Fuzzy Gravitational Search Algorithm

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Abstract—**One of the noticeable topics in fuzzy logic controllers is parameter controlling of heuristic search algorithms. In this paper, one of the parameters of Gravitational Search Algorithm, GSA, is controlled using fuzzy logic controller to achieve better optimization results and to increase convergence rate. Several experiments are performed and results are compared with the results of the original GSA. Experimental results confirm the efficiency of the proposed method.**

Keywords- Gravitational search algorithm; Fuzzy logic controler; Fuzzy rules; Exploration; Exploitation.

I. INTRODUCTION

In recent years, many complicated nonlinear controllers such as adaptive control [1], neural control [2], and fuzzy control [3] have been proposed for controlling systems. Fuzzy logic controller (FLC) is one of the common tools used for parameter controlling.

Fuzzy theory was proposed by Zadeh in 1965 [4]. Major reasons for development of fuzzy theory can be briefly stated by two reasons. Firstly, fuzzy logic can make the difficulties of developing and analyzing complex systems easier. Secondly, fuzzy logic is capable of making human inference from not well-defined concepts/knowledge. The basic parts of a fuzzy logic controller are knowledge base, fuzzy rules, and inference engine [5].

In this paper, FLC is used to control the parameter of a heuristic search algorithm. Today, there is a great interest to control heuristic search algorithms using FLC. For example, Ref [6] proposes an improved genetic algorithm (GA) with two fuzzy controllers based on some heuristics to adaptively adjust the crossover probability and mutation rate during the optimization process. In [7] a fuzzy control method is mixed with GA to adjust the probabilities of crossover and mutation. In [8], Fuzzy logic based controllers are utilized for tuning the crossover and mutation probability of GA to improve the algorithm performance. In [9], an FLC is used to control the values of cross over rate in GA.

In [10], an adaptive fuzzy particle swarm optimization (AFPSO) algorithm utilizes fuzzy set theory to adjust PSO acceleration coefficients adaptively. In [11], a fuzzy ant colony optimization (FACO) is proposed and an FLC is used to adapt the evaporated and deposited value of pheromone trail based on the ant's fitness and pheromone trail age. In [12], fuzzy logic is used to combine the results of the PSO and GA in the best possible way.

Today, with enlargement issues and the importance of responding quickly, classical methods are not able to resolve many issues. Therefore, heuristic random-based search algorithms are used instead. Heuristic search algorithms are inspired from nature or physical processes. The use of these algorithms has grown dramatically in recent years.

Gravitational Search Algorithm (GSA) is proposed motivated by gravitational law and laws of motion [13]. This algorithm has high performances in solving various optimization problems and examined in various problems like filter modeling [14], web services composition[15], parameters identification of hydraulic turbine [16], solving Wessinger's Equation [17], synthesis of ring array antenna [18], classifier design [19], [20], and Multi-objective optimization [21].

By controlling the parameters of GSA, we can control the balance between the power of exploration and exploitation to get better results with less iterations of the algorithm. If there is no well-established balance between exploration and exploitation, the algorithm is not enough efficient. In this paper, fuzzy logic controller is utilized to increase convergence rate and ensure of finding optimum solution.

In the next section, gravitational search algorithm is reviewed. In Section 3, the proposed FGSA is described in detail. Experimental results are in section 4 and finally the paper is concluded in section 5.

II. GRAVITATIONAL SEARCH ALGORITHM

GSA is a heuristic algorithm that has been inspired by the Newtonian laws of gravity and motion [13]. In GSA, a set of agents called objects are introduced to find the optimum solution by simulation of Newtonian laws of gravity and motion [13].

To describe the GSA, consider a system with s objects in which the position of the i^{th} object is defined as follows:

$$X_i = (x_i^1, \ldots, x_i^d, \ldots, x_i^m) \quad , i = 1,2,\ldots,s \tag{1}$$

where x_i^d is the position of i^{th} object in the d^{th} dimension, m is dimension of the search space and s is the population size. Based on [13], the mass of each agent is calculated after computing current population's fitness as follows:

$$q_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \quad (2)$$

$$M_i(t) = \frac{q_i(t)}{\sum_{j=1}^{s} q_j(t)} \quad (3)$$

where $M_i(t)$ and $fit_i(t)$ represent the mass and the fitness value of the agent i at iteration t, and, $worst(t)$ and $best(t)$ are defined as follows (for a minimization problem):

$$best(t) = \min_{j\in\{1,..,s\}} fit_j(t) \quad (4)$$

$$worst(t) = \max_{j\in\{1,..,s\}} fit_j(t) \quad (5)$$

To compute the acceleration of an agent, total force from a set of heavier objects that apply on it should be considered based on law of gravity as Eq. (6) which is followed by calculation of agent acceleration using law of motion as Eq. (7). Afterward, the next velocity of an agent is calculated as a fraction of its current velocity added to its acceleration as Eq. (8). Then, its position could be calculated using Eq. (9).

$$F_i^d(t) = \sum_{j\in kbest, j\neq i} rand_j^d G(t) \frac{M_j(t) M_i(t)}{R_{ij}(t)^{rPower} + \varepsilon} (x_j^d(t) - x_i^d(t)) \quad (6)$$

$$a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} = \sum_{j\in kbest, j\neq i} rand_j^d G(t) \frac{M_j(t)}{R_{ij}(t)^{rPower} + \varepsilon} (x_j^d(t) - x_i^d(t)) \quad (7)$$

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \quad (8)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (9)$$

where $rand_i$ and $rand_j$ are two uniform random in the interval [0, 1], ε is a small value, and $R_{ij}(t)$ is the Euclidian distance between two agents i and j defined as $R_{ij}(t) = \left\| X_i(t), X_j(t) \right\|_2$. In this paper, the value of *rPower* is considered to be 1. *kbest* is the set of first K agents with the best fitness value and biggest mass. *kbest* is a function of time, initialized to K_0 at the beginning and decreasing with time. Here, K_0 is set to s (total number of agents) and is decreased linearly to 1. In GSA, the gravitational constant (G) takes an initial value of G_0 and it will be reduced by time (Eq. (10)). T is the total age of the system (maximum number of iterations).

$$G(t) = G_0 e^{-\alpha \frac{t}{T}} \quad (10)$$

In this paper, a fuzzy logic controller is designed to control the convergence speed of a heuristic search algorithm. The proposed approach attempts to use some measures of population diversity and population progress [22-24].

III. FUZZY GRAVITATIONAL SEARCH ALGORITHM

Till now, many works have been reported that used FLC to control the parameters of heuristic search algorithms [6-12]. Increasing convergence speed and guarantee of finding the approximate optimal solution are two important issues in heuristic algorithms. One of common ways to achieve this desire is to establish the tradeoff between the exploration and exploitation of the algorithm by parameter controlling. In this paper, due to the simplicity and intelligibility of FLC, it is used for controlling the parameters of GSA.

In GSA, the parameter of α in Eq. (10) plays an important role in increasing or decreasing the power of exploration and exploitation. The amount of gravitational constant, G, depends on the parameter of α. The value of G has a great effect in the acceleration of the agents. By controlling the acceleration and velocity of agents, we can increase or decrease diversity and strengthen or weaken the power of exploration and exploitation. Our purpose is to achieve better results by controlling the parameter of α during iterations.

A. Population diversity and population progress

The most effective tools for monitoring the convergence rate of the algorithm are the measurements of population diversity and progress. In this paper, ED is the parameter that is considered to measure the population diversity. ED is calculated using (11).

$$ED = \frac{R^{ave} - R^{\min}}{R^{\max} - R^{\min}} \quad (11)$$

where R^{ave}, R^{max} and R^{min} are the average, maximum and minimum distance between agents and best result respectively.

Beside ED, the parameter of CM is considered for measuring the population progress. In minimization problems, CM is calculated using (12).

$$CM = \frac{fit^{ave}(t-1) - fit^{ave}(t)}{fit^{ave}(t)} \quad (12)$$

If ED approaches to zero, the population has a low level of diversity and if CM approaches to zero, the population has low level of progress.

B. Membership functions

To design an FLC, we use a fuzzy inference system which takes four input variables and produces one output variables: IT, CM, ED, and α(t-1) are the input variables and α(t) is the output variable. The variable of IT is the current iteration number.

When we speak of population diversity in terms of low, medium, or high diversity, we can see that the language becomes a fuzzy variable whose spatial denotation is imprecise. Fuzzy theory can be made to resemble a high level language instead of a mathematical language [25]. To describe this, fuzzy sets with names such as Low, Medium, or High are used to create membership functions. In most cases, membership functions are designed by experts.

In this paper, with regards to IT, CM, ED, α(t-1) and α(t), five membership functions are defined for inputs and output. The set of linguistic labels associated with IT, α(t-1), and α(t) are Low, Medium, and High and those related to CM and ED are Low and High. The membership functions of these labels are illustrated in Figure 1. ED and CM belong to the interval [0,1], IT belongs to the interval [1,1000], and α belongs to the interval [29,31]. The limited values for the parameter of α are determined empirically.

C. Fuzzy rules

In this subsection, the proposed rules in the fuzzy logic controller are explained. The proposed fuzzy system with four input variables such as described in previous subsection, will have (3×2×2×3) =36 rules. Some of proposed fuzzy rules are

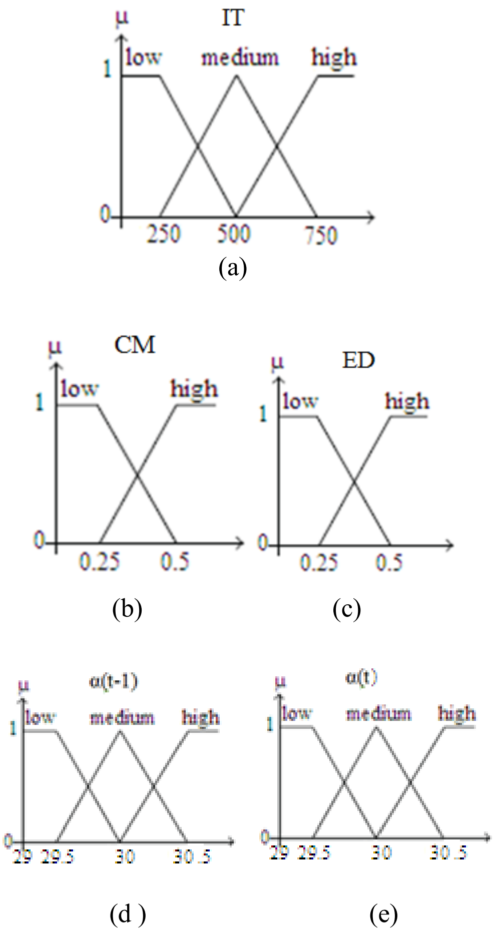

Figure 1. membership functions of a) the first input(IT/iteration), b) the second input (CM) , c) the third input (ED), d) the fourth input (α(t-1)) , and e) the output (α(t)).

presented in Table 1. These rules are designed to prevent algorithm from trapping into local optima and premature convergence. Furthermore, these rules increase the convergence rate after finding approximate optimal solution. Each of these rules targeted and designed by some knowledge.

In Table 1, the first rule has been generated based on this knowledge:

"If there is lack of improvement at the beginning of the algorithm, then decrease α".

Early lack of improvement is a sign of being trapped in local optimum and premature convergence. Through this rule, we can prevent the premature convergence. Notice that by decreasing the parameter of α, the value of G and as a result the acceleration and the velocity of agents is increased. By increasing velocity, the population can escape from local optima.

About the second rule, if algorithm is at the middle of the program and algorithm's diversity is low, particles should be distributed by decreasing α to improve diversity and hope to achieve better results.

The third rule has been generated based on this knowledge:

"If the algorithm is under progress in last iterations, then increase α".

This knowledge insisted that at last iterations, if algorithm is in a natural way in solving the problem, the algorithm's power of exploitation should be strengthened. By increasing α, the value of G is decreased that leads to increase the power of exploitation of the algorithm.

About rule number fourth, if the algorithm is at last iterations and CM is low (lack of improvement in the algorithm) and ED is high (diversity is enormous), it could be argued that the algorithm has failed to reach convergence. Therefore α is likely to be increased. By so doing, convergence rate is increased.

In this paper, inputs are combined logically using the AND operator and to obtain a crisp decision from, the centre of gravity approach is used for defuzzification.

IV. EXPERIMENTAL RESULTS

The proposed algorithm is tested in minimization of some standard functions including some unimodal and some

TABLE I. SOME OF FUZZY RULES FOR CONTROLLING THE PARAMETER OF $\alpha(t)$.

Rule		IT	CM	ED	$\alpha(t-1)$		α(t)
1	IF	low	high	low	medium	Then	low
2		medium	low	low	high		medium
3		high	high	high	low		medium
4		high	low	high	medium		high

multimodal functions that are in table 2 [13]. In these tables, the dimension of search space is 30 (m = 30), except for F_6 and F_7 with m=2.
The minimum values of the functions of Tables 2 are zero, except for F_6 which has a minimum value of 1, and F_7 which has a minimum value of 0.398 [13].

Parameters of GSA are set as [13], in GSA, the value of G_0 is set to 100 and α is set to 20. The population size is 50 and the maximum number of iterations is 1000. In fuzzy GSA, the value of G_0 is set to 100.

Fuzzy logic controller is applied on the gravitational search algorithm to solve some standard functions. The results of applying these methods are shown in table 3 and Figs. 1 and 2. What is seen in these diagrams is significant improvement of the proposed fuzzy GSA.

TABEL II. TEST FUNCTIONS

Test Function	Bounds
$F_1(X)=\sum_{i=1}^{m} x_i^2$	$[-100,100]^m$
$F_2(X)=\sum_{i=1}^{m}\lvert x_i\rvert+\prod_{i=1}^{m}\lvert x_i\rvert$	$[-10,10]^m$
$F3(X)=-20\exp(-0.2\frac{1}{m}\sqrt{\sum_{i=1}^{m}x_i^2})$ $-\exp\frac{1}{m}\sum_{i=1}^{m}\cos(2\pi x_i))+20+e$	$[-32,32]^m$
$F_4(X)=\max_i\{\lvert x_i\rvert,1\le i\le m\}$	$[-100,100]^m$
$F_5(X)=\frac{\pi}{m}\{10\sin(\pi y_1)+\sum_{i=1}^{m-1}(y_i-1)^2$ $[1+10\sin^2(\pi y_{i+1})]+(y_m-1)^2\}+$ $\sum_{i=1}^{m}u(x_i,10,100,4)$ $y_i=1+\frac{x_i+1}{4}$	$[-50,50]^m$
$F_6(X)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}\right)^{-1}$	$[-65.53,65.53]^2$
$F_7(X)=(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6)^2+$ $10(1-\frac{1}{8\pi})\cos x_1+10$	$[-5.10]\times[0.15]$

TABLE III. MINIMIZATION RESULTS OF BENCHMARK FUNCTIONS IN TABLE II WITH m =30. MAXIMUM NUMBER OF ITERATIONS=1000.

		GSA	Fuzzy GSA
F_1	Average best so far	7.3×10-11	2.2×10-21
	Median best so far	7.1×10-11	2.1×10-21
F_2	Average best so far	4.03×10-5	2.5×10-11
	Median best so far	4.07×10-5	2.5×10-11
F_3	Average best so far	6.9×10-6	1.1×10-11
	Median best so far	6.9×10-6	1.1×10-11
F_4	Average best so far	3.7×10-6	4.8×10-11
	Median best so far	3.7×10-6	4.5×10-11
F_5	Average best so far	0.01	0.0035
	Median best so far	4.2 ×10-13	3.8×10-24
F_6	Average best so far	3.70	1.19
	Median best so far	2.07	1.016
F_7	Average best so far	0.3979	0.3979
	Median best so far	0.3979	0.3979

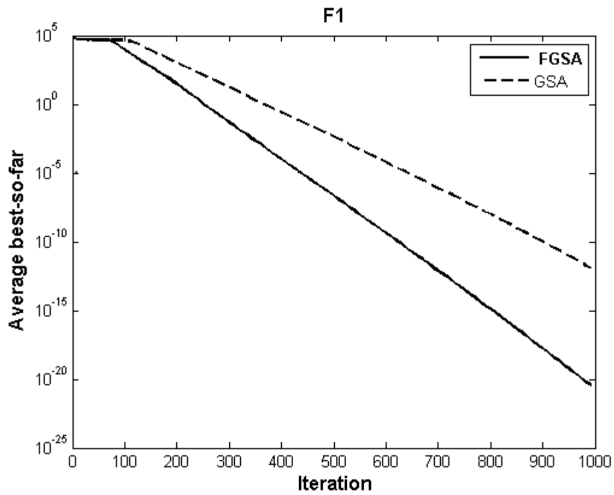

Figure 1. Performance comparison of GSA and FGSA for minimization of F1 with m= 30.

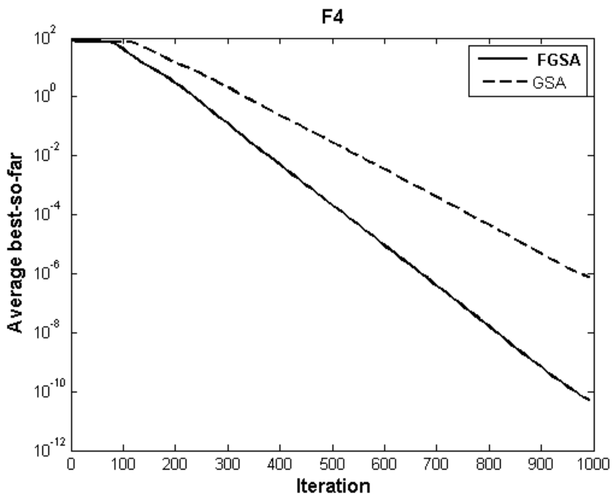

Figure 2. Performance comparison of GSA and FGSA for minimization of F4 with m= 30.

V. CONCLUSION

One of the noticeable topics in the fuzzy logic controllers is the control of the parameters of heuristic search algorithms. In this paper, the gravitational constant is controlled in gravitational search algorithm using fuzzy logic controller to get better results in solving optimization problems. The proposed method examined in solving some standard functions. Experimental results confirm good performance of the proposed method.

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