First Seminar on
Reliability Theory and its Applications
27-28, May 2015

Use Weibull Distribution in Accelerated Life Testing for Computing MTTF Under Normal Operating Conditions

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Abstract

The intensity of the global competition for the development of new products in a short time. Testing under normal operating conditions for compute reliability quatities, requires a very long time. This has led to the development of accelerated life testing (ALT). In this article, We compute MTTF of Bourdon tubes (used as a part of pressure sensors in avionics) in stress conditiion. The failure is leak in the tube. Base on Anderson-Darling test Weibull distribution is appropriate for fitting data under stress condition. We determine MTTF of Bourdon tubes in operating condition base on arrhenius model and mean of Weibull distributions.

Keywords: Acceleration test, Arrheinus model, Mean time to failure , Anderson-Darling test, Weibull distribution.

1 Introduction

The intensity of the global competition for the development of new products in a short time has motivated the development of new methods. Testing under normal operating conditions requires a very long time. This has led to the development of accelerated life testing (ALT), where units are subjected to a more severe environment (increased or decreased stress levels) than the normal operating environment so that failures can be induced in a short period of test time. Information obtained under accelerated conditions is then used in estimate the characteristics of life distributions at normal operating conditions.

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2 Design of Accelerated Life Testing Plans

A detailed test plan is usually designed before conducting an accelerated life test. The plan requires determination of the type of stress, methods of applying stress, stress levels, the number of units to be tested at each stress level, and an applicable accelerated life testing model that relates the failure times at accelerated conditions to those at normal conditions. Stress in ALT can be applied in various ways as decrease, increase, constant or Synthetic. In [2] provide extensive tables and practical guidelines for planning an ALT. In [1] introduce theoritcal discssion on ALT. Three department of sumsung company result of them research in case one pump in washing machine represent in [3].
Assumed that the components are tested at different accelerated stress levels s1, s2, . . . , sn. The failure times at each stress level are then used to determine the most appropriate failure time probability distribution, along with its parameters. Under the parametric statistics-based model assumptions, the failure times at different stress levels are linearly related to each other. Thus

$$t_o = C_A t_s$$

where t_o is the failure time under operating conditions, t_s is the failure time under stress conditions, and C_A is the acceleration factor.

3 Acceleration Model for the Weibull Model

The relationships between the failure time distributions at the accelerated and normal conditions base on weibull distribution can be derived the following:

$$R_s(t) = e^{-(t/\beta_s)^{\alpha_s}} \qquad t \geq 0, \alpha_s \geq 0, \beta > 0$$

where α_s is the shape parameter of the Weibull distribution under stress conditions and β_s is the scale parameter under stress conditions. The CDF under normal operating conditions is

$$R_o(t) = R_s(\frac{t}{C_A}) = e^{-(t/C_A\beta_s)^{\alpha_s}} = e^{-(t/\beta_o)^{\alpha_o}}$$

The underlying linearity assumption $\alpha_s = \alpha_o$, and $\beta_o = C_A\beta_s$. If the shape parameters at different stress levels are significantly different, then either the assumption of true linear acceleration is invalid or the Weibull distribution is inappropriate to use for analysis of such data. Let $\alpha_s = \alpha_o = \alpha \geq 1$. Then the probability density function under normal operating conditions is

$$f_o(t) = (\frac{1}{C_A})f_s(\frac{t}{C_A}) = \frac{\alpha}{C_A\beta_s}(\frac{t}{C_A\beta_s})^{\alpha-1}e^{-(t/C_A\beta_s)^{\alpha}}, \qquad t \geq 0, \beta_s \geq 0$$

The MTTF under normal operating conditions is

$$MTTF_o = \beta_o\Gamma(1 + \frac{1}{\alpha})$$

The failure rate under normal operating conditions is

$$h_o(t) = (\frac{1}{C_A})h_s(\frac{t}{C_A}) = \frac{\alpha}{C_A\beta_s}(\frac{t}{C_A\beta_s})^{\alpha-1} = \frac{h_s(t)}{C_A^{\alpha}}$$

4 Case study

A manufacturer of Bourdon tubes (used as a part of pressure sensors in avionics) wishes to determine its MTTF. The manufacturer defines the failure as a leak in the tube. The tubes are manufactured from 18 Ni (250) maraging steel and operate with dry 99.9fluid as the internal working agent. Tubes fail as a result of hydrogen embrittlement arising from the pitting corrosion attack. Because of the criticality of these tubes, the manufacturer decides to conduct ALT by subjecting them to different levels of pressures and determining the time for a leak to occur. The units are continuously examined using an ultrasound method for detecting leaks, indicating failure of the tube. Units are subjected to three stress levels of gas pressures and the times for tubes to show leak are recorded. Determine the mean lives and plot the reliability functions for design pressures of 80 and 90 psi.
Solution. The result of fit the failure times to Weibull distributions shows in table 1. Base on Anderson-Darling test at level 0f 0.05 error, Weibull distribution is good for fit failure time at every 3 level of stress pressure. P-Value at every 3 level in this test are greater of 0.25.

Table 1: Parameters estimation of weibull distribution in different pressure

parameter	100 psi	120 psi	140 psi
α	2.87	2.67	2.52
β	10392	5375	943

Table 2: Mean of time failure in different pressure

	100 psi	120 psi	140 psi
mean	9236	4777	838

Since $\alpha_1 = \alpha_2 = \alpha_3 \cong 2.65$, then the Weibull model is appropriate to describe the relationship between failure times under accelerated conditions and normal operating conditions. We determine the mean time of the population fails as

$$t = \beta\Gamma(1 + \frac{1}{\alpha})$$

The mean of life time at every 3 level of stress are shown in table 2.
The relationship between the failure time t and the applied pressure P can be assumed to be similar to the Arrhenius model; thus

$$t = ke^{c/P}$$

where k and c are constants. By making a logarithmic transformation, the above expression can be written as

$$ln(t) = ln(k) + \frac{c}{P}$$

Using a linear regression model, we obtain $k = 3.391$ and $c = 811.400$.The estimated mean at 80 psi and 90 psi are 86131 h and 27980 h respectively. The corresponding acceleration factors are 9.33 and 3.03. The failure rates under normal operating conditions are

$$h_{80}(t) = \frac{2.65}{1.63847 \times 10^{13}} t^{1.65}, \qquad h_{90}(t) = \frac{2.65}{8.31909 \times 10^{11}} t^{1.65}$$

The MTTFs for 80 and 90 psi are calculated as

$$MTTF_{80} = \beta\Gamma(1 + \frac{1}{\alpha}) = (1.63847 \times 10^{13})^{1/2.65}\Gamma(1 + \frac{1}{2.65}) = 85807h$$

and

$$MTTF_{90} = 31488 \times 0.885 = 27867h$$

References

[1] Hoang, P. (2003), *Handbook of reliability engineering*, Springer.

[2] Meeker, W. and Hahn, G. (1985), How to plan an accelerated life test-some practical guidelines. *Statistical techniques*, **10**.

[3] Sang-Jun, P., Sang-Deuk, P., Kwang-Suck, K. and Ji-Hyun, C. (2006), Reliability evaluation for the pump assembly using an accelerated test, *International Journal of Pressure Vessels and Piping*, **83**, 283–286.