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The Multi-item capacitated lot-sizing problem with limited outsourcing

*Hossein Alipour
Assistant Professor, Department of industrial Engineering, Khalkhal Branch, Islamic Azad University, Khalkhal, Iran

Abstract

The production planning problems encountered in real- life situations are generally intractable due to a number of practical constraints. The decision maker has to find a good feasible solution in a reasonable execution time rather than an optimal one. The objective is to satisfy the requirements in the production plan with minimum costs over a multi-period planning horizon. This paper proposes a new mixed integer programming model for multi-item capacitated lot-sizing problem with setup times, safety stock and demand shortages. The problem is NP-hard and to solve it, a harmony search (HS) approach is used. To verify and validate the efficiency of the HS algorithm, the results are compared with those of the Lingo 8 software. Results suggest that the HS algorithm have good ability of solving the problem, especially in the case of large and medium-sized problems for which Lingo 8 cannot produce solutions.

Keywords: Lot-sizing, Safety stocks, Harmony search, Safety stock.

1. Introduction

The lot-sizing problem is to find the right balance between these costs so as to minimize the total costs (Hoesel and Wagelmans, 1991). (Wagner and Whitin, 1958) presented a dynamic programming solution algorithm for single product, multi-period inventory lot-sizing problem. The Wagner-Whitin algorithm for dynamic lot-sizing has often been misunderstood as requiring inordinate computational time and storage requirements. (Absi and Kedad-Sidhoum, 2009) addressed a multi-item capacitated lot-sizing problem with setup times, safety stock and demand shortages. They proposed a Lagrangian relaxation of the resource capacity constraints and developed a dynamic programming algorithm to solve the problem. (Süral et al, 2009) considered a lot-sizing problem with setup times where the objective is to minimize the total inventory carrying cost only. They proposed some efficient Lagrangian relaxation based heuristics for a lot-sizing problem. (Abad, 2001) considered the problem of determining the optimal price and lot-size for a reseller. He assumed that demand can be backlogged and that the selling price is constant within the inventory cycle. (Aksen et al., 2003) addressed a profit maximization version of the well-known Wagner–Whitin model for the deterministic single-item uncapacitated lot-sizing problem with lost sales. The authors proposed an $O(T^2)$ forward dynamic programming algorithm to solve the problem. (Brahimi et al., 2006) presented four different mathematical programming formulations of the Single-item lot sizing problems. (Chu et al., 2013) addressed a real-life production planning problem arising in a manufacturer of luxury goods. This problem can be modeled as a single-item dynamic lot-sizing model with backlogging, outsourcing and inventory capacity. They showed that this problem can be solved in $O(T^4 \log T)$ time where T is the number of periods in the planning horizon. (Tang, 2004) provides a brief presentation of simulated annealing techniques and their application in lot-sizing problems. (Sadjadi et al., 2009) proposed an improved algorithm of the Wagner and Whitin method. In so doing, they first assumed that shortage is not permitted and inventory holding and setup costs are fixed, and then assumed the possibility of shortage and variability of setup and holding costs. (Absi et al, 2013) investigated the multi-item capacitated lot-sizing problem with setup times and lost sales. Because of lost sales, demands can be partially or totally lost. To find a good lower bound, They used a Lagrangian relaxation of the capacity constraints. They propose a non-myopic heuristic based on a probing strategy and a refining procedure.

The main contribution of this paper is twofold. First, we develop the uncapacitated multi-item lot-sizing problem with demand shortage and safety stock deficit costs with capacity stock, limited outsourcing and several methods for produce. Then we design Harmony search (HS) algorithm to solve the problem. The remaining of this paper is organized as follows: Section 2 describes an MIP formulation of the multi-item capacitated lot-sizing problem. The solution approach (HS) is presented in Sections 3 and Section 4 presents computational experiments. The conclusions and suggestions for future studies are included in Section 5.

2. Problem formulation

In this section, we present an MIP formulation of the problem. The Multi-item capacitated lot-sizing problem with backlogging, safety stocks and limited outsourcing, is a production planning problem in which there is a time-varying demand for an item over T periods. First the problem, assumptions, parameters, and decision variables have thoroughly been discussed, and then the proposed model has been defined.

2.1. Assumptions

Before the formulation is considered, the other following assumptions are made on the problem.

I. The demand is considered deterministic.
II. Shortage is backlogged.
III. Shortage and inventory costs must be taken into consideration at the end.
IV. Raw material resource with given capacities are considered.
V. The quantity of inventory and shortage at the beginning of the planning horizon is zero.
VI. The quantity of inventory and shortage at the end of the planning horizon is zero.

2.2. Parameters

T: Number of periods, indexed from 1 to T, involved in the planning horizon
N: Number of products, i = 1, …, N
J: Number of production manner, j = 1,…, J
d_{it}: The demand for product i in the period t
L_{it}: The quantity of the safety stock of product i in the period t
C_{ijt}: The production cost of each unit of product i in the period t through the manner j
A_{ijt}: The setup cost of the production of product i in the period t through the manner j
h_{it}^{+} : The unit holding cost of product i in the period t
h_{it}^{-} : Unitary safety stock deficit cost of product i in period t
∂_{it} : Unitary shortage cost of product i in period t
γ_{it} : Unit out-sourcing cost of each unit of product i in the period t
M_i : A large number
v_i : Space needs for per unit of product i
φ_t : The total available space in period t

2.3. Decision Variables

X_{ijt}: Production quantity for product i in the period t through the manner j

y_{ijt}: Binary variable; 1 if the product i is produced in the period t through the manner j, otherwise $y_{ijt}=0$

U_{it}: Out-sourcing level of product i in the period t

I^-_{it} : The quantity of shortage of product i in the period t

S^+_{it} : The quantity of overstock deficit of product i in the period t

S^-_{it} : The quantity of safety stock deficit of product i in the period t

2.4. The proposed Model

$$MinZ=\sum_{t=1}^{T}\sum_{i=1}^{N}\Big(\sum_{j=1}^{J}(C_{ijt}X_{ijt}+A_{ijt}y_{ijt})+\partial_{it}I^-_{it}$$
$$+h^+_{it}S^+_{it}+h^-_{it}S^-_{it}+\gamma_{it}U_{it}\Big) \qquad (1)$$

Subject to:

$$S^+_{i,t-1}-S^-_{i,t-1}-I^-_{i,t-1}+I^-_{i,t}+\sum_{j=1}^{J}X_{ijt}+U_{it}=S^+_{it}-S^-_{it}+d_{it}+L_{it}-L_{i,t-1}$$
$$\forall \quad t=1,2,....,T \quad i=1,2,...,N \qquad (2)$$

$$S^+_{iT}=0 \qquad i=1,2,...,N \qquad (3)$$

$$I^-_{iT}=0 \qquad i=1,2,...,N \qquad (4)$$

$$X_{ijt}\le My_{ijt} \qquad \forall \quad j=1,2,....,J \quad t=1,2,....,T \quad i=1,2,...,N \qquad (5)$$

$$I^-_{it}\le d_{it} \qquad \forall \quad t=1,2,....,T-1 \quad i=1,2,...,N \qquad (6)$$

$$S^-_{it}\le L_{it} \qquad \forall \quad t=1,2,....,T \quad i=1,2,...,N \qquad (7)$$

$$0\le U_{it}\le I^-_{i,t-1}+S^-_{i,t-1}+d_{it}+L_{it}$$
$$\forall \quad t=1,2,....,T \quad i=1,2,...,N \qquad (8)$$

$$v\Big(\sum_{j=1}^{J}X_{ijt}+U_{it}\Big)\le\varphi_t$$
$$\forall \quad t=1,2,....,T \quad i=1,2,...,N \qquad (9)$$

$$y_{ijt}\in\{0,1\}$$
$$\forall \quad j=1,2,....,J \quad t=1,2,....,T \quad i=1,2,...,N \qquad (10)$$

$$X_{ijt},I^-_{it},S^-_{it},S^+_{it}\ge 0$$
$$\forall \quad j=1,2,....,J \quad t=1,2,....T \quad i=1,2,...,N \qquad (11)$$

The objective function (1) shows the total cost. Constraints (2) are the inventory flow conservation equations through the planning horizon. Constraints (3) and (4) define respectively, the demand shortage and the safety stock deficit for item at end period is zero. If we produce an item at period t, then constraints (5) impose that the quantity produced must not exceed a maximum production level M_t. M_t is a large number.

Constraints (6) and (7) define upper bounds on, respectively, the demand shortage and the safety stock deficit for item in period t. Constraints (8) ensure that outsourcing level U_t at period t is nonnegative and cannot exceed the sum of the demand, safety stock of period t and the quantity backlogged, safety stock deficit from previous periods. Constraints (9) are the maximum space available for storage of items in excess. Constraints (10) and (11) characterize y_{jt} is a binary variable and the variable's domains: $X_{jt}, I_t^-, S_t^-, S_t^+$ are non-negative for $j \in$ J and $t \in$ T.

3. Soulotion approach

Harmony search (HS) is a new heuristic method that mimics the improvisation of music players. HS was proposed by (Geem et al, 2001). Inspiration was drawn from musical performance processes that occur when a musician searches for a better state of harmony, improvising the instrument pitches towards a better aesthetic outcome. Because the HS algorithm is based on stochastic random searches, the derivative information is also not necessary. In the HS algorithm, musicians search for a perfect state of harmony determined by aesthetic estimation, as the optimization algorithms search for a best state (i.e., global optimum) determined by an objective function.

Step 1: Initialize the optimization problem and algorithm parameters to apply HS, in the first step, the optimization problem are specified as follows:

$$\begin{gathered} \textit{Minimize (or Maximize) } f(x) \\ \textit{Subject to } \; x_i \in X_i, \quad i = 1, 2, \ldots, N \end{gathered} \tag{12}$$

where f (x) is a objective function to be optimized, x is a solution vector composed of decision variables, x_i e X_i is the set of possible range of values for each decision variable x_i (continuous decision variable), that is ${}_Lx_i \le Xi \le {}_Ux_i$, where ${}_Lx_i$ and ${}_Ux_i$ are the lower and upper bounds for each decision variable respectively, and N is the number of decision variables. Furthermore, the control parameters of HS are specified in this step. These parameters are the harmony memory size (HMS); harmony memory consideration rate (HMCR); pitch adjusting rate (PAR).

Step 2: In the second step each component of each vector in the parental population (harmony memory) is initialized with a uniformly distributed random number between the upper and lower bounds [${}_Lx_i$, ${}_Ux_i$], Where 1 < i < N. The ith component of the jth solution vector is given by

x_i^j = Rand [possible range of values for x_i]

Where j = 1, 2, 3, . . ., HMS. Each row of the consists of a randomly generated solution vector for the formulated optimisation problem, and the objective function value for the jth solution vector is denoted by $f(x^j)$. The matrix formed is governed by

HM (j, 1: N) = x^j

HM (j, N + 1) = f (x^j)

The HM with the size of HMS × (N + 1) can be represented by a matrix, as

$$HM=\begin{bmatrix} X^1 \\ X^2 \\ \vdots \\ \vdots \\ X^{HMS} \end{bmatrix} = \begin{bmatrix} X_1^1 & \cdots & X_N^1 & f(x^1) \\ X_1^2 & \cdots & X_N^2 & f(x^2) \\ \vdots & \cdots & \vdots & \vdots \\ X_1^{HMS} & \cdots & X_N^{HMS} & f(x^{HMS}) \end{bmatrix} \tag{13}$$

Step 3: Improvise a new harmony from the HM

After defining the HM as shown in Equation 13, for the optimization problem, the improvisation of the HM is performed by generating a new harmony vector $x' = (x'_1, x'_2, \dots\dots\dots, x'_N)$ each component of the new harmony vector is generated using

$$x'_i \leftarrow \begin{cases} x'_i \in HM(i) \text{ With probability HMCR} \\ x'_i \in X_i \text{ With probability (1 - HMCR)} \end{cases} \qquad (14)$$

Where HM (i) is the ith column of the HM, HMCR is defined as the probability of selecting a component from the HM members, and (1 - HMCR) is, therefore, the probability of generating a component randomly from the possible range of values. If x'_i is generated from the HM, then it is further modified or mutated according to PAR. PAR determines the probability of a candidate from the HM mutating, and (1 - PAR) is the probability of no mutation. Here the pitch adjustment for the selected x'_i is given by

$$x'_i \leftarrow \begin{cases} x'_i = \text{Rand } \{x'_i \in X_i\} \text{ With probability PAR} \\ x'_i \text{ With probability (1 - PAR)} \end{cases} \qquad (15)$$

Step 4: Update the HM
The newly generated harmony vector (x_0) is evaluated in terms of the objective function value. If the objective function value for the new harmony vector is better than the objective function value for the worst harmony in the HM, then the new harmony is included in the HM, and the existing worst harmony is excluded from the HM.
Step 5: If the stopping criterion (maximum number of improvisations) is satisfied, computation is terminated. Otherwise, steps 3 and 4 are repeated.

4. Experimental results

Computational experiments are conducted to validate and verify the behavior and the performance of the heuristic algorithms employed to solve the considered multi-item capacitated lot-sizing problem with backlogging, safety stocks and limited outsourcing. We try to test the performance of the HS in finding good quality solutions in reasonable time for the problem. For this purpose, 15 problems with different sizes are generated. These test problems are classified into three classes: small size, medium size and large size with backlogging, safety stocks and limited outsourcing problems. The number of manners and periods has the most impact on problem hardness. The proposed model coded with Lingo (ver.8) software using for solving the instances. The approaches are implemented to solve each instance in five times to obtain more reliable data. For implementation HS algorithm, the parameters set as: PAR = 0.2, HMCR = 0.6, HMS = 50 and STOP=120. The best results are recorded as a measure for the related problem. Table 1 shows details of computational results obtained by solution methods for all test problems. The results of running HS is compared with the optimal solution of the instances, obtained from Lingo software. Comparing the CPU times of exact solution confirms that computation time grows exponentially by increasing the dimension of the problem Also, after the tenth row the Lingo software has not reached the optimal solution after 3600s of computation time. The metaheuristic algorithm can find the optimal solution for small size problems. A general review of the results shows that:

- The HS has the ability to obtain solution for all test problems.
- The HS can find good quality solutions for medium and large size problems.
- The objective values obtained by HS are closes to Lingo results.
- The HS has the ability to obtain optimal solution for small size problems.

TABLE 1. Details of computational results for all test problems

No	Class	manner	period	Objective Value			
				Lingo	Time	HS	Time
1	Small size	2	3	974913	0	974913	10
2		2	5	1653660	0	1653660	21
3		3	5	1656647	0	1656647	24
4		2	6	2066948	0	2066948	32
5		3	6	2069934	0	2069934	35
6	Medium size	2	12	5541128	0	5422604	40
7		3	12	5550101	3	5431580	47
8		5	12	5568249	9	5443476	69
9		6	12	5574290	100	5449562	80
10		7	12	5578776	268	5454051	88
11	Large size	8	12	-------	-------	5457017	100
12		8	16	-------	-------	7103642	390
13		8	19	-------	-------	8353860	421
14		8	20	-------	-------	8889188	498
15		8	21	-------	-------	9327974	503

— Means that a feasible solution has not been found after 3600 s of computing time.

$$C_{jt} \in [50, 78]; A_{jt} \in [10000, 20000]; d_t \in [2, 12]; r_t \in [42000, 80000];$$

$$L_t \in [2, 5]; \partial_t \in [12, 20]; h_t^- \in [14, 18]; h_t^+ \in [8, 12]; \gamma_t \in [34000, 54000];$$

$$V = 2; \varphi_t = 30; B_{kt} = 14 \,\&\, k = 2$$

5. Conclusion

In this paper, we propose a mathematical formulation of a new multi-item capacitated lot-sizing problem with backlogging, safety stocks and limited outsourcing. This formulation takes into account several industrial constraints such as shortage costs, safety stock deficit costs and limited outsourcing. Due to the complexity of the problem, HS algorithm is used to solve problem instances. Several problems with different sizes generated and solved by HS and Lingo software. The results show that the HS algorithm able to find good quality solutions in reasonable time. One straightforward opportunity for future research is extending the assumption of the proposed model for including real conditions of production systems such as fuzzy demands and etc. Also, developing a new heuristic or metaheuristic to construct better feasible solutions.

References

[1]. Stan Van, Hoesel. Albert. Wagelmans. (1991).A dual algorithm for the economic lot-sizing problem. European Journal of Operational Research. Vol. 52. pp. 315–325.

[2]. Harvy, Wagner. Thomson, Whitin. (1958).A dynamic version of the economic lit size model. Management Science. Vol.5. 89-9.

[3]. Nabil, Absi. Safia Kedad-Sidhoum. (2009).The multi-item capacitated lot-sizing problem with safety stocks and demand shortage costs. Computer and Operations Research, Vol.36. 2926–2936.

[4]. Haldun, Süral. Meltem, Denizel. Luk N. Van, Wassenhove. (2009).Lagrangean relaxation based heuristics for lot-sizing with setup times. European Journal of Operational Research, Vol. 194. 51–63.

[5]. Parkash, Abad. (2001).Optimal price and Order Size for a Reseller Under Partial Backordering. Computers & Operations Research. Vol.28. 53-65.

[6]. Deniz, Aksen. Kemal, Altinkemer. Suresh, Chand. (2003).The single-item lot-sizing problem with immediate lost sales. European Journal of Operational Research. Vol.147. 558–566.

[7]. Nadjib, Brahimi. Stephane, Dauzere-Peres. Najib, Najid. Atle, Nordli. (2006). Single item lot sizing problems. European Journal of Operational Research. Vol.168. 1–16.

[8]. Chengbin, Chu. Feng, Chu. Jinhong, Zhong. Shanlin, Yang. (2013).A polynomial algorithm for a lot-sizing problem with backlogging, out sourcing and limited inventory. Computers & Industrial Engineering. Vol.64. 200–210.

[9]. Ou, Tang. (2004). Simulated annealing in lot sizing problems. Int. J. Production Economics. Vol.88.173–181.

[10]. Sadjadi. Aryanezhad. Sadeghi. (2009).An Improved WAGNER-WHITIN Algorithm. International Journal of Industrial Engineering & Production Research. No.3. 117-123.

[11]. Nabil, Absi. Boris, Detienne. Stephane, Dauzere-Peres. (2013). Heuristics for the multi-item capacitated lot-sizing problem with lost sales. Computers & Operations Research. Vol. 40. Issue. 1. 264–272.

[12]. Zong Woo, Geem. Joong Hoon, Kim. Loganathan. (2001). A new heuristic optimization algorithm: harmony search. Simulation. 60–8.