A Hybrid Algorithm Based on Gravitational Search Algorithm for Unimodal Optimization

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***Abstract*—Nowadays, utilizing heuristic algorithms is highly appreciated in solving optimization problems. The fundamental of these algorithms are inspired by nature. The gravitational search algorithm (GSA) is a novel heuristic search algorithm which is invented by using law of gravity and mass interactions. In this paper, a new operator is presented which is called "the black hole". This operator is inspired by the concept of an astronomy phenomenon. By adding the black hole operator, the exploitation of the GSA is improved. The proposed algorithm is evaluated by seven standard unimodal benchmarks. The results obtained demonstrate better performance of the proposed algorithm in comparison with those of the standard GSA and other version of GSA which is equipped with the disruption operator.**

Keywords-Heuristic algorithms; the gravitational search algorithm (GSA); the black hole operator; unimodal functions.

I. INTRODUCTION

Many problems need to be optimized in industry, science and environment. Heuristic search algorithms are one of the optimization approaches that are different from classical algorithms since these algorithms are population based and have been established based on stochastic rules.

These algorithms are inspired from the observation of natural phenomenon. It has been shown by many researches that these algorithms are viable candidates as tools to solve complex computational problems.

In the heuristic based search methods two aspects should be investigated, exploration and exploitation. Exploration deals with searching the solution space while exploitation is able to find optimal solution around the best obtained solution so far. For a good performance it is necessary to make a suitable trade-off between exploration and exploitation capabilities.

Furthermore, different operators of heuristic search algorithms have particular tasks for realization of the exploration or the exploitation. Therefore, creating new operators could help for a better performance of algorithms by a good balancing between exploration and exploitation abilities. Hence, the attention of researchers is increased in creating and modifying operators of heuristic algorithms, in order to improve the algorithm performance. The works carried out in [1]-[5] are examples of such improvements in genetic algorithm (GA) and Particle Swarm Optimization (PSO).

The gravitational search algorithm (GSA) is a novel heuristic algorithm based on the laws of gravity and motion which introduced recently by Rashedi and Nezamabadi-pour [8]. The experiments demonstrate the ability of this algorithm on globally search, convergence of solutions and finding the optimal solution [6-9]. Although the ability of GSA is proved by many researches on different problems, there is also the possibility of premature convergence. Since the guiding force attracts the objects to each other, the algorithm performance falls if the objects converged to a non-optimal solution. For this reason, it is essential to create some operators that prevent the premature convergence and leads to balance between the exploration and the exploitation. In view of this, in this paper a new operator is defined to increase the ability of exploitation in unimodal problems which is improved the prior work [10]. This operator is inspired by the astronomical phenomenon and named as "the black hole" operator.

The paper is organized as follows: to make a proper background, the basic concept of the GSA is briefly explained in Section II. The concept of the black hole is given in Section III. The results of the GSA by its new operator are given in Section IV and some conclusions are drawn in Section V.

II. THE GRAVITATIONAL SEARCH ALGORITHM

In this algorithm, the search agents are the objects that can be considered as planets in a solar system. These objects are moving using Newton's law of gravitation and the movement in the exchange of information with each other.

In the GSA, each object understands the locations and situations of the other objects by the gravity force. In GSA, the position of each agent (X_i for agent i) provides a solution. At the first, the objects are placed in search space randomly. The mass value of each object is determined according to its fitness value:

$$M_i = \frac{fit_i(t) - worst(t)}{\sum_{j=1}^{N} fit_j(t) - worst(t)} \ , \qquad (1)$$

where $fit_i\ (t)$ is fitness value of object i at iteration t, $worst\ (t)$ is the worst fitness value of the objects at time t, and N is the number of objects or the size of swarm.

To compute the acceleration of an agent, total forces from a set of heavier masses that apply on it should be considered based on law of gravity which is followed by calculation of agent acceleration using law of motion as in (2). Afterward, the next velocity of an agent is calculated as a fraction of its current velocity added to its acceleration using (3). Then, its position could be calculated by using (4) as below:

$$a_i^d(t) = \sum_{j\neq i} r_j . G(t) \frac{M_j(t)}{R_{ij}(t)+\varepsilon} . (x_j^d(t) - x_i^d(t)). \quad (2)$$

Thus, we have:

$$v_i^d(t+1) = r_i * v_i^d(t) + a_i^d(t), \quad (3)$$

and consequently,

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1). \quad (4)$$

In (2), $G(t)$ is the gravity constant, ε is a very small value, and $R_{ij}(t)$ is the Euclidian distance between two agents i and j, and r_i and r_j are two random numbers in the interval [0,1] that guarantee the stochastic characteristics of the algorithm. The applications of GSA in several problems such as designing of IIR filters, pattern recognition, solving the TSP, multi-objective problems and so on are given in [11-15]. In [16], Sarafrazi and Nezamabadi-pour introduced the "Disruption" as a new operator in the GSA. This operator can give the GSA the ability to more explore the search space and also exploit the good solutions. The pseudo code of the GSA is presented in Fig.1.

III. The Nature of Black Holes

There are many wonderful phenomena in our world. One natural phenomenon in the world of stars is the black hole. In 1783, John Michell, an English clergyman and amateur, considered the applications of Newton's corpuscular theory of light. If light indeed is a stream of particles, it should be influenced by gravity. In particular, he conjectured that the gravity of a star 500 times larger than the Sun, but with the Sun's average density, would be sufficiently strong that even light could not escape from it.

In 1939, American physicists, J. Robert Oppenheimer and Hartland Snyder, described the ultimate gravitational collapse of a massive star that had exhausted its sources of nuclear fusion[18]. In this situation, the star is called "black hole". A black hole is a region of spacetime where prevents any thing, from escaping.

There is a mathematically defined surface around of a black hole called "event horizon". This distance is measured by:

$$R_s = \frac{2GM}{c^2}, \quad (5)$$

where M is the mass of black hole, G is the constant gravity, c is the velocity of light and R_s is called "Schwarzschild radius".

The spherical surface at $r = R_s$ acts as a barrier and prevents our receiving any information from within. For this reason, a star that has collapsed down within the Schwarzschild radius is called a black hole [18].

The objects that are close to the black hole are experiencing very strong forces known as the tidal forces. These forces cause objects to be fragmented and collapsed into the black hole. According to [17], the following equation could explain the movement of the particle which has fallen in the black hole:

$$\frac{dt}{dr} = -\frac{2M}{r-2M}. \quad (6)$$

Thus, we have by integration:

$$t = -2Ml\,n\left(\frac{r}{2M} - 1\right) + const, \quad (7)$$

where t is time and r is distance of the object to black hole. Then, we have:

$$r = 2M(1 + \exp\left(-\frac{t-const}{2M}\right)). \quad (8)$$

IV. Simulation of the Black Hole as a Gravitational Operator

As it is mentioned, a black hole operator is inserted in standard GSA to resolve unimodal problems. We assume some of the heavy objects are as stars in a gravitational system, which become black holes [17]. Each of the black holes has a radius that calculated by the following equation:

$$R_s = M.log(t), \quad (9)$$

Where M is the mass of black hole, and t is the number of iteration algorithm.

In duration of algorithm, the radius of the black hole becomes larger and more objects are encompassed and helps to prevent premature convergence. Then, some of the best objects become black hole, affecting other objects by their strong gravity. The other objects are divided into two categories: heavy agents and light agents. The position of heavy agents and light agents change with (10) and (11), respectively. These influences depend on the distance of the black hole to other masses. This distance is compared with radius R_s, so that if the object is close enough to the black hole, (10) and (11) are simulated according to (8) as below:

$$x_i^d(t+1) \leftarrow \begin{cases} x_i^d(t+1).*\,(1 + exp(-\frac{t}{2M})) & (10) \\ x_i^d(t+1).*\,R(\frac{1}{1+log(t)}) & (11) \end{cases}$$

The pseudo code of the proposed gravitational search algorithm is given by Fig. 2.

1. Search space identification, t=0;
2. Random initialization, $X_i(t)$;
   For $i=1, \ldots, N$
3. Fitness evaluation of objects;
4. Update the parameters of *G, best, worst and M;*
   For $i=1, \ldots, N$
5. Calculation of the force on each object;
6. Calculation of the acceleration and the velocity of each object;
7. Update the position of the agents by (4) to yield $X_i(t+1)$;
   $t=t+1$;
8. Repeat steps 3 to 7 until the stop criteria is reached;
9. end

Figure1. Pseudo code for standard GSA (SGSA).

1. Search space identification, t=0;
2. Random initialization, $X_i(t)$;
   For $i = 1, \ldots, N$
3. Fitness evaluation of objects;
4. Update the parameters of *G, best, worst and M;*
   For $i = 1, \ldots, N$
5. Calculation of the force on each object;
6. Calculation of the acceleration and the velocity of each object;
7. Appling (4) to obtain position of agents;
8. Update the position of the agents by (10, 11) to yield $X_i(t+1)$;
   $t=t+1$;
9. Repeat steps 3 to 8 until the stop criteria is reached;
10. end

Figure2. The Pseudo code for the proposed GSA (IGSA).

V. EXPERIMENTAL RESULTS

Seven benchmark unimodal functions are used to evaluate the proposed method. These functions are given in Table I. The first step to implement the proposed GSA, is generating the initial population (*N=50* objects) where *N* is considered to be population size. Dimension is 30 ($n = 30$) and maximum iteration is 1000 for functions.

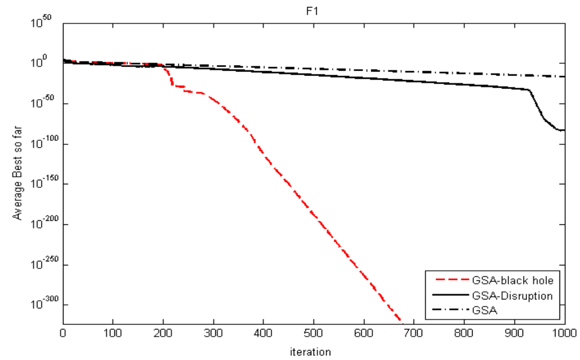
The results obtained are averaged over 25 runs and the average best-so-far is reported. The performance of the proposed algorithm is shown through Table II and Fig. 3, where it is compared with standard GSA (SGSA) and disruption GSA.

The results show the significant performance of the proposed GSA comparing to SGSA and disruption GSA. This is due to black hole operator that provides a better exploitation for the algorithm and prevents premature convergence.

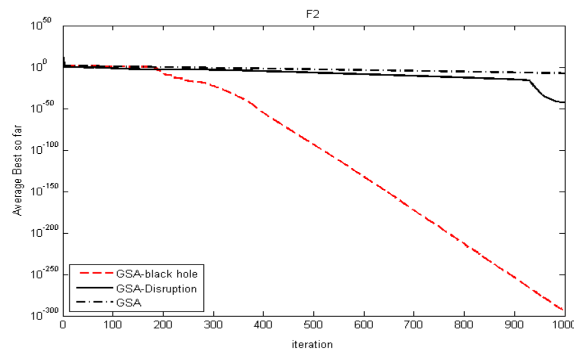
As shown in Table II, the black hole algorithm has efficiently outperformed the GSA and the Disruption GSA algorithms in the cases of functions F1, F2, F3 and F4. This is mostly due to the fact that the feature of this algorithm is its high accuracy in finding the optimal solution. Disruption GSA algorithm performs better than the black hole and GSA algorithms on F5 due to its strength in the exploration of search space.

TABLE I. UNIMODAL TEST FUNCTIONS.

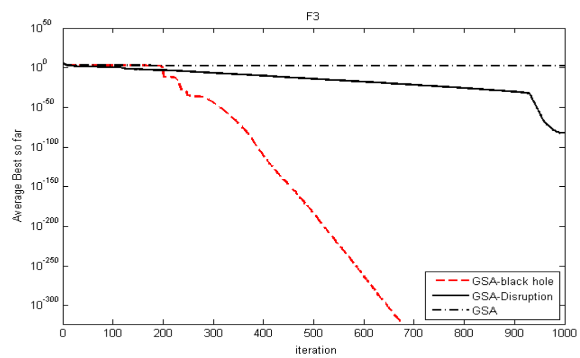
Test function	***S***	f_{opt}
$F_1(x) = \sum_{i=1}^{n} x_i^2$	$[-100,100]^n$	0
$F_2(x) = \sum_{i=1}^{n}\lvert x_i\rvert + \prod_{i=1}^{n}\lvert x_i\rvert$	$[-10,10]^n$	0
$F_3(x) = \sum_{i=1}^{n}(\sum_{j=1}^{i} x_j)^2$	$[-100,100]^n$	0
$F_4(x) = \max_i\{\lvert x_i\rvert, 1 \le i \le n\}$	$[-100,100]^n$	0
$F_5(x) = \sum_{i=1}^{n-1}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-30,30]^n$	0
$F_6(x) = \sum_{i=1}^{n}([x_i + 0.5])^2$	$[-100,100]^n$	0
$F_7(x) = \sum_{i=1}^{n} ix_i^4 + random[0,1)$	$[-1.28,1.28]^n$	0



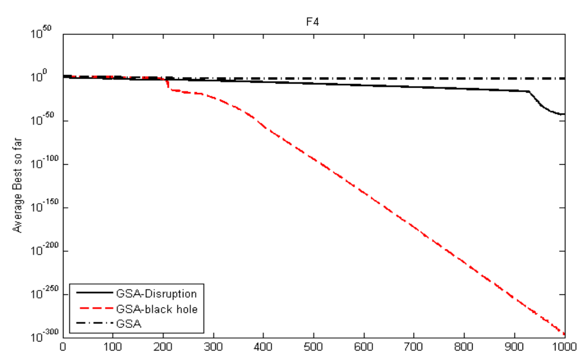
(a)



(b)



(c)



(d)

Figure 3. Comparison of performance of Black hole GSA, Disruption GSA and GSA for minimization of F1-F4.

TABLE II. THE RESULTS OF UNIMODAL FUNCTIONS.

Function	Average best so far		
	SGSA	***Disruption***	***Black hole***
F1	0.8×10-16	1.28×10-54	0
F2	4.5×10-8	2.31×10-43	0
F3	250.5	3.78×10-82	0
F4	3.42×10−9	2.32×10−43	0
F5	29.76	0.6714	27.31
F6	2.07×10−17	0.0055	0
F7	0.0165	2.55×10−5	3.37×10−5

VI. CONCLUSION

Nowadays the use of heuristic algorithms has been widely developed to solve different optimization problems. Most of these algorithms are inspired by physical phenomena and the natural behavior of organisms. GSA is a new heuristic algorithm has been created that utilizes the law of gravity and motion. In this paper, the GSA is improved by adding a new operator which is based on the astronomical phenomenon. This operator is called “the black hole”. The effectiveness of the algorithm is shown by solving unimodal problems.

Currently, the authors are working on algorithm to be suitable in solving multimodal functions as well.

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