivaltne to U-statistic given by

$$U = \frac{1}{\begin{pmatrix} n \\ 2 \end{pmatrix}}\sum_{i<j}\phi(X_i^*, X_j^*).$$

The following theorem gives the large sample properties of $\hat{\delta}$ or U.
Theolem 1. As $n \to \infty$, $\sqrt{n}(\hat{\delta} - \delta)$is asymptotically normal whit mean 0 and variance

$$Var\left(\frac{1}{2}\left\{\frac{2}{\mu^*}\{X_1^* F(X_2^*) + \mu^* \bar{F}(X_2^*)\} - \frac{X_1^*}{2\mu^*} - \frac{1}{2}\right\}\right).$$

Under the null hypothesis H_0 ,$X_i^* \sim Exp(1)$, the variance is calculated as $\sigma_0^2 = \frac{7}{48}$.

To carry out this test, we calculate $\sqrt{n}\hat{\delta}\sigma_0^{-1}$ and reject H_0 if this value is larger than $Z_{1-\alpha}$.

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