Decoupled phase voltages control of three phase four-leg voltage source inverter via state feedback

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***Abstract*— the four-leg voltage source inverter (VSI) is becoming increasingly popular in power applications due to its superior performance characteristics such as relatively low dc-bus voltage and capability to handle unbalanced or nonlinear load currents. With unbalanced load condition, to balance such a system, a neutral connection is needed to control each phase voltage independently. Average large signal model in ABC coordinates is obtained and state space equation is presented. This paper presents new decoupled control strategy based on state feedback with simple proportional controller thus each phase is controlled independently in order to track references properly. Simulation results with MATLAB/Simulink show acceptable performance in unbalanced load conditions.**

Keywords — four leg voltage source inverter, decoupled control, large signal model, unbalanced load.

I. INTRODUCTION

Four-leg voltage source inverter (VSI) is becoming increasingly common in power electronic applications due to its superior characteristics such as relatively low dc-bus voltage and capability to handle unbalanced loads which are becoming nonlinear [1-5]. Such applications of four-leg voltage source inverters are mentioned in literature including active power filter, unified power supply, power electronic converter in distributed generation, etc. [6-7].

In this topology neutral connection is provided by fourth-leg; in fact the four-leg VSI is combination of three full-bridge inverters with shared neutral leg [6]. Fig. 1 shows generalized four-leg VSI topology with output filter.

To control converter the main purpose is to maintain output voltages in sinusoidal waveform with generic unbalanced / nonlinear loads that is needed to be controlled each phase voltage independently. Modeling of the four-leg VSI has been discussed in the literatures [1-4]. Based on obtained model, there is unlike coupling between channels causes multi-input multi-output system that is difficult to control. Most papers neglect coupling and propose classical control strategies, such as PI controller in qd0 stationary reference frame [3] to control the system, but has some drawbacks such as it can't lead to a precise response in time domain, etc. in the other hand, much papers consider load as impedance that is not generic [2-3].

This paper solves all the problems as bellow:

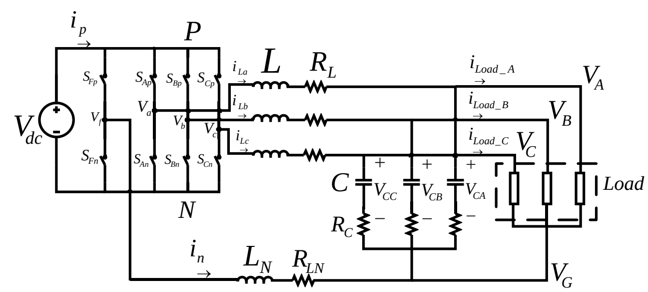


Figure 1. Generalized four leg voltage source inverter topology

1. It proposed particular pole placement based on system matrix that decouples system in rotatory coordinate then controls each phase independently by simple proportional controller to make the transient performance of the system greatly adjustable.
2. It models load currents as disturbances and then eliminates its effects on output voltages to have generalized control with any kind of loads (unbalanced or nonlinear).
3. It demonstrates that proposed controller is simple to design and implement.

After this introduction, the average large signal model of the inverter is presented and state space model of system is realized in section II. Section III describes proposed input-output decouple control via state feedback. The assessment the effects of load current disturbances is mentioned in section IV. To test the performance of the proposed control strategy, simulation is performed using MATLAB/ SIMULINK and is presented in section V that shows acceptable performance in certain unbalanced load conditions.

II. REALIZED STATE SPACE MODEL IN ABC COORDINATES

Generalized topology of three phase four leg voltage source with LC output filter is shown in Fig. 1. The average large signal model in ABC coordinates is obtained by averaging parameters.(Fig 2.).

From Fig.2 the output voltages and currents are represented as following:

$$\begin{bmatrix} V_{af} & V_{bf} & V_{cf} \end{bmatrix}^T = \begin{bmatrix} d_{af} & d_{bf} & d_{cf} \end{bmatrix}^T \times V_{PN} \tag{1}$$

$$I_p = \begin{bmatrix} d_{af} & d_{bf} & d_{cf} \end{bmatrix} \times \begin{bmatrix} I_a & I_b & I_c \end{bmatrix}^T \quad (2)$$

Where V_{if} *(i=a,b,c)* are line-to-neutral output voltages, I_i *(i=a,b,c)* are inductance currents, I_p is source current, V_{pn} is input DC voltage and d_{if} *(i=a,b,c)* are line-to-neutral duty ratios.

The duty ratios d_{if} *(i=a,b,c)* are controlled signals. The output voltages of four-leg inverter should be sinusoidal irrespective load current however input control signals are in the way so as produce output voltages as required:

$$\begin{bmatrix} V_{ref\_A} \\ V_{ref\_B} \\ V_{ref\_C} \end{bmatrix} = V_m \begin{bmatrix} \cos(\omega t) \\ \cos(\omega t - 120) \\ \cos(\omega t + 120) \end{bmatrix} \quad (3)$$

Where V_{ref_i} *(i=a,b,c)* are the inverter output reference voltages (line-to-neutral).

From Fig. 2 the output LC filter and loads are modeled by following state equations by driving Kirchhoff's laws, where the load currents can be considered as disturbances; that is,

$$\dot{X} = AX + BU + EW \quad (4)$$

Where X is the state variable vector and is define:

$$X = \begin{bmatrix} i_{LA} & V_{CA} & i_{LB} & V_{CB} & i_{LC} & V_{CC} \end{bmatrix}^T \quad (5)$$

And U is control input vector as mentioned in (1), also W is disturbance as load currents ([I_{load-A} I_{load-B} I_{load-C}]). Other matrixes in (4) can be represented by:

$$A = \begin{bmatrix} -2R/L & -4/5L & 0 & 1/5L & 0 & 1/5L \\ 1/C & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/5L & -2R/L & -4/5L & 0 & 1/5L \\ 0 & 0 & 1/C & 0 & 0 & 0 \\ 0 & 1/5L & 0 & 1/5L & -2R/L & -4/5L \\ 0 & 0 & 0 & 0 & 1/C & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4/5L & -1/5L & -1/5L \\ 0 & 0 & 0 \\ -1/5L & 4/5L & -1/5L \\ 0 & 0 & 0 \\ -1/5L & -1/5L & 4/5L \\ 0 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 4R/5L & -R/5L & -R/5L \\ -1/C & 0 & 0 \\ -R/5L & 4R/5L & -R/5L \\ 0 & -1/C & 0 \\ -R/5L & -R/5L & 4R/5L \\ 0 & 0 & -1/C \end{bmatrix}$$

Where L_N=L/2 and parasitic component resistors are considered equal; R_L=R_C=R_{LN}=R (Fig. 2).

Output of system is line-to-neutral voltages thus, output equation can be express as:

$$Y = CX + DU + FW \quad (6)$$

Where Y=[V_{AG} V_{BG} V_{CG}]^T and

$$C = \begin{bmatrix} R & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & R & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & R & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad F = \begin{bmatrix} -R & 0 & 0 \\ 0 & -R & 0 \\ 0 & 0 & -R \end{bmatrix}$$

III. Proposed Input-Output Decoupled Control via State Feedback

System with (5) and (6) equations is multi-input multi-output (MIMO) with three different disturbances; let the load currents considered to be zero initially. Input-output transfer function TF can be express as:

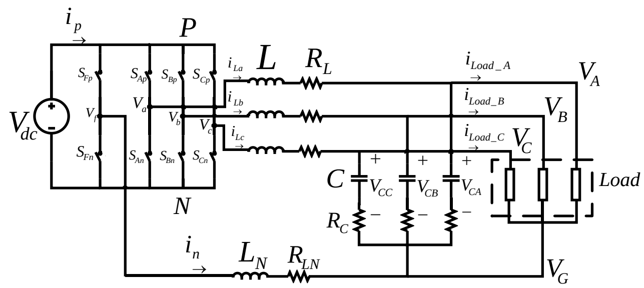


Figure 2. The average large signal model of three-phase four-leg VSI in abc coordinates

$$TF = \frac{Y}{U} = C(sI - A)^{-1}B + D \quad (7)$$

Where the C, A, B and D are system matrixes from (4) and (6). TF here is a matrix with 3×3 dimensions, that is:

$$TF = \frac{1}{D(s)} \begin{bmatrix} N_{1-1}(s) & N_{1-2}(s) & N_{1-3}(s) \\ N_{2-1}(s) & N_{2-2}(s) & N_{2-3}(s) \\ N_{3-1}(s) & N_{3-2}(s) & N_{3-3}(s) \end{bmatrix} \quad (8)$$

Where

$$D(s) = (\mathrm{CLs}^2 + 2\mathrm{CRs} + 1)(5\mathrm{CLs}^2 + 10\mathrm{CRs} + 2)^2$$

$$N_{i-j}(s) = 2(\mathrm{CRs} + 1)(2\mathrm{CLs}^2 + 4\mathrm{CRs} + 1)(5\mathrm{CLs}^2 + 10\mathrm{CRs} + 2), \mathrm{i} = \mathrm{j}$$

$$N_{i-j}(s) = -\mathrm{Cs}(\mathrm{Ls} + 2\mathrm{R})(\mathrm{CRs} + 1)(5\mathrm{CLs}^2 + 10\mathrm{CRs} + 2), \mathrm{i} \neq \mathrm{j}$$

From (8), TF is symmetrical but not diagonal, that causes coupling between phases. Required decoupled transfer function matrix can be considered as:

$$TF^* = \begin{bmatrix} h_{1-1}(s) & 0 & 0 \\ 0 & h_{2-2}(s) & 0 \\ 0 & 0 & h_{3-3}(s) \end{bmatrix} \quad (9)$$

Where $h_{i-i}(s)$ (i=1,2,3) is required transfer function such that the response of closed-loop system has desired characteristics, tracking required outputs as mentioned in (3).

A. State Feedback Controller

Consider linear system (4) and (6) without disturbances. Let define the controller with the linear state feedback form:

$$U = -KX + Gr \quad (10)$$

Where in this case:

$$r = \begin{bmatrix} V_{ref\_A} \\ V_{ref\_B} \\ V_{ref\_C} \end{bmatrix} \quad K = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \end{bmatrix}$$

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix}$$

Substituting (10), (4) and (6) yields closed-loop system equation

$$\begin{cases} \dot{X} = (A - BK)X + BGr \\ Y = CX \end{cases} \quad (11)$$

The control problem is to determine the control law (10) such that the closed-loop transfer function (9) to be obtained. Falb and Wolovich [8] first proved the theorem for input-output decoupling via state feedback. General theorem was presented In [8]. Here we use the theorem in our case, system explained in section 2.

B. Decoupling Via State Feedback

System can be decoupled via state feedback, i.e. there is pair of K and G (10), which decouples the system if and only if following matrix is regular:

$$B^* = \begin{bmatrix} C_1 A^{d_1} B \\ \cdots \\ C_2 A^{d_2} B \\ \cdots \\ C_3 A^{d_3} B \end{bmatrix} \quad (12)$$

Where C_i (i=1,2,3) is i-th row of matrix C in (6) and d_i (i=1,2,3) is integer, which in our case is defined as:

$$d_i = \begin{cases} \min j : C_i A^j B \neq 0 & j = 0,1,2,\ldots,5 \\ 5 & if : C_i A^j B = 0 \quad for \quad all \quad j \end{cases}$$

To decouple system, a pair of matrixes K and G is motioned as

$$K = \left(B^*\right)^{-1} A^* \qquad G = \left(B^*\right)^{-1} \quad (13)$$

Where

$$A^* = \begin{bmatrix} C_1 A^{d_1+1} \\ \cdots \\ C_2 A^{d_2+1} \\ \cdots \\ C_3 A^{d_3+1} \end{bmatrix}$$

That is

$$B^* = \begin{bmatrix} \frac{4R}{5L} & \frac{-R}{5L} & \frac{-R}{5L} \\ \frac{-R}{5L} & \frac{4R}{5L} & \frac{-R}{5L} \\ \frac{-R}{5L} & \frac{-R}{5L} & \frac{4R}{5L} \end{bmatrix}$$

$$A^* = \begin{bmatrix} \frac{1}{C} - \frac{2R^2}{L} & \frac{-4R}{5L} & 0 & \frac{R}{5L} & 0 & \frac{R}{5L} \\ 0 & \frac{R}{5L} & \frac{1}{C} - \frac{2R^2}{L} & \frac{-4R}{5L} & 0 & \frac{R}{5L} \\ 0 & \frac{R}{5L} & 0 & \frac{R}{5L} & \frac{1}{C} - \frac{2R^2}{L} & \frac{-4R}{5L} \end{bmatrix}$$

$$K = \begin{bmatrix} \frac{3L}{2CR} - 3R & -1 & \frac{L}{2CR} - R & 0 & \frac{L}{2CR} - R & 0 \\ \frac{L}{2CR} - R & 0 & \frac{3L}{2CR} - 3R & -1 & \frac{L}{2CR} - R & 0 \\ \frac{L}{2CR} - R & 0 & \frac{L}{2CR} - R & 0 & \frac{3L}{2CR} - 3R & -1 \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{3L}{2CR} & \frac{L}{2CR} & \frac{L}{2CR} \\ \frac{L}{2CR} & \frac{3L}{2CR} & \frac{L}{2CR} \\ \frac{L}{2CR} & \frac{L}{2CR} & \frac{3L}{2CR} \end{bmatrix}$$

Closed-loop transfer function of system (11) can be express:

$$TF^* = C(sI - A + BK)^{-1} BG = \begin{bmatrix} \frac{1}{s} & 0 & 0 \\ 0 & \frac{1}{s} & 0 \\ 0 & 0 & \frac{1}{s} \end{bmatrix} \quad (14)$$

From (14), closed-loop system is decoupled, i.e. each output is affected by only one input. Fig. 3(b) shows the realization of system. System (14) can be decomposed to three subsystems to complete the control law, (Fig. 3(c)) that is:

$$Y_1 = \frac{1}{s} r_1, Y_2 = \frac{1}{s} r_2, Y_3 = \frac{1}{s} r_3 \quad (15)$$

Systems (15) are first order and easy to control; to obtain overall required characteristics for tracking references in each phase, proportional controller is employed, (Fig. 3(d))

Overall input-output transfer function at each phase from Fig. 3(d) can be express as:

$$\frac{V_{AG}}{V_{ref\_A}} = \frac{K_{p\_A}}{s + K_{p\_A}} \qquad \frac{V_{BG}}{V_{ref\_B}} = \frac{K_{p\_B}}{s + K_{p\_B}} \qquad \frac{V_{CG}}{V_{ref\_C}} = \frac{K_{p\_C}}{s + K_{p\_C}} \quad (16)$$

Where K_{p-i}(i=A,B,C) is proportional controller that can be considered to yield required response of system.

IV. Assessment The Effects Of Load Current Disturbances

As mentioned before we supposed the input disturbances (load currents) to be zero, in the other words, closed-loop transfer function (16) is obtained in no-load condition. Let rewrite the overall system-output as:

$$Y = \left.\frac{Y}{r_{ref}}\right|_{W=0} \times r_{ref} + \left.\frac{Y}{W}\right|_{r_{ref}=0} \times W \quad (17)$$

Where, the first term is (16). To assessment the effects of load currents, it is sufficient to obtain second term of (17). In our case, it is required that the output-to-disturbance transfer function to be zero or near to zero.

Substituting (10), (4) and (6) yields closed-loop system equations with disturbance:

$$\begin{cases} \dot{X} = (A - BK)X + BGr + EW \\ Y = CX + FW \end{cases} \quad (18)$$

From Fig. 3(d) control law can be express:

$$r = K_p (r_{ref} - Y) \quad (19)$$

Where

$$K_P = \begin{bmatrix} K_{p\_A} & 0 & 0 \\ 0 & K_{p\_B} & 0 \\ 0 & 0 & K_{p\_C} \end{bmatrix} \qquad r_{ref} = \begin{bmatrix} r_{ref\_A} \\ r_{ref\_B} \\ r_{ref\_C} \end{bmatrix}$$

Substituting (18) and (19) with considering r=0 and simplifying result terms, can yield the output-to-disturbance transfer function, that is:

$$\left.\frac{Y}{W}\right|_{r_{ref}=0}=\begin{bmatrix}\frac{-\left(CLRs+L-0.8CR^2\right)}{CL\left(K_{P\_A}+s\right)} & \frac{-\left(R^2\right)}{5L\left(K_{P\_A}+s\right)} & \frac{-\left(R^2\right)}{5L\left(K_{P\_A}+s\right)}\\ \frac{-\left(R^2\right)}{5L\left(K_{P\_B}+s\right)} & \frac{-\left(CLRs+L-0.8CR^2\right)}{CL\left(K_{P\_B}+s\right)} & \frac{-\left(R^2\right)}{5L\left(K_{P\_B}+s\right)}\\ \frac{-\left(R^2\right)}{5L\left(K_{P\_C}+s\right)} & \frac{-\left(R^2\right)}{5L\left(K_{P\_C}+s\right)} & \frac{-\left(CLRs+L-0.8CR^2\right)}{CL\left(K_{P\_C}+s\right)}\end{bmatrix} \quad (20)$$

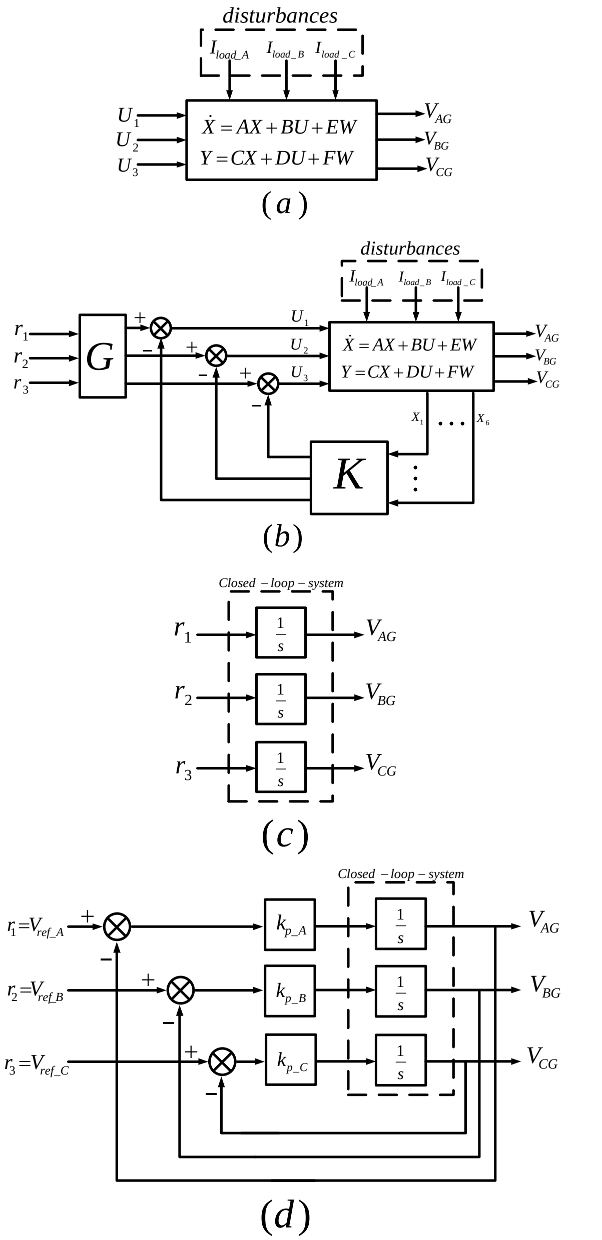


Figure 3. Propoed decoupled control. (a) open loop system with disturbances; (b) closed-loop system via linear decouling state feedback, disturbances are considered to be zero; (c) equivalent transfer function of closed-loop system of (b); (d) proposed porportional decoupled control of each phase.

From (20), it can be seen that the output-to-disturbance transfer function matrix is not decoupled but because of term R^2 (R is parasitic resistor and small) in numerator of fraction in non-diagonal elements of matrix, and also term K_{p-i} in denominator, for K_{p-i} to be enough large, we have:

$$\left.\frac{Y}{W}\right|_{r_{ref}=0}\approx\begin{bmatrix}\frac{-\left(CLRs+L-0.8CR^2\right)}{CL\left(K_{P\_A}+s\right)} & 0 & 0\\ 0 & \frac{-\left(CLRs+L-0.8CR^2\right)}{CL\left(K_{P\_B}+s\right)} & 0\\ 0 & 0 & \frac{-\left(CLRs+L-0.8CR^2\right)}{CL\left(K_{P\_C}+s\right)}\end{bmatrix} \quad (21)$$

To cancel fully the effects of disturbances; from (21), K_{p-i} in each phase should be chosen enough large however, the equivalent magnitude of (21) to be neglected. Thus

$$\left|\frac{-\left(CLRs+L-0.8CR^2\right)}{CL\left(K_{P\_i}+s\right)}\right|\approx 0\Rightarrow K_{P\_i}>>1, i=A,B,C \quad (22)$$

V. Simulation Study

The performance of proposed control strategy is investigated by simulation using MATLAB/SIMULINK.

The system parameters are as follow:

$$\begin{aligned}L&=333\mu H\\ C&=100\mu F\\ R&=10m\Omega\end{aligned} \quad (23)$$

And output required parameters are (considering unbalanced condition):

$$\begin{aligned}&V_m=380\sqrt{2},\omega=2\pi\times 50\\ &R\_load\_A=2\Omega, L\_load\_A=5e-3H\\ &R\_load\_B=200\Omega, L\_load\_C=500e-3H\\ &R\_load\_c=200\Omega, L\_load\_C=500e-3H\end{aligned} \quad (24)$$

A. *Desired control system*

To test the proposed closed-loop system (18) with control input (19), in this case, (22) can be represented as:

$$\left|\frac{-\left(3.3300\text{e-}010(100\pi j)+3.3299\text{e-}004\right)}{3.3300\text{e-}008\left(K_{P\_i}+100\pi j\right)}\right|=1e-3, K_{P\_i}=1e7 \quad (25)$$

Fig. 4 shows step response of output-to-disturbance (21).

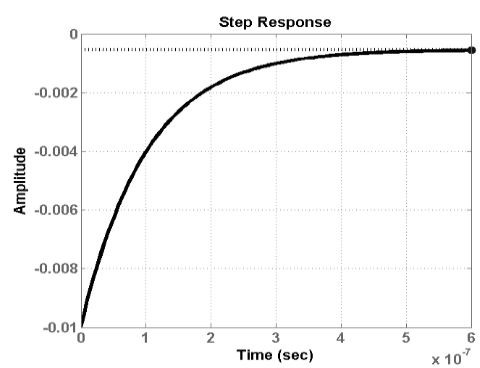


Figure 4. Step response of output-to-disturbance, K_{P_i}=1e7

B. Decoupling performance with unbalance loaded condition

Considering K_{p_i} from (25), the performance of output-to-reference input transfer function can be examined by simulation. Fig. 5 shows step response of close-loop system (18) with the control law of (19) in each channel (disturbances are zero). From Fig. 5, the equivalent system has good performance for tracking system, i.e. has small rise-time without any over-shoot and steady-state error of zero.

It is required that the three-phase output voltages of converter be balanced and sinusoidal when it is loaded in unbalanced condition such as one-phase loaded. Fig. 6(a) shows the output voltages and Fig. 6(b) shows the load currents of converter with the parameters of (23) and (24).

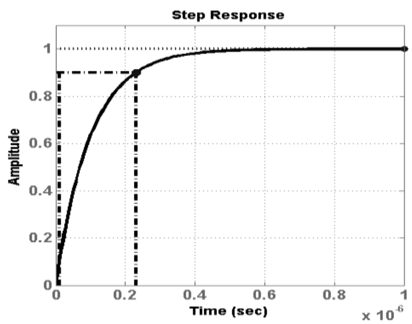
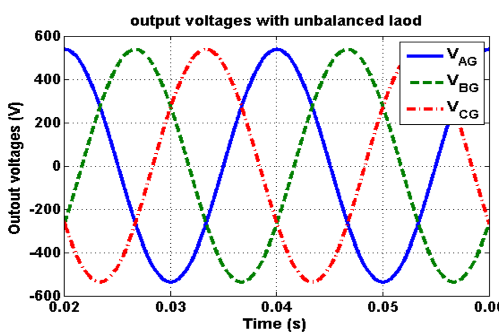
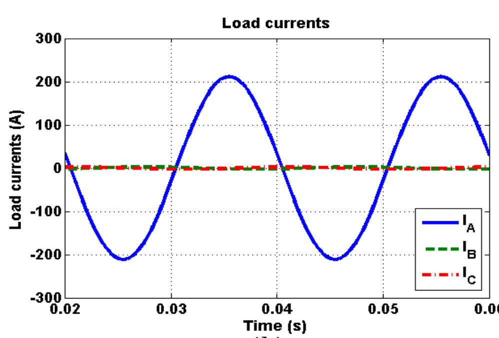


Figure 5. step response of close-loop system (18) with the control law of (19) in each channel, disturbances are zero



(*a*)



(*b*)

Figure 6. Output voltages and current of converter; (a) three-phase terminal voltages. (b) load currents, phase a is loaded

From Fig. 6(a), with proposed decoupled control strategy output voltages are obtained sinusoidal and balanced.

VI. Conclusion

This paper adopts four-leg voltage source inverter in order to solve unbalanced load problem instead of three-leg VSI. Average large signal model is expressed in ABC coordinates and state space equations are obtained. From equations, coupling within phases is observed however based on state feedback strategy certain pole assignment base on system matrix is proposed to decouple phases. Finally simple proportional controller is presented to attain required performance of system. Simulation results are provided under typical unbalanced conditions and verify the validity of proposed control strategy.

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