A new fuzzy complete ranking of DMUs with undesirable outputs

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Abstract

Efficiency analysis is performed not only to estimate the current level of efficiency, but also to provide information on how to remove inefficiency, that is, to obtain benchmarking information. Data Envelopment Analysis (DEA) was developed in order to satisfy both objectives and the strength of its benchmarking analysis gives DEA a unique advantage over other methodologies of efficiency analysis. Data envelopment analysis is an efficiency estimation technique, but it can be used for solving many problems of management such as ranking of DMUs. Ranking DMUs is an important issue in DEA studies. Traditional data envelopment analysis models do not deal with imprecise data an assume that the data for all inputs and outputs are known exactly. In real world situations, however, this assumption may not always be true. The motivation of this study is to develop a new fuzzy ranking model with undesirable outputs. We use possibility approach is chosen to convert the fuzzy model to a crisp model.

Keywords: Data envelopment analysis, Ranking, Fuzzy

1.Introduction

Data envelopment analysis (DEA) is a powerful technique in productivity management. It is a linear programming based methodology introduced by Charnes et al. for measuring the relative efficiency of decision making units. DEA identifies an efficient frontier where all DMUs have a unity score (Charnes et al., 1978). In order to discriminate the performance among efficient DMUs, based upon the CCR model, a super-efficiency DEA model in which a DMU under evaluation is excluded from the reference set was first developed by Banker and Gifford and Banker et al. (Banker and Gifford, 1988; Banker et al., 1989). Cook et al. presented a general model for aggregating votes from a preferential ballot(Cook et al., 1992). In the real world, it might not be possible to adjust all inputs and outputs of ine6cient units based on the DEA results, therefore, Kao presented a modified version of DEA in which bounds are imposed on inputs and outputs. The results from his proposed model provide efficiency improvement for inefficient units, which is feasible in practice (Kao, 1994). A DEA analysis provides a variety of valuable information. It assigns a single score to each DMU that makes the comparison easy. The method has the ability to simultaneously handle multiple inputs and outputs without requiring any judgments on their relative importance, so it does not need a parametrically driven input and output production function. It establishes a best practice frontier among the units based on a comparison process. The units on this frontier are efficient units with an efficiency score of 1.0 and the rest are deemed inefficient. The level of inefficiency is measured by the unit’s distance from this frontier. One of the important advantages of DEA is its ability to identify performance targets for inefficient units and indicate what improvements can be made to achieve pareto-efficiency(Charnes et al., 1997; Cooper et al., 2000). Alirezaee and Afsharian studied the complete ranking of DMUs proposing a new index, namely Balance Index, based on DEA model. In their model, they take two-stage steps to rank all DMUs. Firstly, they rank all DMUs depending on different DEA efficiency scores; secondly, they rank the DMUs with the same DEA efficiency scores according to the proposed Balance Index(Alirezaee and Afsharian, 2007). The efficiency measure of the Least-Distance Measure, developed by Chulwoo and Jeong-dong can provide a welldefined measure of efficiency, while providing the most relevant and easily attainable benchmarking information(Chulwoo and Jeong-dong, 2009). Wu et al. illustrate that the proposed Balance Index is not stable. So, the corresponding rankings are also unstable. Thus, they develop a modified model by introducing the Maximal Balance Index, which can determine a unique ranking of DMUs (Wu et al., 2010). Guo and Wu extended a DEA model considering undesirable outputs using restrictions is presented to realize a unique ranking of DMUs through the new “Maximal Balance Index” based on the optimal shadow prices (Guo and Wu, 2013). Puri and Yadav proposed a fuzzy DEA model with undesirable fuzzy outputs which can be solved as crisp linear program for each a in (0,1] using α-cut approach (Puri and Yadav, 2014). Liu proposed a methodology for a fuzzy two-stage DEA model, where the weights are restricted in ranges and input-output data are treated as fuzzy numbers(Liu, 2014). Based on Zadeh’s extension principle, a pair of two-level mathematical programs is formulated to calculate the upper bound and lower bound of the fuzzy efficiency score. Then he transform this pair of two-level mathematical programs into a pair of conventional one-level mathematical programs to calculate the bounds of the fuzzy efficiency scores. The rest of the work is organized as follows. Section 2 introduces the proposed ranking method. In Section 3 concluding remarks are presented.

2. Methodology

In real world situations involving undesirable outputs, a common method to deal with undesirable outputs in DEA models is to treat undesirable variables as inputs, because of the economic argument that both inputs and undesirable outputs incur costs for a DMU and thus DMUs usually want to reduce both types of variables as much as possible(Guo and Wu, 2013). Korhonen and Luptacik suggested the following DEA model in which negative weights are taken for undesirable outputs(Korhonen and Luptacik, 2004):

Model-1.

$$\max E_k = \frac{\sum_{r=1}^{s_1} u_{rk}^{g} y_{rk}^{g} - \sum_{p=1}^{s_2} u_{pk}^{b} y_{pk}^{b}}{\sum_{i=1}^{m} v_{ik} x_{ik}}$$

subject to :

$$E_j = \frac{\sum_{r=1}^{s_1} u_{rk}^{g} y_{rj}^{g} - \sum_{p=1}^{s_2} u_{pk}^{b} y_{pj}^{b}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \leq 1, \forall j = 1, 2, \ldots, n$$

$$u_{rk}^{g} \geq \varepsilon \forall r, \quad u_{pk}^{b} \geq \varepsilon \forall p, \quad v_{ik} \geq \varepsilon \forall i, \quad \varepsilon \geq 0$$

Where u_{rk}^{g}, u_{pk}^{b} and v_{ik} are wights for rth desirable output, pth undesirable output and ith input of the kth DMU respectively, and ε is the non-Archimedean infinitesimal. In the above model, the weighted sum of all the desirable and undesirable outputs is used, but with negative weights for undesirable outputs. However, due to these negative weights it may happen that for any optimal solution $(u_k^{g*}, u_k^{b*}, v_k^{*})$ for DMU_k of Model-1, there exists some DMUj for which the efficiency Ej becomes negative, i.e., for some DMU_j,

$$E_j = \frac{\sum_{r=1}^{s_1} u_{rk}^{g} y_{rj}^{g} - \sum_{p=1}^{s_2} u_{pk}^{b} y_{pj}^{b}}{\sum_{i=1}^{m} v_{ik} x_{ij}}, \forall j = 1, 2, \ldots, n$$

This can happen while using the cross-efficiency technique in which each DMU is evaluated by using the optimal weights of the other DMUs. Therefore, in order to make efficiency non-negative for every DMUj, we propose a new DEA model in which we include additional constraints $E_j \geq 0$, j = 1, 2, 3,... ,n in Model-1 and hence, the Model-1 becomes:

Model-2.

$$\max E_k = \frac{\sum_{r=1}^{s_1} u_{rk}^g y_{rk}^g - \sum_{p=1}^{s_2} u_{pk}^b y_{pk}^b}{\sum_{i=1}^{m} v_{ik} x_{ik}}$$

subject to :

$$E_j = \frac{\sum_{r=1}^{s_1} u_{rk}^g y_{rj}^g - \sum_{p=1}^{s_2} u_{pk}^b y_{pj}^b}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1, \forall j = 1, 2, \ldots, n$$

$$u_{rk}^g \ge \varepsilon \, \forall r, \quad u_{pk}^b \ge \varepsilon \forall p, \quad v_{ik} \ge \varepsilon \forall i, \quad \varepsilon \ge 0$$

Puri and Yadav By using Charnes–Cooper transformation (Puri and Yadav, 2014), transformed Model-2 into the linear programming problem (LPP) given by:

$$MaxE_k = \sum_{r=1}^{s_1} U_{rk}^g y_{rk}^g - \sum_{p=1}^{s_2} U_{pk}^b y_{pk}^b$$

$$s.t:$$

$$\sum_{i=1}^{m} V_{ik} x_{ik} = 1$$

$$\sum_{r=1}^{s_1} U_{rk}^g y_{rj}^g - \sum_{p=1}^{s_2} U_{pk}^b y_{pj}^b - \sum_{i=1}^{m} V_{ik} x_{ij} \le 0$$

$$\sum_{r=1}^{s_1} U_{rk}^g y_{rj}^g - \sum_{p=1}^{s_2} U_{pk}^b y_{pj}^b \ge 0$$

$$U_{rk}^g \ge \varepsilon$$

$$U_{pk}^b \ge \varepsilon$$

$$V_{ik} \ge \varepsilon$$

$$\varepsilon \ge 0$$

So we can assess the relative efficiency of each DMU using the above model. we use Guo and Wu(2013). model to rankings all DMUs:

$$\max\left(\sum_{i=1}^{m} v_i w_i + \sum_{t=1}^{k} \eta_t h_t - \sum_{r=1}^{s} u_r q_r\right)$$

$$s.t:$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{t=1}^{k} \eta_t b_{tj} \le 0, \quad \forall j$$

$$\sum_{i=1}^{m} v_i x_{ip} + \sum_{t=1}^{k} \eta_t b_{tp} = 1$$

$$\sum_{r=1}^{s} u_r y_{rp} = EFF_p$$

$$u_r, v_i, \eta_t \ge 0, \quad \forall r \forall i \forall t$$

where w_i (i = 1, . . . ,m), q_r (r = 1, . . . , s), h_t (t = 1, . . . , k) are respectively the amount of *i* th input, *r* th desirable output and *t* th undesirable output and η_t is weight of undesirable output for all DMUs.

The fuzzy set theory was first proposed by Zadeh (Zadeh, 1965). It is a mathematical tool to describe imprecision in a fuzzy environment. Imprecision refers to the sense of vagueness rather than the lack of knowledge about the value of parameters. The vagueness is due to the unique experiences and judgments of decision makers. Fuzzy mathematical programming or fuzzy optimization proposed by Zimmermann is one application of the fuzzy set theory(Zimmermann, 1996).

we develop Guo and Wu model when all outputs are undesirable. then

$$\max\left(\sum_{i=1}^{m} v_i w_i + \sum_{t=1}^{k} \eta_t h_t\right)$$

$$s.t:$$

$$\sum_{i=1}^{m} v_i x_{ij} + \sum_{t=1}^{k} \eta_t b_{tj} \geq 0, \quad \forall j$$

$$\sum_{i=1}^{m} v_i x_{ip} + \sum_{t=1}^{k} \eta_t b_{tp} = 1$$

$$v_i, \eta_t \geq 0, \quad \forall i \, \forall t$$

In order to close the gap between the conditions of the problem and real world conditions, we extend above model to fuzzy environment in which all inputs and undesirable outputs are taken as fuzzy numbers, in particular triangular fuzzy numbers (TFNs).

$$\max\left(\sum_{i=1}^{m} v_i \tilde{w}_i + \sum_{t=1}^{k} \eta_t \tilde{h}_t\right)$$

$$s.t:$$

$$\sum_{i=1}^{m} v_i \tilde{x}_{ij} + \sum_{t=1}^{k} \eta_t \tilde{b}_{tj} \geq 0, \quad \forall j$$

$$\sum_{i=1}^{m} v_i \tilde{x}_{ip} + \sum_{t=1}^{k} \eta_t \tilde{b}_{tp} = 1$$

$$v_i, \eta_t \geq 0, \quad \forall i \, \forall t$$

The possibility approach in the context of the fuzzy set theory was introduced by Zadeh (Zadeh,1987) to deal with non-stochastic imprecision and vagueness. In this section, the possibility approach is used to convert the fuzzy model to the equivalent crisp model.

In this paper, chance-constrained programming (CCP) proposed by Charnes and Cooper (Charnes and Cooper, 1959), which is normally used to confront stochastic linear programming (SLP), is adopted as a way to convert the fuzzy ranking model to the equivalent crisp ranking model. The concept of CCP guarantees that the probability of stochastic constraints is greater than or equal to a pre-specified minimum probability. Lertworasirikul et al. (Lertworasirikul et al., 2003) proved and proposed the following Lemma:

Let $\tilde{a}_i (i=1,\ldots,n)$ be fuzzy variables with normal and convex membership functions and b be a crisp variable. The lower and upper bounds of the α-level set of $\tilde{a}_i$ are denoted by $(\tilde{a}_i)_\alpha^L$ and $(\tilde{a}_i)_\alpha^U$, respectively. Then, for any given possibility levels α_1, α_2 and α_3 with $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$, (Wu et al, 2010):

(i) $p(\tilde{a}_1 + \cdots + \tilde{a}_n \le b) \ge \alpha_1$ iff $(\tilde{a}_1)^L_{\alpha_1} + \cdots + (\tilde{a}_n)^L_{\alpha_1} \le b$,

(ii) $p(\tilde{a}_1 + \cdots + \tilde{a}_n \ge b) \ge \alpha_2$ iff $(\tilde{a}_1)^U_{\alpha_2} + \cdots + (\tilde{a}_n)^U_{\alpha_2} \ge b$,

(iii) $p(\tilde{a}_1 + \cdots + \tilde{a}_n = b) \ge \alpha_3$ iff $(\tilde{a}_1)^L_{\alpha_3} + \cdots + (\tilde{a}_n)^L_{\alpha_3} \le b$ and $(\tilde{a}_1)^U_{\alpha_3} + \cdots + (\tilde{a}_n)^U_{\alpha_3} \ge b$.

The ranking model with fuzzy parameters is transformed into the equivalent crisp ranking model by the Lemma 1, as follows:

$$\max\left(\sum_{i=1}^{m} v_i\,(\alpha(\tilde{w}_i)^U_1 + (1-\alpha)(\tilde{w}_i)^U_0) + \sum_{t=1}^{k} \eta_t\,(\alpha(\tilde{h}_t)^U_1 + (1-\alpha)(\tilde{h}_t)^U_0)\right)$$

$$s.t:$$

$$\sum_{i=1}^{m} v_i\,(\alpha(\tilde{x}_{ij})^U_1 + (1-\alpha)(\tilde{x}_{ij})^U_0) + \sum_{t=1}^{k} \eta_t\,(\alpha(\tilde{b}_{tj})^U_1 + (1-\alpha)(\tilde{b}_{tj})^U_0) \ge 0, \quad \forall j$$

$$\sum_{i=1}^{m} v_i\,(\alpha(\tilde{x}_{ip})^U_1 + (1-\alpha)(\tilde{x}_{ip})^U_0) + \sum_{t=1}^{k} \eta_t\,(\alpha(\tilde{b}_{tp})^U_1 + (1-\alpha)(\tilde{b}_{tp})^U_0) \le 1$$

$$\sum_{i=1}^{m} v_i\,(\alpha(\tilde{x}_{ip})^U_1 + (1-\alpha)(\tilde{x}_{ip})^U_0) + \sum_{t=1}^{k} \eta_t\,(\alpha(\tilde{b}_{tp})^U_1 + (1-\alpha)(\tilde{b}_{tp})^U_0) \ge 1$$

$$v_i, \eta_t \ge 0, \quad \forall i\, \forall t$$

3. Conclusion

Models that have been proposed so far have been considered desirable and undesirable outputs together but in real life applications, maybe we only have undesirable outputs which needs to be minimized. Therefore, an extended DEA model with undesirable outputs is presented in this paper and an empirical study are utilized to illustrate the usefulness of this proposed model. In the empirical study, this model realizes a stable and unique ranking among DMUs and a complete ranking of DMUs is determined in the case of incorporating undesirable outputs.

References

[1]. Alirezaee, M.R. Afsharian, M. (2007). A complete ranking of DMUs using restrictions in DEA models. Applied Mathematics and Computation, 189, 1550–1559.

[2]. Banker, R. D. Gifford, R. D. (1988). A relative efficiency model for the evaluation of public health nurse productivity. School of Urban and Public Affairs, Carnegie Mellon.

[3]. Banker, R. D. Das, S. Datar, S. M. (1989). Analysis of cost variances for management control in hospitals. Research in Governmental and Nonprofit Accountin, 5, 268–291.

[4]. Charnes, A. Cooper, W.W. (1959). Chance constrained programming. Management Science, 6, 73-79.

[5]. Charnes, A. Cooper, W.W. Rhodes, E. (1978). Measuring the Efficiency of Decision Making Units.," European Journal of Operational Research, 2, 429-444.

[6]. Charnes, A. Cooper, W.W. Lewin, A.Y. Seiford, L. M. (1997). Data envelopment analysis: theory, methodology and applications. Dordrecht: Kluwer Academic Publisher.

[7]. Chulwoo, B. Jeong-dong, L. (2009). The relevance of DEA benchmarking information and the least-distance measure. Mathematical and Computer Modelling, 49, 265–275.

[8]. Cook, W. Kress, M. Seiford, L. (1992). Prioritization models for frontier decision making units in DEA. European Journal of Operational Research, 59, 319–323.

[9]. Cooper, W. W. Seiford, L. M. Tone, K. (2000). Data envelopment analysis—a comprehensive text with models, applications, references and DEA-solver software. Dordrecht: Kluwer Academic Publisher.

[10]. Guo, D. Wu, J. (2013). A complete ranking of DMUs with undesirable outputs using restrictions in DEA models. Mathematical and Computer Modeling, 58, 1102-1109.

[11]. Kao, C. (1994). Efficiency improvement in data envelopment analysis. European Journal of Operational Research, 73, 487–94.

[12].Korhonen, P. J. Luptacik, M. (2004). Eco-Efficiency Analysis of Power Plants: An Extension of Data Envelopment Analysis. EJOR, 154, 437-446.

[13].Lertworasirikul, S. Fang, S. C. Joines, J. A. Nuttle, H.L.W. (2003). Fuzzy data envelopment analysis (DEA): A possibility approach. Fuzzy Sets and Systems, 139, 379-394.

[14].Liu, S.T. (2014). Restricting weight flexibility in fuzzy two-stage DEA. Computers & Industrial Engineering, 74, 149-160.

[15].Puri, J. Yadav, S.P. (2014). A fuzzy DEA model with undesirable fuzzy outputs and its application to the banking sector in India. Expert Systems with Applications, 41, 6419-6432.

[16].Wu, J. Yang, F. Liang, L. (2010). A modified complete ranking of DMUs using restrictions in DEA models. Applied Mathematics and Computation, 217, 745–751.

[17]. Zadeh, L. A. (1965). Fuzzy sets Information and Control, 8, 338-353.

[18].Zimmermann, H. J. (1996). Fuzzy set theory and its application. London: Kluwer Academic Publishers.

[19].Zadeh, L. A. (1987). Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 1978, 1, 3-28.