Adaptive Interval Type-2 Fuzzy PI Sliding Mode Control with Optimization of Membership Functions Using Genetic Algorithm

Mostafa Ghaemi,
Mo.Ghaemi@yahoo.com

Mohammad-R. Akbarzadeh-T,
akbarzadeh@ieee.org

Mohsen Jalaeian-F,
m.jalaeian@yahoo.com

Center of Excellence on Soft-Computing and Intelligent Information Processing,
Ferdowsi University of Mashhad, Mashhad, Iran

Abstract— **A new stable Adaptive Interval Type-2 Fuzzy Proportional Integral Sliding Mode Controller (AI2FPISMC) is introduced here to control a class of nonlinear systems. The proposed method is based on interval type-2 fuzzy logic system (IT2FLS) whose antecedent and consequent membership functions are interval type-2 fuzzy sets. IT2FLS is utilized to approximate unknown nonlinear functions. To achieve high performance, optimizing membership functions (MFs) of interval type-2 fuzzy sets (IT2FS) is required. Genetic algorithm (GA) is a parallel search optimization method; that here contributes to optimize the MFs. In order to cope with the chattering of sliding mode controller, PI control law is proposed and Lyapunov analysis is utilized to prove asymptotic stability of the proposed approach. The adaptation laws are derived using Lyapunov approach. Two nonlinear system simulation examples are presented to verify the effectiveness of the proposed method, and their results confirm the optimization merits.**

Keywords- Proportional Integral (PI), Adaptive interval type-2 fuzzy sliding mode, Uncertain nonlinear systems, Interval type-2 fuzzy set (IT2FS), Interval type-2 fuzzy logic system (IT2FLS).

I. Introduction

Recently, nonlinear control, in the presence of uncertainties, is a wide research area. Generally there are two kinds of uncertainties, one caused by lack of information about structure and parameters of a system and the other caused by internal and external disturbances and noises. Uncertainties in plant dynamics can be addressed via Sliding Mode Control (SMC) which is a robust nonlinear discontinuous feedback control technique, with the drawback of chattering [1]. The chattering is the main drawback of SMC, which can excite undesirable high-frequency dynamics. To reduce chattering phenomenon, a small boundary layer is introduced around the sliding surface. However, the state trajectory of the resulting system may not converge on a narrow bound around to the sliding surface[1]. A way to eliminate chattering is to use a proportional integral (PI) control term [11] which is used in this paper. The other method which can handle uncertainty in plant dynamics is adaptive control. In adaptive control, an adaptation law is introduced that adjusts the parameters of the controller against system uncertainties and disturbances. It should be noted that adaptive control methods generally guarantee parameter convergence only if parameter changes are slow enough [2].

In the recent decades some techniques based on fuzzy logic systems (FLS) have been emerged for control of nonlinear systems [2-5]. Fuzzy logic provides a tool for using human expert knowledge in additional to mathematical knowledge. This is mainly useful for employing fuzzy knowledge-based control to deal with systems dynamic obscurities, conveniently non-modeled systems [1] or difficulties in dynamic mathematically analysis. Quite often, the FLS rules construct information is uncertain [6, 7]. Type-1 FLSs are unable to directly handle rule uncertainties, because their membership functions are type-1 fuzzy sets. In contrast, type-2 FLSs involved in this paper can handle rule uncertainties. Therefore, hybrid combinations of the SMC, type-2 fuzzy logic and adaptive control are an attractive approach for designing robust control systems with high degrees of nonlinearities and uncertainties. To improve performance, optimizing MFs of IT2FSs is required. One of the best optimization methods is GA that is a parallel search population based method.

In this paper, we introduce an adaptive interval type-2 fuzzy logic sliding mode control for a class of uncertain SISO nonlinear systems which was originally proposed in [8] to an adaptive sliding mode control based on type-1 fuzzy sets. We use IT2FLS to approximate the unknown nonlinear term which their antecedent and consequent sets are interval type-2 fuzzy sets. The proposed controller uses advantages of both IT2FLS and adaptive sliding mode controller to handle uncertainties and use PI term to eliminate chattering; Lyapunov synthesis is used to guarantees system asymptotic stability, and to achieve a good response, GA tunes the MFs of AIT2FPISMC.

II. Type-2 Fuzzy Logic Systems

Generally an interval type-2 fuzzy set in universal set X is denoted as $\tilde{A}$ is characterized in following form:

$$\tilde{A} = \int_{x\in X} \mu_{\tilde{A}}(x)\,/x = \int_{x\epsilon X}[\int_{u\epsilon J_x} 1/u]/x \quad , \quad J_x\epsilon[0,1] \qquad (1)$$

where $\mu_{\tilde{A}}(x)$ is called a secondary MF and the domain of a secondary MF is called the primary membership of x is noted by J_x. where $J_x\epsilon[0,1]$ for $\forall\ x\epsilon X$; $u\epsilon[0,1]u$ is a fuzzy set.

(IT2FSs) demonstrate an uniform uncertainty at the primary membership of x. Many researchers use this kind of type-2 fuzzy sets because of their simplicity of calculation especially in the type reduction [9-11].

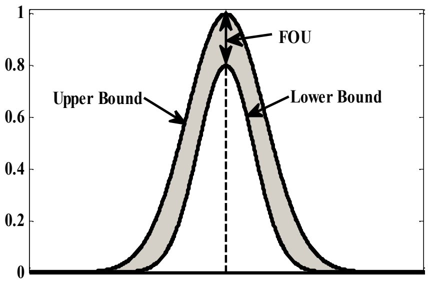

FIGURE 1. Interval type-2 fuzzy MF

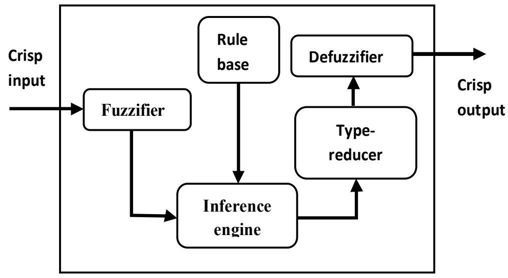

FIGURE 2. Structure of an ITFLS

An IT2FS $\tilde{A}$ is described by its lower $\underline{\mu}_{\tilde{A}}(x)$ and upper $\overline{\mu}_{\tilde{A}}(x)$ membership functions. The Footprint of Uncertainty (FOU) for an IT2FS is described in terms of these MFs as

$$\text{FOU }(\tilde{A})=\cup_{x\epsilon X}[\underline{\mu}_{\tilde{A}}(x),\overline{\mu}_{\tilde{A}}(x)] \quad (2)$$

Figure .1 shows a type-2 fuzzy MF with its FOU, upper and lower bounds. Generally, an IT2FLS consists of fuzzifier; fuzzy rule base; fuzzy inference engine; type reducer and defuzzifier. Figure 2 shows the general structure of an IT2FLS. Type reducer block is main difference between type-1 and type-2 fuzzy logic systems.

Since the output of the inference engine is a type-2 fuzzy set, its type must be reduced before defuzzification. Type reducer was first introduced by Karnik and Mendel [6, 12]. In [12] five different type reducers are described which are based on computing the centroid of an IT2FS. Center of sets type reducer can be expressed as:

$$Y_{cos} = (Y^1 ,\dots, Y^M , F^1 ,\dots, F^M)= [y_l , y_r] = \int_{y^1}\dots\int_{y^M}\int_{f^1}\dots\int_{f^M} 1/\frac{\sum_{i=1}^{M} f^i y^i}{\sum_{i=1}^{M} f^i} \quad (3)$$

where $f^i\epsilon F^i = [\underline{f}^i(X),\overline{f}^i(X)]$, $y^i\epsilon Y^i = [y_l^i, y_r^i]$.

Whereas, in this paper we use IT2FS then Y_{cos} is the interval set determined with its left end point, y_l, and its right end point y_r. We defuzzify the set obtained from type reducer by using the average of y_l and y_r [9], therefore we can obtain a crisp output:

$$y(x)= (y_l(x) + y_r(x))/2 \quad (4)$$

In general, there are no closed form formula for computing y_l and y_r; however Mendel and Karnik developed two algorithm for calculating these two end points in [13]. If we use singleton fuzzifier, product inference engine and COS type reducer y_l and y_r can be illustrated as:

$$y_l=\frac{\sum_{i=1}^{M} f^i y_l^i}{\sum_{i=1}^{M} f^i} = \theta_l^T \xi_l \quad (5)$$

Where $\theta_l^i = y_l^i$ and $\theta_l = [\theta_l^1, \dots, \theta_l^M]^T$, $\xi_l^i=\frac{f^i}{\sum_{i=1}^{M} f^i}$ and $\xi_l= [\xi_l^1, \dots,\xi_l^M]^T$

$$y_r=\frac{\sum_{i=1}^{M} f^i y_r^i}{\sum_{i=1}^{M} f^i} = \theta_r^T \xi_r \quad (6)$$

Where $\theta_r^i = y_r^i$ and $\theta_r = [\theta_r^1, \dots, \theta_r^M]^T$, $\xi_r^i=\frac{f^i}{\sum_{i=1}^{M} f^i}$ and $\xi_r= [\xi_r^1, \dots,\xi_r^M]^T$

Now the defuzzified crisp output obtain as:

$$y(x) = \frac{y_r + y_l}{2} = \frac{1}{2}(\theta_l^T \xi_l + \theta_r^T \xi_r) \quad (7)$$

III. SLIDING MODE CONTROL

Consider a general class of SISO nonlinear system as

$$x^{(n)}=f(\underline{x},t) + g(\underline{x},t)u + d(t) \quad (8)$$
$$y = x$$

Where f and g are unknown bounded nonlinear functions where the bounds need not be known, $d(t)$ is the unknown bounded external disturbance, $u\epsilon R$ and $y\epsilon R$ are input and output of the system, respectively, $\underline{x} = [x, \dot{x}, \dots, x^{(n-1)}]\epsilon R^n$ is the state vector of the system which is assumed to be available for measurement. We have following assumptions:

Assumption 1. We assume external disturbance $d(t)$ is bounded by a known constant D, i.e.

$$d(t) \le D \quad (9)$$

Assumption 2. System (8) is controllable, it means $g(\underline{x},t) \neq 0$. Without loss of generality, we assume $g(\underline{x},t) > 0$, i.e. can be negative and the control can be similarly derived.

The control objective is to determine a feedback control $u(\underline{x})$ such that the state $\underline{x}$ of the system follows the desired state vector $\underline{x_d} = [x_d, \dot{x}_d, \dots, x_d^{(n-1)}]$, therefore the tracking error is:

$$\underline{e} = \underline{x} - \underline{x_d} = [e, \dot{e}, \dots, e^{(n-1)}]^T \quad (10)$$

Then a sliding surface in the space of the error state can be defined as

$$s(\underline{e}) = \underline{c}^T \underline{e} = e^{(n-1)} + c_{n-1}e^{(n-2)} + \dots + c_1 e \quad (11)$$

where $\underline{c}$ are the coefficients of the Hurwitz polynomial $h(r) = \lambda^{(n-1)} + c_{(n-1)}\lambda^{(n-2)} + \dots + c_1$, i.e. all the roots are in the open left-hand (λ is a Laplace operator). A sufficient condition for stability of the system controlled is given in [1] as:

$$\dot{s} \le -\eta sgn(s), \qquad \eta > 0 \quad (12)$$

By taking the time derivative of (11), we obtain:

$$\dot{s} = \sum_{i=1}^{n-1} c_i e^i + x^{(n)} - x_d^{(n)} = \sum_{i=1}^{n-1} c_i e^i + f(\underline{x},t) + g(\underline{x},t)u + d(t) - x_d^{(n)} \quad (13)$$

Substituting (13) into (12), sliding condition can be re-expressed as:

$(\sum_{i=1}^{n-1} c_i e^i + f(\underline{x},t) + g(\underline{x},t)u + d(t) - x_d^{(n)}) \leq - \eta sgn(s)$ (14)

Therefore optimal control u^* is:

$u^* = \frac{1}{g(\underline{x},t)}[-\sum_{i=1}^{n-1} c_i e^i - f(\underline{x},t) - d(t) + x_d^{(n)} - \eta_\Delta sgn(s)]$ (15)

where $\eta_\Delta \geq \eta > 0$.

IV. Adaptive Sliding Mode Control Based on IT2FLS

Since $(\underline{x},t)$, $g(\underline{x},t)$ and $d(t)$ in (15) are unknown, we replace $f(\underline{x},t)$ and $g(\underline{x},t)$ by the IT2FLS $\hat{f}(\underline{x}|\theta_f)$ and $\hat{g}(\underline{x}|\theta_g)$ which are in the form (7). Dou to the sliding control law (15) is discontinuous across the sliding surface $s(t)$ and leads to chattering, we employ a PI control term to eliminate the chattering where PI controller is in the form of:

$$P = K_P s + K_I \int s dt = \theta_p^T \psi(\underline{z}) \quad (16)$$

Where $\theta_p = [K_P, K_I]^T$ is an adjustable parameter vector, and $\psi(\underline{z}) = [z_1, z_2] = [s, \int s dt]^T$ is a regressive vector. Therefore, around the sliding surface, control law is introduced as:

$$\hat{u}_p(\underline{z}|\theta_p) = \begin{cases} \hat{u}_1 = \theta_p^T \psi(\underline{z}) & if\ |s| < \varphi \\ \hat{u}_2 = (D + \eta_\Delta) sgn(s) & if\ |s| \geq \varphi \end{cases} \quad (17)$$

Where φ is the thickness of the boundary layer, D is obtained from (9). The resulting control input is:

$$u = \frac{1}{\hat{g}(\underline{x}|\theta_g)}\left[-\sum_{i=1}^{n-1} c_i e^i - \hat{f}(\underline{x}|\theta_f) + x_d^{(n)} - \hat{u}_p(\underline{z}|\theta_p)\right] \quad (18)$$

Where:

$$\hat{f}(\underline{x}|\theta_f) = \frac{\hat{f}_l + \hat{f}_r}{2} = \frac{1}{2}(\theta_{fl}^T \xi_l + \theta_{fr}^T \xi_r) = \theta_f^T \xi_f \quad (19)$$

$$\hat{g}(\underline{x}|\theta_g) = \frac{\hat{g}_l + \hat{g}_r}{2} = \frac{1}{2}(\theta_{gl}^T \xi_l + \theta_{gr}^T \xi_r) = \theta_g^T \xi_g \quad (20)$$

Theorem1. Consider the nonlinear SISO system (8) and the control input u in (18), if the fuzzy based adaptive laws are chosen as:

$$\dot{\theta}_{fl} = \gamma_1 s \xi_{fl} \quad (21)$$
$$\dot{\theta}_{fr} = \gamma_2 s \xi_{fr} \quad (22)$$
$$\dot{\theta}_{gl} = \gamma_3 s \xi_{gl} u \quad (23)$$
$$\dot{\theta}_{gr} = \gamma_4 s \xi_{gr} u \quad (24)$$
$$\dot{\theta}_p = \gamma_5 s \psi(\underline{z}) \quad (25)$$

The closed loop system signals will be bounded and the tracking error will converge to zero asymptotically.

Proof . The optimal parameters of fuzzy systems are defined as:

$$\theta_f^* = \arg min_{\theta_f \epsilon \Omega_f}[\sup_{\underline{x}}|\hat{f}(\underline{x}|\theta_f) - f(\underline{x},t)|] \quad (26)$$
$$\theta_g^* = \arg min_{\theta_g \epsilon \Omega_g}[\sup_{\underline{x}}|\hat{g}(\underline{x}|\theta_g) - g(\underline{x},t)|] \quad (27)$$
$$\theta_p^* = \arg min_{\theta_f \epsilon \Omega_s}[\sup_{\underline{z}}|\hat{u}_p(\underline{z}|\theta_p) - \hat{u}_2|] \quad (28)$$

Where Ω_f , Ω_g and Ω_s are defined as:

$$\Omega_f = \{\theta_f \epsilon R^n | |\theta_f| \leq M_f\} \quad (29)$$
$$\Omega_g = \{\theta_g \epsilon R^n | 0 < |\theta_g| \leq M_g\} \quad (30)$$
$$\Omega_p = \{\theta_p \epsilon R^2 | 0 < |\theta_p| \leq M_p\} \quad (31)$$

where M_f , M_g and M_p are positive constant. We can define the minimum approximation error as:

$$\omega = [f(\underline{x},t) - \hat{f}(\underline{x}|\theta_f^*)] + [g(\underline{x},t) - \hat{g}(\underline{x}|\theta_g^*)]u \quad (32)$$

Then from substituting (18) and (32) into (13), derivative of sliding surface is:

$$\dot{s} = \sum_{i=1}^{n-1} c_i e^i + f(\underline{x},t) + g(\underline{x},t)u + d(t) - x_d^{(n)}$$
$$= f(\underline{x},t) - \hat{f}(\underline{x}|\theta_f) + (g(\underline{x},t) - \hat{g}(\underline{x}|\theta_g))u + d(t) - \hat{u}_p(\underline{z}|\theta_p)$$
$$= [\hat{f}(\underline{x}|\theta_f^*) - \hat{f}(\underline{x}|\theta_f)] + [\hat{g}(\underline{x}|\theta_g^*) - \hat{g}(\underline{x}|\theta_g)]u + d(t) + [\hat{u}_p(\underline{z}|\theta_p^*) - \hat{u}_p(\underline{z}|\theta_p)] + \omega - \hat{u}_p(\underline{z}|\theta_p^*)$$
$$= (\theta_f^{*T}\xi_f - \theta_f^T \xi_f) + (\theta_g^{*T}\xi_g - \theta_g^T\xi_g)u + (\theta_p^{*T}\psi - \theta_p^T\psi) + d(t) - \hat{u}_p(\underline{z}|\theta_p^*) + \omega$$
$$= \emptyset_f^T\xi_f + \emptyset_g^T\xi_f u + \emptyset_p^T\psi + d(t) - \hat{u}_p(\underline{z}|\theta_p^*) + \omega$$
$$= \frac{1}{2}(\emptyset_{fl}^T\xi_{fl} + \emptyset_{fr}^T\xi_{fr}) + \frac{1}{2}(\emptyset_{gl}^T\xi_{gl} + \emptyset_{gr}^T\xi_{gr})u + \emptyset_p^T\psi + d(t) - \hat{u}_p(\underline{z}|\theta_p^*) + \omega \quad (33)$$

Where $\emptyset_f = \theta_f^* - \theta_f$, $\emptyset_g = \theta_g^* - \theta_g$ and $\emptyset_p = \theta_p^* - \theta_p$

Now the Lyapunov function is defined as:

$$V = \frac{1}{2}s^2 + \frac{1}{4\gamma_1}\emptyset_{fl}^T\emptyset_{fl} + \frac{1}{4\gamma_2}\emptyset_{fr}^T\emptyset_{fr} + \frac{1}{4\gamma_3}\emptyset_{gl}^T\emptyset_{gl} + \frac{1}{4\gamma_4}\emptyset_{gr}^T\emptyset_{gr} + \frac{1}{2\gamma_5}\emptyset_p^T\emptyset_p \quad (34)$$

Where γ_1 ,γ_2, γ_3 ,γ_4 and γ_5 are positive constant. The time derivative of V is:

$$\dot{V} = s\dot{s} + \frac{1}{2\gamma_1}\emptyset_{fl}^T\dot{\emptyset}_{fl} + \frac{1}{2\gamma_2}\emptyset_{fr}^T\dot{\emptyset}_{fr} + \frac{1}{2\gamma_3}\emptyset_{gl}^T\dot{\emptyset}_{gl} + \frac{1}{2\gamma_4}\emptyset_{gr}^T\dot{\emptyset}_{gr} + \frac{1}{\gamma_5}\emptyset_p^T\dot{\emptyset}_p$$
$$= s(\frac{1}{2}(\emptyset_{fl}^T\xi_{fl} + \emptyset_{fr}^T\xi_{fr}) + \frac{1}{2}(\emptyset_{gl}^T\xi_{gl} + \emptyset_{gr}^T\xi_{gr})u + \emptyset_p^T\psi + d(t) - \hat{u}_p(\underline{z}|\theta_p^*) + \omega) + \frac{1}{2\gamma_1}\emptyset_{fl}^T\dot{\emptyset}_{fl} + \frac{1}{2\gamma_2}\emptyset_{fr}^T\dot{\emptyset}_{fr} + \frac{1}{2\gamma_3}\emptyset_{gl}^T\dot{\emptyset}_{gl} + \frac{1}{2\gamma_4}\emptyset_{gr}^T\dot{\emptyset}_{gr} + \frac{1}{\gamma_5}\emptyset_p^T\dot{\emptyset}_p$$
$$= \frac{1}{2\gamma_1}\emptyset_{fl}^T(\dot{\emptyset}_{fl} + \gamma_1 s\xi_{fl}) + \frac{1}{2\gamma_2}\emptyset_{fr}^T(\dot{\emptyset}_{fr} + \gamma_2 s\xi_{fr}) + \frac{1}{2\gamma_3}\emptyset_{gl}^T(\dot{\emptyset}_{gl} + \gamma_3 s\xi_{gl}u)$$
$$+ \frac{1}{2\gamma_4}\emptyset_{gr}^T(\dot{\emptyset}_{gr} + \gamma_4 s\xi_{gr}u) + \frac{1}{\gamma_5}\emptyset_p^T(\dot{\emptyset}_p + \gamma_5 s\psi) + sd(t) - \hat{u}_p(\underline{z}|\theta_p^*) + s\omega$$
$$\leq \frac{1}{2\gamma_1}\emptyset_{fl}^T(\dot{\emptyset}_{fl} + \gamma_1 s\xi_{fl}) + \frac{1}{2\gamma_2}\emptyset_{fr}^T(\dot{\emptyset}_{fr} + \gamma_2 s\xi_{fr}) + \frac{1}{2\gamma_3}\emptyset_{gl}^T(\dot{\emptyset}_{gl} + \gamma_3 s\xi_{gl}u) + \frac{1}{2\gamma_4}\emptyset_{gr}^T(\dot{\emptyset}_{gr} + \gamma_4 s\xi_{gr}u) + \frac{1}{\gamma_5}\emptyset_p^T(\dot{\emptyset}_p + \gamma_5 s\psi) + sd(t) - s(\eta_\Delta + D)sgn(s) + s\omega \quad (35)$$

where $\dot{\emptyset}_{fr} = -\dot{\theta}_{fr}$, $\dot{\emptyset}_{fl} = -\dot{\theta}_{fl}$, $\dot{\emptyset}_{gr} = -\dot{\theta}_{gr}$ and $\dot{\emptyset}_{gl} = -\dot{\theta}_{gl}$, substituting (21)-(25) into (35), then we have:

$$\dot{V} = sd(t) - s(\eta_\Delta + D)sgn(s) + s\omega = sd(t) - |s|(\eta_\Delta + D) + s\omega \leq -|s|(\eta_\Delta) + s\omega \quad (36)$$

to be based on the approximation theorem [14], it can be anticipated that the term $s\omega$ should be very small if it not equals to zero in the IT-2 FLS, we obtain:

$$\dot{V} = -|s|(\eta_\Delta) \leq 0 \quad (37)$$

Since $\underline{c} = [c_1, \dots, c_{(n-2)}, c_{(n-1)}, 1]^T$ in which the c_i's are all real and chosen such that $h(\lambda) = \sum_{i=1}^{n} c_i \lambda^{(i-1)}, c_n = 1$ is a Hurwitz polynomial, we have $\lim_{t\to\infty} |e(t)| = 0$, therefore $\lim_{t\to\infty} |s(e)| = 0$, then, the system is stable and the error will asymptotically converge to zero. the proof is completed. ■

V. Genetic Algorithm Optimization Procedure

In this work, we propose using Genetic algorithm (GA) to optimize the membership functions of the interval type-2 fuzzy logic system. GA was first introduced by Holland in 1975 [20]. GA is a population based parallel search algorithm which is based on the mechanism of natural selection and natural genetics that operate without knowledge of the task domain and utilize only the fitness of evaluated individuals. In general, three basic operators of GA are reproduction, crossover and mutation. GA can be considered as a general purpose optimization method and has been successfully applied to search and optimization [26-29]. Here, Binary GA method is used as the aim of optimization as describe below.

Optimization emphasized on the parameters of MFs. Let us consider θ as a vector of parameters which we are going to optimize, so determined θ contains standard division (δ_{1i}, δ_{2i}), mean (m_i) and amplitude of lower MF (Am_i) (figure 1), it means $\theta = [m_i, \delta_{i1}, \delta_{i2}, Am_i]$, $i = 1, 2, \dots, n$,where n is the number of MFs, so the number of parameters of θ is quadruple of n.

GA operates on chromosomes, which are binary strings, but the main problem must work out with real numbers, hence artificial genetic algorithm has two spaces, genotype and phenotype. Genotype, the space GA work on, consists in chromosomes and phenotype consists of real numbers; which each number decodes from one gene and vise versa. Here, each individual adjust the membership functions of the IT2FLS. The fitness of the related chromosome is evaluated by monitoring the overall mean squared error of that closed loop control system. Also the mutation and crossover operate on individuals of each generation and generate new child for constitute the new generations. Some constraints are considered based on GA-convergence speed:

C1: $0.01 < \delta_{1i} < \delta_{2i} < 2 \quad , i = 1, 2, \dots, n$

C2: $-3 < m_1 < m_2 < \dots < m_n < 3$

C3: 0.1< amplitudes of lower MFs (Am_i)<1

For tuning the GA parameters we determined the detail as below table1:

Table1 : GA's characteristics:

character	Value
Max number of generation	400
Population in each generation	40
Probability of crossover (Pc)	0.6
probability of mutation (Pm)	0.4

In next section we apply the optimized MFs and uniform MFs for two nonlinear systems to verify the advantages of the optimization procedure.

VI. Simulation Examples

In this section, we want to apply our proposed adaptive sliding mode controller for two examples. The first example is a regulation problem to let the output of a first order nonlinear system to track a constant trajectory. The second example is to let a Duffing forced oscillation system to track a sin-wave trajectory.

Example 1. Consider a first order system as follow

$$\dot{x} = \frac{1-e^{-x}}{1+e^{x}} + u(t) + d(t), \quad d(t) = \cos(3t) + 4 \quad (38)$$

Where $u(t)$ is the input control signal and $d(t)$ is a bounded disturbance with known bound D=6. Assume $K_P(0) = 100$, $K_I(0) = 80$, $\eta_\Delta = 0.5$, $\varphi = 0.2$ and The desired trajectory is $x_d = 0$. We use GA to optimize three MFs over interval [-3,3] where each MF has 4 parameters, it means each individual has 12 adjustable parameters. Table 2 shoes parameters of optimized MFs.

Table 2: Optimized Parameters

Parameter	μ_N	μ_Z	μ_P
m	-2.87	-0.44	0.04
$[\delta_1, \delta_2]$	[0.1 , 0.1]	[0.17 , 0.17]	[0.01 , 0.01]
Am	0.76	0.74	1

The initial consequent parameters $\theta_{fl}(0)$ and $\theta_{fr}(0)$ are chosen uniformly over interval [0.1, 2] and [0.6, 2.5], respectively. Let the learning rate $\gamma_1 = 50$, $\gamma_2 = 60$ and $\gamma_5 = 10$. The results are simulated for the initial state $x(0) = 1.5$ and step size 0.01.

Figure 3 (a) shows the system response, and compare AIT2FPISMC based on uniform MFs and Optimized MFS. Simulation results show effectiveness of optimized method. From Figure 3 (b) it can be seen that the Integral squared error (ISE) of the optimized AIT2FPISMC is less than uniform one.

Example 2. Consider the Duffing forced-oscillation system in the form of

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos(t) + u(t) + d(t)$$

$$d(t) = 5\cos(4t)$$

Table 3: Optimized Parameters

Parameter	$\mu_N(x1)$	$\mu_Z(x1)$	$\mu_P(x1)$	$\mu_N(x2)$	$\mu_Z(x2)$	$\mu_P(x2)$
m	-2.31	0	2.2	-0.79	3	3
$[\delta_1,\delta_2]$	[0.5, 0.5]	[0.44 , 0.44]	[0.35 , 0.35]	[1.4,1.4]	[0.01,0.01]	[0.07,0.07]
Am	0.77	0.25	0.22	0.84	0.26	0.94

where $d(t)$ is an unknown disturbance with known bound D=7. The desired trajectory is $x_d = \sin(t)$ and we assume $K_P(0) = 40$, $K_I(0) = 20$, $\eta_\Delta = 0.2$ and $\varphi = 0.5$. Three membership functions for each of the system states x_1 and x_2 are chosen as in table 3, then there are 9 rules to approximate the system function f. Figure 4 show the optimized MFs. The initial consequent parameters $\theta_{fl}(0)$ and $\theta_{fr}(0)$ are chosen uniformly over interval [-2, 2] and [-1.9, 2.1], respectively. Let the learning rate $\gamma_1 = 10, \gamma_2 = 20, \gamma_5 = 10$. Choose the sliding surface as $s = \dot{e} + 5e$. The results are simulated for the initial states $x_1(0) = 2$, $x_2(0) = 2$ and step size 0.01.

Figure 5 shows the system response for the AFPISMC. Based on the simulation results it can be seen that the proposed controller achieves a good performance and using GA optimization is helpful to improve the performance.

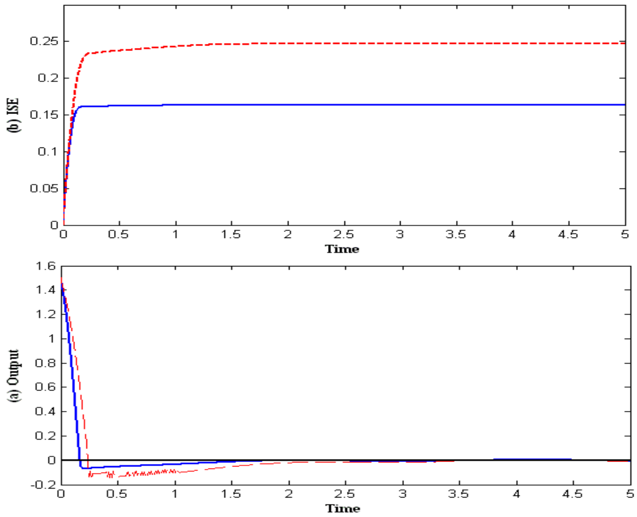

Fig 3. (a) System output for optimized MFs (solid) and uniform MFs (dashed); (b)The tracking performance $\int_{t=0}^{5} e^2\,dt$ for optimized MFs (solid) and uniform MFs (dashed).

VII. Conclusions

In this paper an adaptive interval type-2 fuzzy PI sliding mode controller for a class of nonlinear systems is designed. We used an interval type-2 fuzzy logic system to approximate the unknown nonlinear terms. Chattering problem is undesirable in most SMC applications; to eliminating this phenomenon a PI term is used near the sliding surface. To prove asymptotic stability of the proposed controller, Lyapunov analysis is used. The simulation results confirm the controller high performance and system stability. Also it is seen that the performance is improved using GA.

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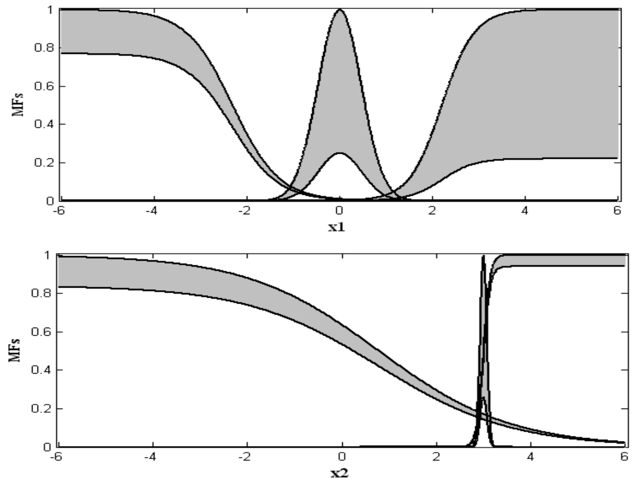

Figure 4. Optimized MFs.

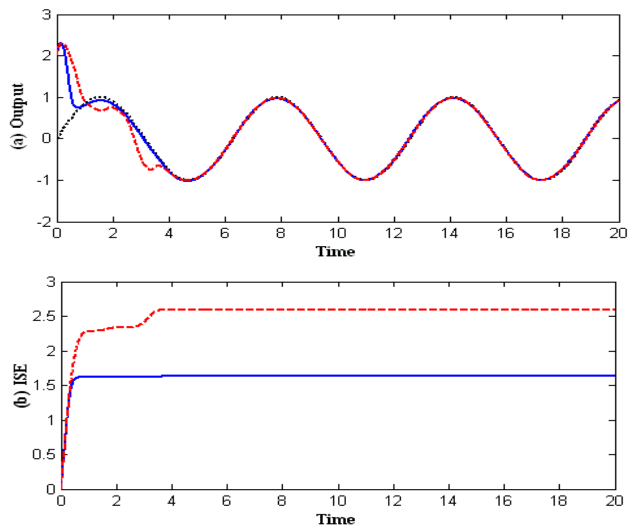

Figure 5. (a) Desired output (dot), Output for optimized MFs (solid) and uniform MFs (dashed); (b)The tracking performance $\int_{t=0}^{5} e^2\,dt$ for optimized MFs (solid) and uniform MFs (dashed).