The 9th Seminar on Commutative Algebra and Related Topics
Ferdowsi University of Mashhad, November 7-8, 2012

Bounds for the regularity of local cohomology of bigraded modules

Ahad Rahimi
Department of Mathematics,
Razi University, Kermanshah, Iran
ahad.rahimi@razi.ac.ir

Abstract

Let K be a field and $S = K[x_1, \ldots, x_m, y_1, \ldots, y_n]$ be the polynomial ring in the variables $x_1, \ldots, x_m, y_1, \ldots, y_n$. We consider S to be a standard bigraded K-algebra with $\deg x_i = (1, 0)$ and $\deg y_j = (0, 1)$ for all i and j. Let $I \subset S$ be a bigraded ideal and set $R = S/I$ and $Q = (y_1, \ldots, y_n)$. For all integers j we set $H^n_Q(R)_j = \bigoplus_i H^n_Q(R)_{(i,j)}$. Notice that $H^n_Q(R)_j$ is a finitely generated graded S_0-module where S_0 is the polynomial ring $K[x_1, \ldots, x_m]$. We prove that a certain power of the content ideal of I annihilates $H^n_Q(R)_j$, and use this fact to bound the regularity of $H^n_Q(R)_j$ in some cases. Next, we consider S as standard $\mathbb{Z}^m \times \mathbb{Z}^n$-graded K-algebra and let M be a finitely generated $\mathbb{Z}^m \times \mathbb{Z}^n$-graded S-module. Then $H^i_Q(M)_j$ are finitely generated $\mathbb{Z}^n$-graded S_0-modules. We show that there exists an integer c such that $|\operatorname{reg} H^i_Q(M)_j| \leq c$ for all i and all j. In particular, if $I \subseteq S$ is a monomial ideal, then there exists an integer c such that $|\operatorname{reg} H^i_Q(S/I)_j| \leq c$ for all i and all j.

Joint work: Jürgen Herzog.