Autonomous Parallel Parking of a Vehicle in a Limited Space Using a RBF Network and a Feedback Linearization Controller

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***Abstract*—In this paper, the issue of autonomous parallel parking of a vehicle in a limited space has been considered based on a parked vehicle in front. A Radial Basis Function Network (RBFN) has been developed to online path generation according to the measured distance from the parked vehicle. These measurements are obtained by employing two sonar sensors mounted in the front left corner of the vehicle. Fifth order polynomial reference paths for three different initial positions have been used to generate training data for the network. In order to use feedback linearization controller for tracking the desired parking path, two timing laws have been proposed for forward and backward maneuvers and the control laws have been designed in timing laws-domain. These timing laws have been considered such that zero velocity is obtained for the vehicle at the initial and goal points of the parking maneuver, as well as, at the point that direction of motion must change, without occurrence singularity in the control laws. Furthermore, due to change of the desired path and direction of motion based on the vehicle's configuration, the vehicle can be parked in limited area. Simulation results show the effectiveness of the proposed approach to use the nonlinear controller in order to intelligently perform the parallel parking in a limited space without knowing the parking space width and only based on distance measurements from the parked vehicle.**

Keywords-Kinematic model; Nonholonomic constraints; Parallel parking; Feedback linearization controller; Radial basis function network.

I. INTRODUCTION

In the last decade, autonomous parking problems of vehicle have attracted much attention from research organization and automobile industries. Parking problems can be classified into garage parking problem and parallel parking problem that are highly limited motion control problems, because of nonholonomic constraints of vehicle and complexity of the task to realize. Nonholonomic constraints are nonintegrable equations involving the derivative of the configuration parameters, which make that admissible path in the configuration space can be nonfeasible trajectories for the vehicle [1].

Various control strategies were proposed for parallel parking problem. For instance, in [6], an automated parking system was proposed that works in three phases, scanning, positioning, and maneuvering. In [7], parking problem was solved by using a time-varying state feedback control based on Lyapunov direct method. Most of these controllers were designed based on tracking and posture stabilization method [2].In general, tracking method is often easier than posture stabilization method for a nonholonomic vehicle [3].The posture stabilization method is to stabilize the vehicle to a desired final posture from any initial posture. The tracking method is done in two steps: first a reference path is planned and then a controller is designed to tracking the path. The reference path planner must satisfy the nonholonomic constraints and environmental model. Many researchers have been worked on automatic parking path planning problems and proposed types of path curves such as circular arcs with straight lines, clothoid curves, arctangent curves, and cubic polynomial curves [4]. Path tracking controller design for a vehicle is to find control laws such that it follows a reference path. For instance, a feedback linearization tracking controller for car like mobile robot was designed in [5], however, because of happening singularity in the control laws in zero velocity, the case that the robot should stop and reverse the direction of motion is not allowed with this control scheme.

Most of the tracking methods that use the nonlinear control laws as the tracking controller, suffer from a serious drawback. Because of offline generation of the parking path, they need parking space dimensions to assign the goal point to create the path, so they fail when the parking space dimensions cannot be identified. In order to overcome this inconvenience, a neuro-fuzzy sensor-based controller has been developed for parallel parking problem in [9] that controls the steering angle of the robot based on sonar measurements for parking the robot in unlimited parking space, but, driving velocity has been considered constant during the maneuver.

In this paper, the problem of parallel parking of a vehicle in limited space has been considered such that forward and backward maneuvers are required to accomplish Parking task. In the proposed method, a RBF network has been used to reference path generation of the parking maneuver based on distance measurements from the parked vehicle in front, also, in order to use feedback linearization controller for tracking the desired path, two timing laws for forward and backward maneuvers have been proposed and control laws have been designed in timing laws-domain. These timing laws have been considered such that zero velocity is obtained for the vehicle at the initial and goal points of the parking maneuver, as well as, at the point that direction of motion must change, without happening singularity in the control laws. Furthermore, due to change of the desired path and direction of motion according to the vehicle's configuration, the vehicle can be parked in limited area. The simulation results exhibit the effectiveness of the proposed approach to use the nonlinear controller in order to enable the autonomous vehicle to intelligently perform parallel parking task in unknown parking space.

II. KINEMATIC MODEL OF A VEHICLE

The motion of vehicle in parking maneuver is generally slow. For small velocities, dynamics is not so important, but kinematic of the system mainly governs the system's behavior [12]. Therefore in this paper, kinematic model has only been considered as basic equations of motion.

Consider a kinematic model of the vehicle shown in Figure 1. The front wheels can turn to left or right. The rear wheels are fixed parallel to car body and allowed to roll or spin but not slip. The no-slipping condition causes the nonholonomic constraint of the vehicle. The vehicle has been described by the kinematic model as

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v\tan\varphi}{l} \\ \dot{\varphi} &= w \end{aligned} \qquad (1)$$

Where (x, y) defines the position of the rear axle's midpoint, θ is the car's orientation relative to the x-axis, φ represents the steering angle with respect to the vehicle body, l is the distance of rear and front axles, $v = \pm\sqrt{\dot{x}^2 + \dot{y}^2}$ is the driving velocity that its sign depends on the direction of motion and w is the steering velocity. In practical case, the steering angle and angular velocity are restricted by

$$|\varphi| \leq \varphi_{max}, 0 < \varphi_{max} < \frac{\pi}{2} \quad ; \quad |w| \leq w_{max} \qquad (2)$$

III. REFERENCE FIFTH-ORDER POLYNOMIAL PATH

Due to the restricted range of the steering angle, the curvature of the vehicle's path is limited by $|k| \leq k_{max} = \frac{\tan \varphi_{max}}{l}$. Furthermore, derivative of the curvature k depends on the steering velocity w and is also restricted by $|\sigma| \leq \sigma_{max}$, $\sigma = \dot{k}$.These constraints must be considered in path planning for parking maneuver.

Among polynomial curves it has been shown that a fifth-order polynomial is the least polynomial behaving the parallel parking [9]. The general form of a fifth-order polynomial is given by:

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 \qquad (3)$$

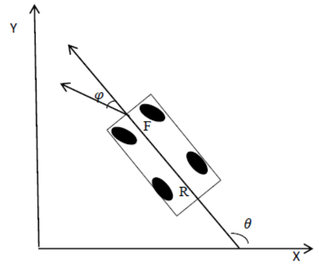

Figure 1. Kinematic model

In order to use the polynomial as the reference path for the parallel parking, the six coefficients in (3) are found to satisfy the position, zero slope and zero curvature constraints at the start and end points of the path. Also, there should exist a minimum horizontal distance between the two end points, so that the resulting curvature does not violate the peak value [4].

IV. PATH PLANNING FOR PARKING MANEUVER USING RBF NETWORK

In this section, path generation for parallel parking is explained in detail. The idea of the proposed scheme is to use RBF network for constructing the fifth-order polynomial path to perform parallel parking based on distance measurements from the parked vehicle in front and without identifying the parking space.

A. *Generation of training data for RBF network*

AS shown in Figure 2, two ultrasonic sensors (S_1, S_2) have been placed in the front left corner of the vehicle. This placement works when the car is parked on the left side of the road. Because the path is generated based on sonar measurements, the number and the orientation of the sensors are important and are done in such a way as to get continuous readings from the parked vehicle in front during the maneuver without having any blind spots. The orientations of the sensors are 151◦, 211◦ for S_1, S_2, respectively and each sensor has a beam width of ±30 degrees. This placement is able to not only obtain the information necessary for car navigation but also eliminate the need for tackle the shortcomings of radial imprecision and angular uncertainty of the ultrasonic sensors [11].

A RBF network has been used to create the desired path for parking maneuver based on measurements of the sensors. This network is an artificial neural network (ANN) that has been recently acknowledged as one of the most efficient

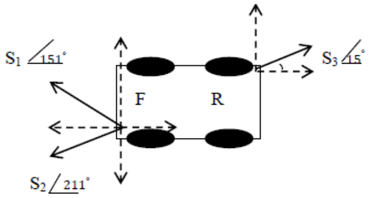

Figure 2. Sensors directions

and promising architecture for the approximation of continuous mappings [10]. Three paths corresponding to three different y-initial positions have been used for training and a path has been used for model validation. These paths are shown in Figure 3. Each path was discretized into 486 sampling points and the distance measurements from the parked vehicle (d_1, d_2) were recorded at each point. Furthermore, two variables (d_3, d_4) based on the increasing or decreasing nature of the sensor measurements were created at each point. The distance measurements obtained by S_1, S_2 during the maneuver corresponding to y-initial position of 310 cm are shown in Figure 4. The reason for creating d_3, d_4 is that our network should be able to provide us the proper point of the path, when we have the same distance measurements by a sensor at different points of the path. For example by considering S_2 readings shown in Figure 4, we can see that at some sampling points we have the same distance measurements but the coordinates corresponding to these points of the path are different. As it can be seen in Figure 4, when the sensor is not able to measure the distance from the parked vehicle, it will show a large value of 5m. The inputs to the network at each point along the path are x-coordinate of the point as well as the information acquired by the sensors at previous point (d_1-d_4) and the output of the network is y- coordinate of the point. After the 5-input 1-output training data set was prepared, the network was trained by using the following parameters:

Mean squared error goal=0
Spread of radial basis functions=3
Maximum number of neurons (MN) =50
Number of neurons to add between displays (DF) =1

The idea is that for the initial position that has not been used in training, the RBFN should be able to interpolate between the three paths used for training and create the appropriate path, by processing the sonar data without knowing the width of the parking space.

V. CONTROLLER DESIGN

The tracking controller used in this article is modified model of the designed controller in [5]. In order to tracking a path starting and ending with zero velocity and also perform forward and backward maneuver by using the controller, geometric has been separated from timing information in the control laws.

Consider a path that the horizontal distance between its start and end points is L. To track the path with zero velocity at the start and end points, the following steps has been considered:

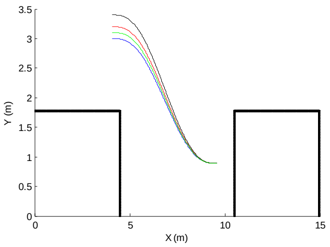

Figure 3. Paths used for training and checking

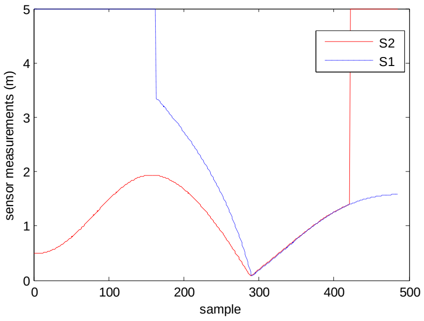

Figure 4. Sensor measurements along the fifth order polynomial path

1) Two timing laws have been proposed for backward and forward maneuvers, respectively as follows:

$$p(t) = \left(\frac{L}{2} - \frac{L}{2} cos\left(\frac{\pi t}{T}\right)\right) + x_0 \qquad t \in [0\ T] \qquad (4)$$

$$p(t) = -\left[\left(\frac{L}{2} + \frac{L}{2} cos\left(\frac{\pi t}{T}\right)\right) + (x_0 - L)\right] \qquad t \in [0\ T] \qquad (5)$$

Where x_0 is the x-coordinate of the start point of the path and T is required time to traverse the path. Attribution of these laws to forward or backward maneuver depends on the range of the state variable θ (in this paper$\pi \le \theta < \frac{\pi}{2}$). These timing laws are increasing function in the specified interval and satisfy the following conditions:

$$p(0) = 0 \qquad p(T) = L \qquad \dot{p}(0) = \dot{p}(T) = 0 \qquad (6)$$

2) The kinematic model (1) has been obtained in p-domain:

$$\begin{aligned} \acute{x} &= u_1 cos\theta \\ \acute{y} &= u_1 sin\theta \\ \acute{\theta} &= \frac{u_1 tan\varphi}{l} \\ \acute{\varphi} &= u_2 \end{aligned} \qquad (7)$$

and the notational abbreviation $(\acute{.}) = d(.)/dp$ is used. Also, the relation between the actual velocity commands *v*, *w* and the new inputs u_1, u_2 are:

$$\begin{aligned} v &= u_1(p(t))\dot{p}(t) \\ w &= u_2(p(t))\dot{p}(t) \end{aligned} \qquad (8)$$

3) Feedback linearization controller has been designed for model (7) in p-domain:

$$r_i = \frac{d^3 z_{di}}{dp^3} + k_{ai}\left(\frac{d^2 z_{di}}{dp^2} - \frac{d^2 z_i}{dp^2}\right) + k_{vi}\left(\frac{dz_{di}}{dp} - \frac{dz_i}{dp}\right) + k_{pi}(z_{di} - z_i), \qquad i = 1,2 \qquad (9)$$

where $z = \binom{z_1}{z_2} = \binom{x}{y}$

$$u_{22} = (r_2 - r_1 tan\theta - 3\xi_1\xi_2 tan\varphi / lcos^3\theta)/{\xi_1}^2$$
$$u_{11} = \xi_1$$
$$u_1 = u_{11}/\cos\theta$$
$$u_2 = \frac{-3u_{11} sin\theta sin^2\varphi}{l} + lu_{22} cos^3\theta cos^2\varphi \tag{10}$$
$$\acute{\xi}_1 = \xi_2$$
$$\acute{\xi}_2 = r_1$$

4) The actual velocity commands *v*, *w* to tracking the desired path has been obtained by eqs(8).

VI. PARKING PROBLEM

The problem addressed in this paper is the parallel parking of a vehicle in limited area such that forward and backward maneuvers are required to accomplish Parking task. The reference path (path of the rear axle midpoint) for the parking problem is shown in Figure 5. As can be seen, at starting point (point A), the vehicle must stand parallel to vehicle **I** such that their rear bumpers are collinear so the length of the parked vehicle dose not effect on the parking algorithm. It has been supposed that the vehicle is equipped with incremental encoders for reconstruction of the current vehicle configuration. In the first step of the maneuver, the vehicle moves straight backward until reaches the configurations of $[\, x_0{+}50cm \;\; y_0 \;\; \theta_0 \;\; \varphi_0 \,]^t$(point B) that $[\, x_0 \; y0 \; \theta_0 \;\; \varphi_0]^t$ is the configuration at the starting point , afterwards, it moves backward on the fifth order polynomial path which is obtained based on measurements of the sensors by using RBFN. In order to improve the performance of the controller, a moving average filter is used in the output of the RBFN. In these two parts, *p* has been considered as eq.(4) with L_1=6.04m, T_1=18s and x_d have been considered as $x_d(t)=p(t)$. When the vehicle reaches the configurations of $[x_1 \;\; y_1 \;\; \theta_1 \;\; \varphi_1]^t$ with $x_1 > 8.06m$, $0.83\text{m} \le y_1 \le 0.92m$, $178^\circ \le \theta_1 \le 182^\circ$or measured distance from vehicle **II** by using the back sensor (that is shown in Figure 2) is less than 10cm (point C), it stops and moves straight forward and finally stops at the distance of 50 cm from vehicle **I** (point D).

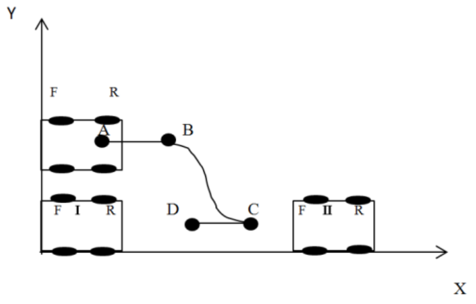

Figure 5. Reference path for parking maneuver

In this part, *p* has been considered as the eq.(5) with L_2=(front sensor's reading at point C) – 50cm, T_2=3s and x_d has been considered as $x_d(t) = -p(t)$.

It is obvious that the vehicle's velocity at the three points A, C, D of the path must be zero. According to eq. (8), we have: $v = 0 \leftrightarrow (u_1 = 0 \;\; or \;\; \dot{p} = 0)$. If $u_1 = 0$ then based on eqs (10) ξ_1 is zero and in this case, the first control input (i.e., u_{22} in eqs. (10)) could diverge so u_1 should never be zero. By choosing x-coordinate of the desired path as mentioned before, the nominal control input $u_{1d} = \pm\sqrt{(\frac{dx_d}{dP})^2 + (\frac{dy_d}{dP})^2}$ does not decay to zero. Because the gains of the controller are selected such that exponential convergence to the desired trajectory is guaranteed, the command $u_1$will be bounded away from zero. However, in order to avoid the singularity in the control laws, the state ζ_1 is reset whenever its value falls below a given threshold. In order to have zero velocity at the three points, *p* has been selected such that it has zero derivatives at these points. Therefore, by performing the time/space separation and choosing proper timing laws, the zero-velocity at the initial and goal points of the parking maneuver, as well as, at the point that direction of motion must change, is obtained without happening singularity. Furthermore, due to change of the desired path and direction of motion based on the vehicle's configuration, the vehicle can be parked in limited area.

The nonlinear dynamic controller (8-10) has been used to track the desired path. For the stabilizing part of the controller, the same gains have been chosen for both input-output channels as $k_{ai} = 12$, $k_{vi} = 48$, $k_{pi} = 64$, $i = 1; 2$. Figure 6 shows the schematic diagram of the vehicle control system.

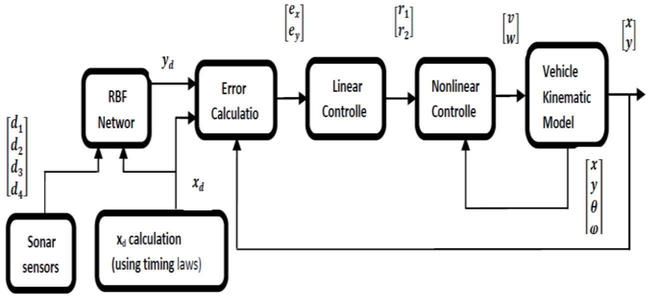

Figure 6. Schematic diagram of the vehicle control system

TABLE I. PARAMETERS USED IN THE SIMULATIONS

length	4.46m
width	1.78m
wheelbase	2.65m
Maximum steering angle	47°
Maximum steering velocity	0.7rad/s
length of the parking space	6m

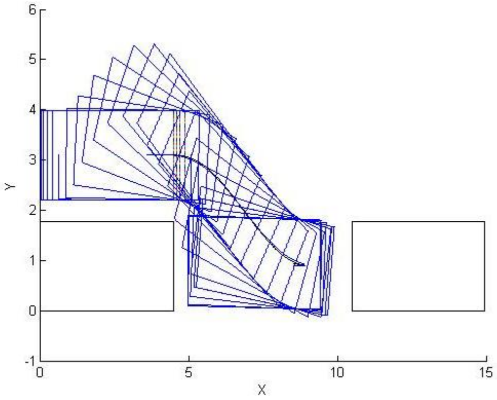

Figure 7. Simulation result for y-initial position=3.1m

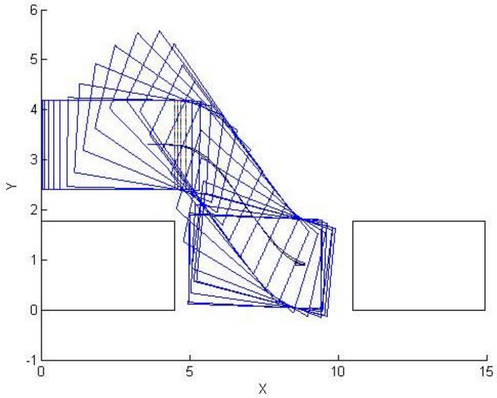

Figure 8. Simulation result for y-initial position=3.3m

VII. SIMULATION RESULTS

Parameters of a typical vehicle and the length of the parking space used in this simulation are shown in Table I. In order to exhibit the feasibility of the proposed method, two different start positions corresponding to y-initial position=310 cm, 330 cm have been used for simulation. The results are shown in Figures 7-8. The reference path generation error for y-initial position=310cm, 330cm are 0.28cm, 0.72cm, respectively, and the maximum vertical distance error between the actual path and the reference path are 5.26cm, 5.3cm, respectively. This shows a very good performance and illustrates that the proposed method enables the vehicle to intelligently perform parallel parking task solely based on distance measurements from the parked vehicle and using feedback linearization controller.

Moreover, because of change of the desired path and the direction of motion according to the vehicle's configuration, the vehicle is able to perform parking task between two parked vehicles.

VIII. CONCLUSION

In this paper, a solution for autonomous parallel parking of a vehicle in limited space based on tracking method and under the guidance of sensor data is proposed. This approach uses the features of RBF neural network and feedback linearization tracking controller to design a system to accomplish Parking task without identifying the parking space. The RBF network has been developed to path generation for tracking controller based on distance measurements from parked vehicle. These measurements are obtained by using two sonar sensors mounted in the front left corner of the vehicle. Fifth order polynomial reference paths for three different initial positions have been used to generate training data for the network. Thus, by using the RBF network, there is no need to any offline path planner. Two timing laws for forward and backward maneuvers have been proposed and feedback linearization tracking controller has been designed in timing laws-domain. Therefore, by performing the time/space separation and choosing the proper timing laws, zero-velocity at the initial and goal points of the parking maneuver, as well as, at the point that direction of motion must change, is obtained without happening singularity in the control laws. The simulation results show the effectiveness of the proposed approach to use the nonlinear controller in order to intelligently perform the parallel parking in limited unknown parking space. Furthermore, due to the generation of the driving and steering velocities by using the nonlinear controller, a fully automatic parking system has been obtained.

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