Solving a fuzzy fixed charge solid transportation problem by two meta-heuristic algorithms

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Abstract

This paper considers a fixed-charge transportation problem (FCTP). In most real world application and problems, a homogeneous product is carried from an origin to a destination using different transportation modes (e.g., road, air, rail and water). This paper investigates a fixed charge solid transportation problem (FCSTP) under a fuzzy environment, in which the both direct and fixed costs are supposed to be fuzzy numbers. To solve such a hard problem, two meta-heuristic algorithms, namely imperialist competitive algorithm (ICA) and simulated annealing (SA), are utilized. To tune up their parameters, various problem sizes are generated at random and then a robust calibration is applied to the parameters using the Taguchi method. Then, computational results are presented and analyzed.

Keywords: fixed-charge transportation, fuzzy environment, meta-heuristics, Taguchi method.

1. Introduction

A fixed-charge transportation problem (FCTP) is an extension of the classical transportation problem, which was first formulated by Balinski (1961). In a transportation problem, products need to be carried from a number of sources to a number of destinations. Decisions need to be made on the amount of products transported between each two sites to reduce the total transportation cost (Steinberg, 1970; Danzig, 1968). The fixed cost has made the problem difficult to be solved. Then, various solution methods have been proposed in the optimization literature (Adlakha et al., 2006). Among these methods, a branch-and bound method is one of the most effective ones and has had many applications (Caramia & Guerriero, 2009). For example, Adlakha, et al. (2010) developed a helpful branching method for the FCTP. Their method starts with a linear formulation, and then sequentially divides the fixed cost and finds a direction to improve the advantage. The method solves the FCTP by decomposing the problem into smaller sub-problems, which can be useful to researchers solving any problem size. In the previous studies, there have been many researchers reported new methods to determine the transportation that can give the least cost. For the first time, Michalewicz et al. (1991) discussed the use of a genetic algorithm (GA) for solving transportation problems. Another GA approach for solving the solid TP was provided by Li et al. (1998). Jimenez and Verdegay (1996) investigated an interval multi-objective solid transportation problem by a GA. It has been shown that this fixed cost transportation problem is NP-hard one (Hirsch & Dantzig, 1968).

Klose (2008) showed a particular case of the FCTP, namely single source FCTP, is also NP-hard. Bit et al. (1993), Bit (2005), Li and Lai (2000), Lee and Li (1993), and Waiel (2001) presented the fuzzy compromise programming approach to a multi-objective transportation problem. Li et al. (1997) designed a neural network approach for the multi-criteria STP. They also presented an improved GA to solve a multi-objective STP with fuzzy numbers in their paper written in the same year (Li et al.. 1997). Yang and Liu (2007) presented a hybrid algorithm that is designed based on the fuzzy simulation technique and tabu search (TS) algorithm for the fuzzy fixed charge STP (FFCSTP). In this paper, we consider the fuzzy fixed charge solid transportation problem (FFCSTP). Up to now, no one has considered the imperialist competitive algorithm (ICA) for any kind of STPs. We develop and use ICA and simulated annealing (SA) for solving the STP for the first time. Besides, to find the best solution, various new neighborhood structures related to the problem are firstly being proposed and used in this paper.

The rest of this paper is organized as follows. In Section 2, some basic knowledge of integral value fuzzy transportation costs are reviewed. Section 3 describes the mathematical model and formulation of the FCTP. Section 4 presents two meta-heuristic algorithms. Section 5, describes the experimental design and compares the computational results. Finally Section 6 provides the conclusion.

2. Preliminaries

The fuzzy logic concept was conceived by Lotfi Zadeh (1973) and presented not as a control methodology, but as a way of processing data by allowing partial set membership rather than a crisp set membership or non-membership. In conventional FCTPs, it is assumed that the decision maker (DM) is sure about the precise values of the transportation cost, fixed cost, and supply and demand of the product. Since this is not a realistic assumption, it seems necessary to use fuzzy numbers and fuzzy variables further in order to make the FCTP more applicable in the real world. Here, we assume that all of the parameters (i.e., transportation cost from the *i*-th source to the *j*-th destination, fixed cost to open a route (i, j), supply of the product at the *i*-th source, and demand of the product at the *j*-th destination) are not deterministic numbers, however, they are triangular fuzzy numbers (TFNs). So, the total transportation costs become fuzzy as well. The fuzzy fixed charge transportation problem (FFCTP) is presented in the following mathematical form:

The triangular fuzzy number is the fuzzy number with the membership function $\mu_{\tilde{A}}(x)$, a continuous mapping: $\mu_{\tilde{A}}(x)$: R → [0, 1]

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & -\infty < x < a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0 & a_3 < x < \infty \end{cases} \tag{1}$$

The corresponding inverse functions of $\mu_{\tilde{A}}(x)$ can be expressed by:

$$g_{\tilde{A}}(y)^L = a_1 + (a_2 - a_1)y \tag{2}$$

$$g_{\tilde{A}}(y)^R = a_3 - (a_3 - a_2)y = a_3 + (a_2 - a_3)y \tag{3}$$

where $y \in [0, 1]$. Thus, the left and right integral values are formulated as follows:

$$I(\tilde{A})^L = \int_0^1 g_{\tilde{A}}(y)^L dy = \frac{1}{2}(a_1 + a_2) \tag{4}$$

$$I(\tilde{A})^R = \int_0^1 g_{\tilde{A}}(y)^R dy = \frac{1}{2}(a_2 + a_3) \tag{5}$$

The total integral value of the TFN $\tilde{A} = (a_1, a_2, a_3)$ is:

$$I_T^{\alpha}(\tilde{A}) = \alpha\, I(\tilde{A})^R + (1-\alpha)I(\tilde{A})^L = \frac{1}{2}(\alpha\, a_3 + a_2 + (1-\alpha)\, a_1) \tag{6}$$

where a degree of optimism $\alpha \in [0, 1]$ is given. When the decision degree of optimism α is 0.5, the above integral value is the same as ordinary representatives (Kaufmann and Gupta, 1988).

3. Problem Formulation and Description

In this section, we provide a mathematical programming model for the fixed charge STP (FCSTP) problem. The aim is to determine in which routes are to be opened and the size of the shipment on those routes using conveyances in such a way that the total cost of the met demand is minimized while satisfying the supply and shipment capacity constraints. In this model, there are *m* suppliers, *n* customers and *K* conveyances with particular demands. Each of *m* suppliers can ship demands to any of *n* customers using any of *K* conveyances at a shipping cost per unit c_{ijk} (i.e., unit cost for shipping from supplier *i* to customer *j* by means of the *k*-th conveyance) plus a fixed cost f_{ijk}, assumed for opening this route. The mathematical model is presented as follows.

$$\text{Min } Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}(c_{ijk}x_{ijk} + f_{ijk}y_{ijk}) \tag{7}$$

s.t.

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le a_i \qquad i = 1, \dots, m \tag{8}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge b_j \qquad j = 1, \dots, n \tag{9}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} x_{ijk} \le e_K \qquad k = 1, \dots, K \tag{10}$$

$$x_{ijk} \ge 0 \qquad \forall i, j, \mathrm{k} \tag{11}$$

$$y_{ijk} = 0 \qquad if\ x_{ijk} = 0 \tag{12}$$

$$y_{ijk} = 1 \qquad if\ x_{ijk} > 0 \tag{13}$$

where x_{ijk} is the unknown quantity to be transported on the route (i, j) that from plant i to consumer j by means of conveyance k, c_{ijk} is the shipping cost per unit from plant i to consumer j by means of conveyance k. a_i is the number of units available at plant i, b_j is the number of units demanded at consumer j, and e_k is the unit of this product called conveyances that can be carried by K different transportation modes. The transportation cost for shipping per unit from plant i to consumer j using conveyance k is $c_{ijk} \times x_{ijk}.f_{ijk}$ is the fixed cost associated, respectively. In this paper, we assume a balanced transportation problem because the unbalanced transportation problem can be converted to a balanced transportation problem by introducing a dummy plant, dummy consumer or dummy conveyance. When the c_{ijk} and f_{ijk} are imprecise, the objective function of the considered FFCSTP can be shown below:

$$\text{Min } \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (\tilde{c}_{ijk} x_{ijk} + \tilde{f}_{ijk} y_{ijk}) \quad (14)$$

Since three-point fuzzy numbers (i.e., triangular) are used to represent the fuzzy transportation cost, the total transportation cost will also be three-point fuzzy number. Consider a simple example of two suppliers (a_1 = 100, a_2 = 200), two customers (b_1 = 200, b_2 = 100) and two conveyances (e_1 = 150, e_2 = 150).

4. Proposed Meta-heuristics

4.1. Simulated annealing

Simulated annealing (SA) is a popular local search algorithm for solving combinatorial optimization problems. It was originally inspired from the process of annealing in metal work. Annealing involves heating and cooling a material to alter its physical properties due to the changes in its internal structure. As the metal cools its new structure becomes fixed, consequently causing the metal to retain its newly obtained properties. In simulated annealing we keep a temperature variable to simulate this heating process. We initially set it high and then allow it to slowly cool as the algorithm runs. While this temperature variable is high the algorithm will be allowed, with more frequency, to accept solutions that are worse than our current solution. This gives the algorithm the ability to jump out of any local optimums it finds itself in early on in execution. We check if the neighbor solution is better than our current solution. If it is, we accept it unconditionally. However, if the neighbor solution is not better the current solution, we need to consider a couple of factors. For better optimization, when initializing the temperature variable we should select a temperature that will initially allow for practically any move against the current solution. This gives the algorithm the ability to better explore the entire search space before cooling and settling in a more focused region.

4.2. Imperialist competitive algorithm

An imperialist competitive algorithm (ICA) is proposed by Atashpaz and Lucas (2008) who showed the algorithms capability in dealing with different types of optimization problems (Atashpaz et al. 2008). This algorithm starts with an initial population. Each population in the ICA is called country. The algorithm starts with N initial countries and the $Nimp$ best of them (i.e., countries with minimum cost) chosen as the imperialists. The remaining countries are colonies, in which each of them is belong to an empire. The initial colonies are belong to imperialists in convenience with their powers. The imperialist countries absorb the colonies towards themselves using the absorption policy. The absorption policy shown in Fig. 1 makes the main core of this algorithm and causes the countries move towards to their minimum optima. The imperialists absorb these colonies towards themselves with respect to their power that described in Eq. (15). The total power of each imperialist is determined by the power of its both terms (i.e., the empire power plus percent of its average colonies power).

$$TC_n = Cost\ imperialist_n + Cost\ colonies\ of\ empire_n \quad (15)$$

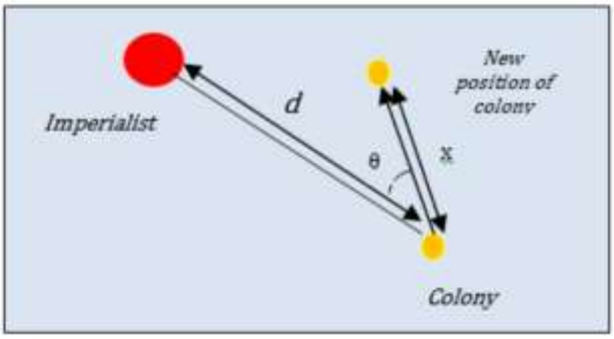

Figure 1. Moving colonies toward their imperialist.

5. Experimental Design

5.1. Data generation

The effectiveness of the ICA and SA algorithms greatly depends on the correct choice of parameters and operators (Ruiz & Maroto, 2006). In this section, we study the behavior of the different parameters and operators of the both proposed algorithms. In order to calibrate the parameters of algorithms, there are several ways to design the experimental investigation. One of these methods used in the most studies is the full factorial design. The more the number of required experiments for tuning are, the more the time and cost will be spent. As it will be explained clearly later, for the SA there are nine test problems, three 3-level factors in our case that each of them should be run three times. So, the total number of running the problem in SA is $9 \times 3^3 \times 3$, which is equal to 729. In the ICA, there are 27 test problems, five 3-level factors in our case that each of which should be run three times. Hence, the total number of running the problem in ICA is $27 \times 5^3 \times 3$, which is equal to 6075. In the related studies, to be more efficient, several experimental designs have been proposed to decrease the number of experiments. Among several experimental design methods, the Taguchi experimental design method has been successfully employed for a systematic approach for optimization (Phadke, 1989). In the Taguchi method, the orthogonal arrays are used to analyze a large number of decision variables with a small number of experiments.

Taguchi has created a transformation of the repetition data to another value, which is the measure of variation. The transformation is the signal-to-noise (S/N) ratio that explains why this type of the parameter design is called robust design. The aim is to maximize the (S/N) ratio. In the Taguchi method, the (S/N) ratio of the minimization objectives is as such (Molla-Alizadeh-Zavardehi et al., 2010):

$$(S/N) \text{ ratio} = -10 \log10 \text{ (objective function)}^2 \quad (16)$$

The control factors of SA are T_0, β, N_{max} and the control factors of the ICA are N_{max}, N_{count}, N_{imp}, β, $epsilon$. Levels of these factors are illustrated in Table 1.

Table 1. Factors of the SA and ICA.

Factors	SA Levels	ICA Levels
T0	3500	
	4500	
	5000	
β	0.91	0.3
	0.94	0.8
	0.96	0.9
Nmax	750	50
	800	100
	950	200
Ncount		40
		120
		200
Nimp		5
		15
		25
epsilon		0.05
		0.1
		0.4

Tables 2 and 3 show the modified orthogonal array L18 for SA and Modified orthogonal array L27 for ICA, respectively. Because the scale of objective functions in each instance is different, they cannot be used directly. To solve this problem, the relative percentage deviation (RPD) is used for each instance, which is obtained by:

$$RPD = \frac{Alg_{sol} - min_{sol}}{min_{sol}} \times 100 \tag{17}$$

Table 2. Modified orthogonal array L18 for SA.

	T0	Beta	Nmax
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

Table 3. Modified orthogonal array L27 for ICA.

	Nmax	Ncount	Nimp	Beta	Epsilon
1	1	1	1	1	1
2	1	1	1	1	2
3	1	1	1	1	3
4	1	2	2	2	1
5	1	2	2	2	2
6	1	2	2	2	3
7	1	3	3	3	1
8	1	3	3	3	2
9	1	3	3	3	3
10	2	1	2	3	1
11	2	1	2	3	2
12	2	1	2	3	3
13	2	2	3	1	1
14	2	2	3	1	2
15	2	2	3	1	3
16	2	3	1	2	1
17	2	3	1	2	2
18	2	3	1	2	3
19	3	1	3	2	1
20	3	1	3	2	2
21	3	1	3	2	3
22	3	2	1	3	1
23	3	2	1	3	2
24	3	2	1	3	3
25	3	3	2	1	1
26	3	3	2	1	2
27	3	3	2	1	3

5.2. Parameter tuning

In order to present the efficiency of the proposed SA and ICA algorithms for solving the given problem, a plan is utilized to generate the test data consisting of the number of suppliers, customers and conveyances, total demand, and range of variable costs and fixed costs (Hajiaghaei-Keshteli, 2011). The (S/N) ratio and RPD of both algorithms are shown in Figures 2 to 5.

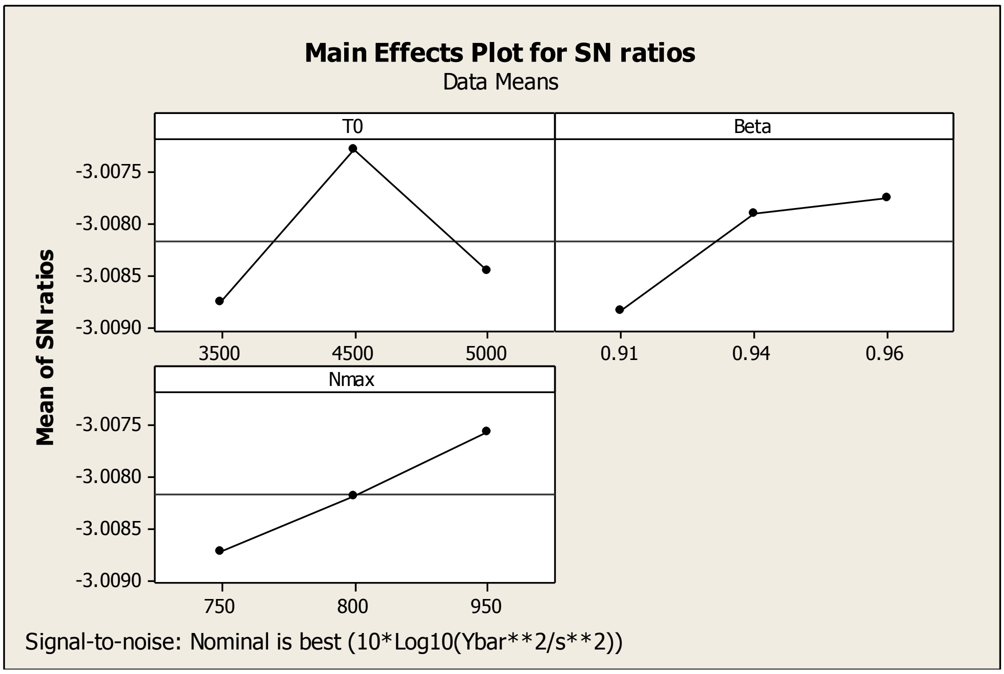

Figure 2. ***S/N*** **ratio ratios for SA**

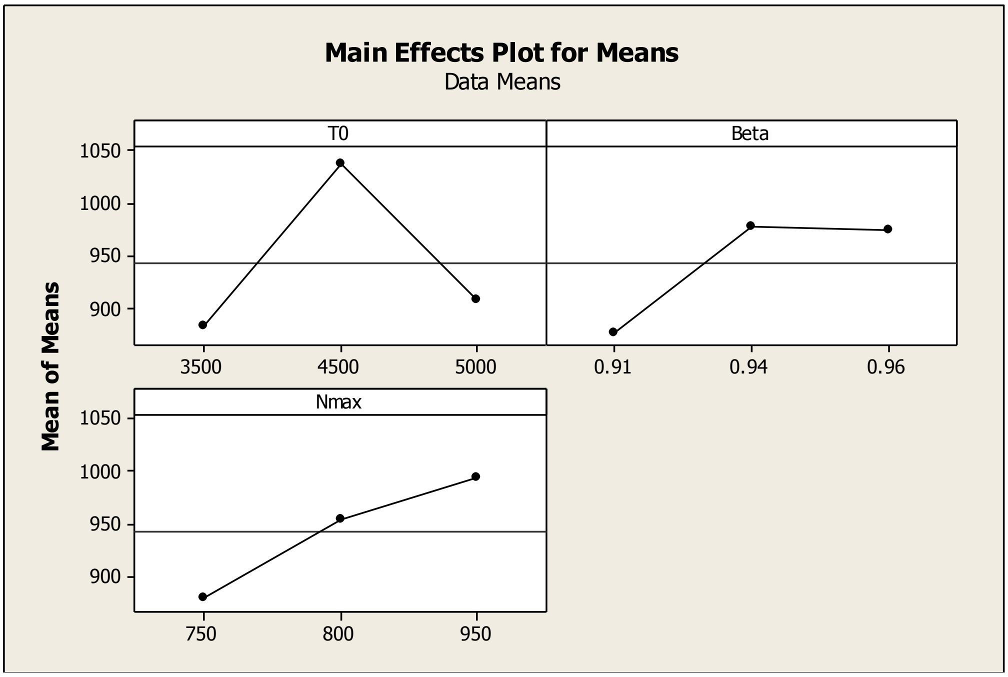

Figure 3. RPD ratio for SA

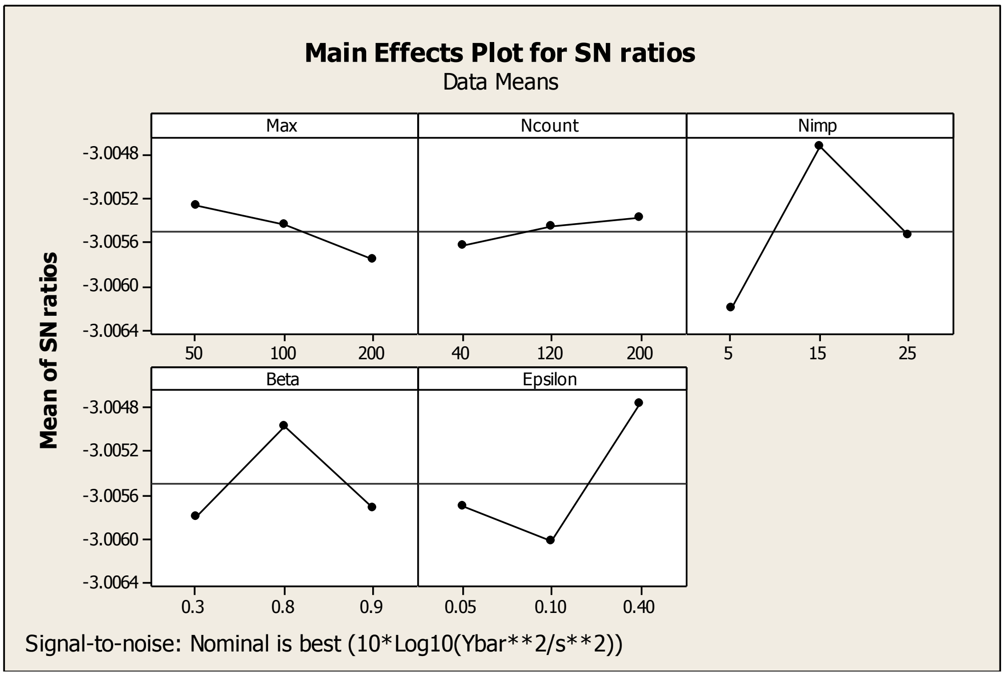

Figure 4. ***S/N*** **ratio for ICA**

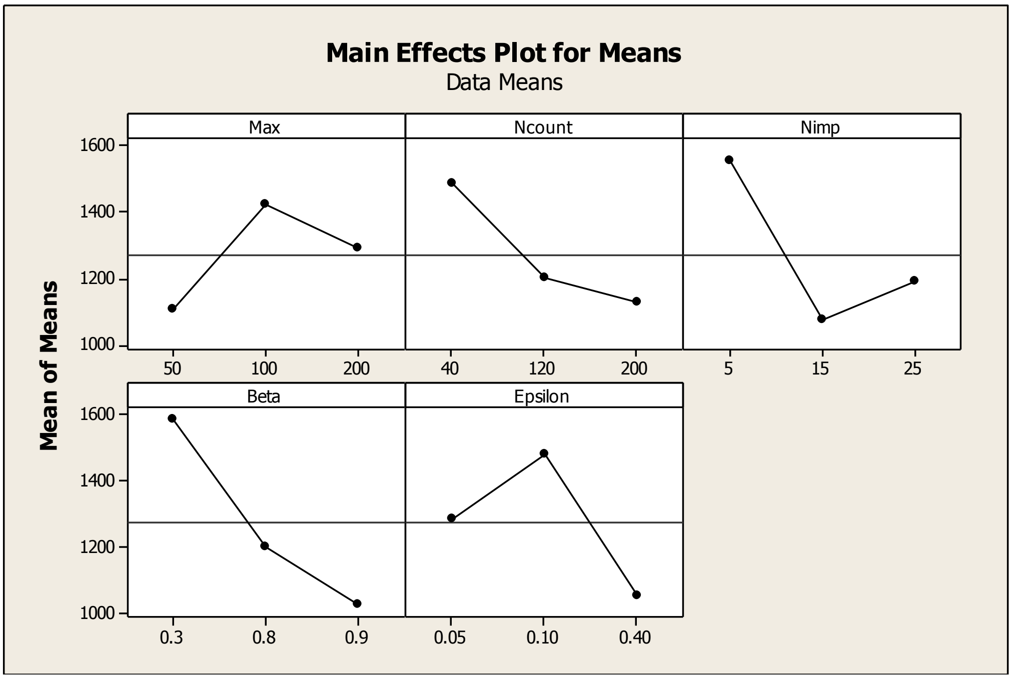

Figure 5. RPD ratio for ICA

5.3. Experimental Results

We set the computational time to be identical for both algorithms. Hence, this criterion is affected by both *n* and *m*. The more the number of DCs or number of customers are, the more the rise of the computational time. The objective function values for SA and ICA are obtained for each of the seven problem sizes as shown in Table 4. The reciprocal between the capability of the algorithms and the size of problems are illustrated in Figure 6. It shows that ICA exhibits robust performance, meanwhile the size of problems increases. The results obtained from ICA are better than SA, except in the first problem size. However, for all the remaining problem sizes, ICA can find the better results.

Table 4. Test problems characteristics

problem size	Total demand	fitness(SA)	fitness(ICA)
10*10*4	10000	2456	3165
10*20*4	15000	3830	3506
15*15*6	10000	4734	3987
10*30*6	12000	3950	3041
50*50*8	14000	6735	5926
30*100*8	18000	7279	4873
50*200*10	20000	13457	10482

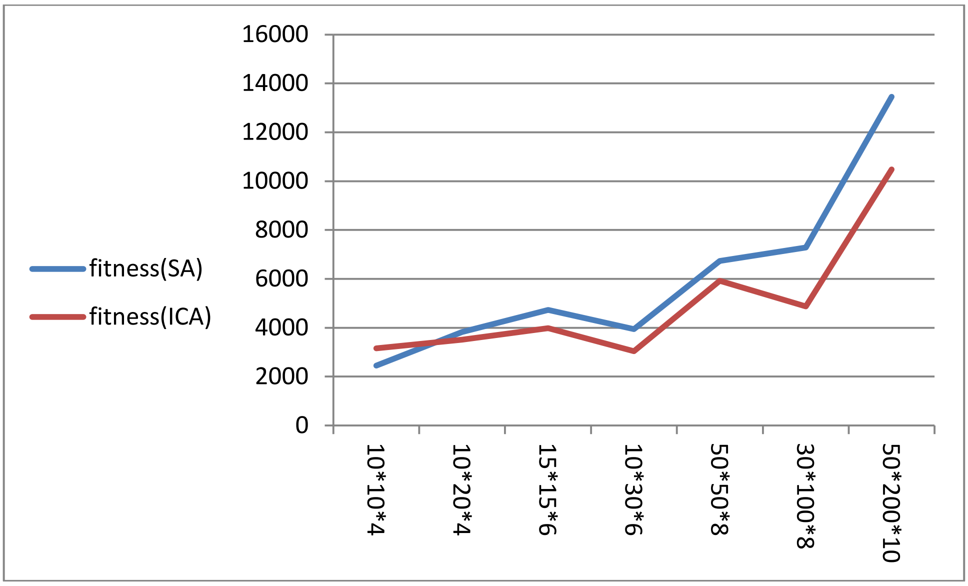

Figure 6. Means plot for the interaction between each algorithm and problem size

6. Conclusions

In this paper, we have solved a fixed charge transportation problem under a fuzzy environment by simulated annealing (SA) and imperialist competitive algorithm (ICA). In order to adjust the parameters and operators of the proposed algorithms, the Taguchi parameter design method has been used. The best levels of each factors for both algorithms according to the RPD have been selected. The computational results have shown the superiority of the proposed ICA in comparison with the proposed SA. There still exist rich opportunities for researchers to study further in this area. For future research, it can be interesting to investigate and develop new algorithms based on other meta-heuristics and compare them with our algorithms.

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