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A New Investigation About Parallel $(2, n-2)$ System Using FGM Copula

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Abstract

Redundancy is a highly used technique to increase the systems lifetimes and availability. Recently, employing standby units in systems has received great attention. Here, parallel system of n units and two-unit parallel system supported by $(n-2)$ cold standbys are considered where units lifetimes are assumed to be dependent in terms of Farlie-Gumbel-Morgenstern copula structure. Applicable formulas of mean time to system failure are given and the impact of dependence parameter on systems lifetimes are investigated.

Keywords: Cold standby FGM copula Mean time to system failure Parallel system.

1 Introduction

Parallel systems are known as the first redundant systems, later application of standby units extended redundant models. In recent studies correlated units lifetimes are also assumed. Papageorgiou and Kokolakis [3] evaluated reliability of two-unit parallel system supported by $(n-2)$ standbys. Papageorgio and Kokolakis [4] extended the main results of their previous study and developed the system reliability and mean time to system failure (MTSF). Eryilmaz and Tank [1] employed copula function to model the reliability and MTSF of a series system with a single cold standby unit. Here, we use copula function to express the reliability and MTSF of two-unit parallel system supported by $(n-2)$ cold standbys which is briefly expressed as parallel $(2, n-2)$ system. The results are specifically given in terms of Farlie-Gumbel-Morgenstern (FGM) copula.

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2 Parallel system with dependent units

Assuming $T_1, T_2, \ldots, T_n$ represent the lifetimes of n units of the parallel system and $T_{i:n}$, $i = 1, 2, \ldots, n$, denotes the ith ordered unit lifetime, $T_{n:n}$ implies the parallel system lifetime, *i.e.*, $T_{P,n} = T_{n:n}$. We expect that, failure of one unit leads more pressure to active ones, therefore, the sooner one unit fails the second unit failure occurs earlier, and this process is expected to be continued. Based on this issue, units lifetimes are considered to be positively dependent. The following proposition presents the reliability of a parallel system with dependent units lifetimes. The reliability of a parallel system containing n units with dependent lifetimes is

$$R_{P,n}(t) = P(T_{P,n} > t) = 1 - C(F_1(t), F_2(t), \ldots, F_n(t)),$$

where $F_i(t)$, $i = 1, 2, \ldots, n$ is the df of ith unit lifetime and C is a n-copula family which models the appropriate positive dependence structure of units lifetimes.

The next proposition delivers the general form of reliability and MTSF of parallel system while the relationship between units lifetimes is modelled via FGM n-copula which has been introduced by Johnson and Kotz [2]. Suppose that lifetimes of n units in a parallel system are identically distributed and the units lifetimes can be modelled via FGM n-copula with the same non-negative dependence parameters. Then, reliability function of the system is,

$$R_{P,n}(t) = 1 - F^n(t)\left[1 + \alpha \sum_{i=2}^{n} \binom{n}{i} \bar{F}^i(t)\right], \quad 0 \le \alpha \le 1.$$

The following example illustrates the reliability and MTSF of parallel system containing different numbers of units.

Example 1. *Let $F(t) = 1 - e^{-\lambda t}$, $\lambda, t > 0$. Then under the assumptions of Proposition 2, one easily concludes the following formulas.*

(i) If $n = 2$; $\quad MTSF_{P,2} = \int_0^\infty R_{P,2}(t)dt = \frac{1}{\lambda}(\frac{3}{2} - \frac{\alpha}{12})$.

(ii) If $n = 3$; $\quad MTSF_{P,3} = \int_0^\infty R_{P,3}(t)dt = \frac{1}{\lambda}(\frac{11}{6} - \frac{\alpha}{6})$.

(iii) If $n = 4$; $\quad MTSF_{P,4} = \int_0^\infty R_{P,4}(t)dt = \frac{1}{\lambda}(\frac{25}{12} - \frac{29\alpha}{120})$.

The trend of MTSFs are decreasing in terms of dependence parameter α. Figure 1 shows the trend of MTSFs for different possible values of dependence parameter α while $\lambda = 0.1$ in marginal df.

3 Parallel (2,n-2) system with dependent units

Consider a parallel $(2, n-2)$ system which in fact is a two-unit parallel system supported by $(n-2)$ cold standbys where $n \ge 3$, is a fixed number of non-repairable units. The failed active unit is replaced upon its failure instantaneously by one of the standbys. This process is continued until all the standbys are used in the system and the system operates if at least one unit is active. Assume that T_i, $i = 1, 2$, represents the lifetime of the ith initial active unit and S_j, $j = 1, \cdots, n-2$, represents the jth cold standby lifetime in the system. The next trivial proposition investigates this system lifetime. Parallel $(2, n-2)$ system lifetime, $T_{P,(2,n-2)}$, for some n is expressed as follows.

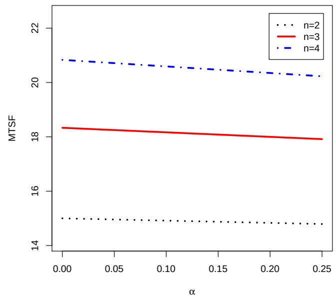

Figure 1: MTSF trend of parallel system for different possible values of dependence parameter α.

Let $n = 3$; $\quad T_{P,(2,1)} = \min(T_1, T_2) + \max(T_1^*, S_1)$, where T_1^* is the residual life of T_1 after the first unit is failed.

Let $n = 4$; $\quad T_{P,(2,2)} = \min(T_1, T_2) + \min(T_1^*, S_1) + p\max(T_1^{**}, S_2) + (1-p)\max(S_1^*, S_2)$, where S_1^* is defined similarly as T_1^*; and T_1^{**} is the residual life of T_1^*.

Let C be a copula that models the joint distribution of units lifetimes. Then we have

$$MTSF_{P,(2,1)} = 2\int_0^\infty \bar{F}(t)dt + \int_0^\infty C(F(t), F(t))dt - \int_0^\infty C(F^*(t), F(t))dt,$$

and

$$\begin{aligned} MTSF_{P,(2,2)} = 3\int_0^\infty \bar{F}(t)dt - \int_0^\infty F^*(t)dt + \int_0^\infty C(F(t), F(t))dt \\ + p\int_0^\infty \big[C(F^*(t), F(t)) - C(F^{**}(t), F(t))\big]dt. \end{aligned}$$

The following example presents the MTSFs of parallel $(2, n-2)$ system for some n and specific marginal distribution.

Example 2. *Let $F(t) = 1 - e^{-\lambda t}$, $\lambda, t > 0$, and dependence structure of units lifetimes is modelled by FGM 2-copula. Hence, the corresponding MTSF of parallel $(2, n-2)$ system is attained as follows.*

(i) If $n = 3$; $\quad MTSF_{P,(2,1)} = \frac{1}{\lambda}(2 - \frac{\alpha}{9} - \frac{\alpha^2}{180} - \frac{22\alpha^3}{945})$.

(ii) If $n = 4$; $\quad MTSF_{P,(2,2)} = \frac{1}{22680\lambda(\alpha^2+5\alpha-45)^2}\Big[114817500 - alpha(29342250 + 3402000p) - \alpha^2(2835000 + 1256850p) + \alpha^3(689850 + 38430p) + \alpha^4(37800 - 28620p) - \alpha^5(1890 + 951p) + 120p\alpha^6 + 31p\alpha^7\Big]$.

The MTSFs are decreasing in terms of dependence parameter α. For better intuition, see Figure 2 where $\lambda = 0.1$ in marginal df.

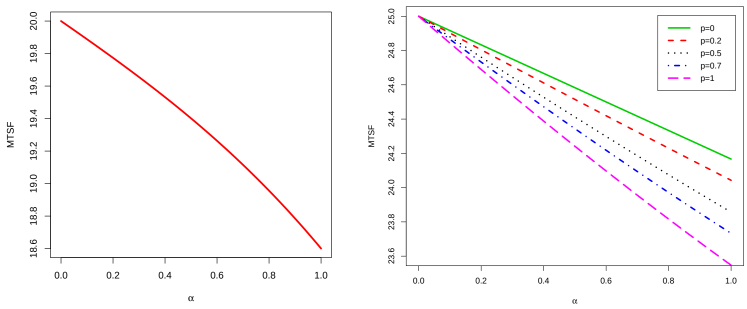

Figure 2: MTSF trend of parallel $(2, 1)$ system (left) and parallel $(2, 2)$ system (right) for different possible values of dependence parameter α and probabilities p.

References

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