
# The multi-level capacitated lot sizing problem with linked lot sizes and non-deterministic demands

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## Abstract

The multi-level capacitated lot-sizing problem with linked lot sizes (MLCLSP-L) is the problem of determining production quantities in multi-stage production settings, such that the sum of setup and holding costs is minimized. The multi-level capacitated lot-sizing problem (MLCLSP) does not consider any setup carryover between two successive periods. As an extension, the MLCLSP-L for short allows the setup state of a resource to be carried over from the current period to the next period. In this paper, we develop and solve the MLCLSP-L by a real-world problem that arises in production. Here, the MLCLSP-L is extended to include resource and stock capacity constraint, shortages backlogging and lost sales, outsourcing and fuzzy demands. Also, we used Delgado et al. (1998) approach to convert fuzzy demands into crisp demands. The aim is minimizing sum of inventory holding costs, overtime costs of resource, setup costs, outsourcing costs, shortages backlogging and lost sales costs. Finally we implement LINGO software to solving this model.

**Keywords:** Production planning, MLCLSP-L, Lot sizing, Mathematical programming

## 1. Introduction

Production planning is one of the most important decisions for manufacturers. It determines how many units of each component/final product should be produced internally or procured from outside suppliers in each period over a given planning horizon, with the objective to minimize the total cost, while meeting customer demand on time. The MLCLSP-L, which is an extension of the big-bucket MLCLSP, allows to carry over the setup state of a resource to the next periods following the setup. This leads to more efficient setup patterns and shorter planning-induced flow times. In addition, due to its more realistic book keeping of setups, it allows to find a feasible solution in cases when the standard MLCLSP formulation would fail. This is particularly likely in situations with high utilization (Sahling et al., 2009). Chen and Thizy have proved that multi-item capacitated lot-sizing problem (MCLSP) with setup times is strongly NP-hard (Chen and Thizy, 1990). Barbarosoğlu and Özdamar describe an analysis of different neighbourhood transition schemes and their effects on the performance of a general purpose simulated annealing procedure in solving the dynamic multi-level capacitated lot sizing problem with general product structures (Barbarosoğlu and Özdamar, 2000). Özdamar and Barbarosoğlu proposed a heuristic approach for the solution of the dynamic multi-level, multi-item capacitated lot sizing problem with general product structures. The difficulty in solving MLCLSP is to provide capacity-feasible lot-sizes while maintaining the non-negativity of the inventories belonging to the items in the lower levels of the product structures. The proposed technique aims to resolve this issue by combining the capability of the Lagrangean relaxation to decompose the hard-to-solve problems into smaller subproblems and the intensive search capability of the simulated annealing. As the first attempt, two Lagrangean relaxation schemes are designed and different versions of simulated annealing are incorporated into relaxation designs as the Lagrangean heuristic (Özdamar and Barbarosoğlu, 2000). Berretta and Rodrigues presented a heuristic approach to solve a complex problem in production planning, the multistage lot-sizing problem with capacity constraints. It consists of determining the quantity to be produced in different periods in a planning horizon, such that an initially given demand forecast can be attained. They considered setup costs and setup times. Due the complexity to solve this problem, They developed methods based on evolutionary metaheuristics, more specifically a memetic algorithm (Berretta and Rodrigues, 2004). Pitakaso et al. proposed to decompose the MLCLSP with respect to products and periods. The form of the decomposition, which results in a sequence of subproblems comprising different sub-rectangles of the product–period matrix, is governed by an ant colony optimization (ACO) meta heuristic (Pitakaso et al., 2006). Billington et al. proposed a line of research on capacity constrained multi stage production scheduling problems. Their formulations compute the required production lead times according to the demands on available capacity, thereby reducing in-process inventory compared to the usual practice in Material requirements planning(Billington et al., 1983). Tempelmeier and Buschkuhl consider an MLCLSP-L with at most one setup carry-over and proposed a highly problem-specific extension of the Lagrangean heuristic of Tempelmeier and Derstroff (1996) (Tempelmeier and Buschkuhl, 2008). Helber and Sahling presented an optimization-based solution approach for the dynamic multi-level capacitated lot sizing problem with positive lead times. The key idea is to solve a series of mixed-integer programs in an iterative fix-and-optimize algorithm(Helber and Sahling, 2010). Zhao et al., combined variable neighborhood decomposition search and accurate mixed integer programming (VNDS-MIP) to solve MLCLSP. This method is based on the variable neighborhood search, and with the use of exact LP/MIP solvers ILOG CPLEX, it's proved to be very efficient in solving MLCLSP problem (Zhao et al., 2012).

Ramezanian et al. proposed a new mixed-integer programming (MIP) model and formulate the problem with sequence-dependent setups and availability constraints. The objective is to find a production and preventive maintenance schedule that minimizes production, holding and setup costs.Also, three MIP-based heuristics with rolling horizon framework are developed to generate the integrated plan (Ramezanian et al., 2013). Almeder et al. presented two classical multi-level capacitated lot-sizing problem; one considering batch production and the other one allowing lot-streaming. As well as comparisons with traditional models demonstrate the capability of the new approach in delivering more realistic results(Almeder et al., 2015). Chen proposed a new fix-and-optimize (FO) approach for two dynamic MLCLSP, the MLCLSP without setup carryover and the MLCLSP with setup carryover (Chen, 2015). we develop Chen (2015) model. we adopt the formulation of Chen with resource and stock capacity constraint, shortages backlogging and lost sales, outsourcing and fuzzy demands.

The rest of this paper is organized in five sections. Section 2 formulate the MLCLSP-L. Section 3 described fuzzy theory and approach to convert fuzzy demands into crisp demands. Section 4 presented computational experiments. The conclusions and suggestions for future research are included in Section 5.

## 2. The mathematical models

In this section, we formulate the problems mathematically.

### 2.1. Numbers, indices, and index sets

N: Set of items (including final products, intermediate products/components, and raw materials) to be produced or procured, with N = {1, 2, …, N}, where N is the number of items considered;

T: Set of periods in the planning horizon, with T = {1, 2, …, T}, where T is the number of periods considered;

K: Set of resources (machines) used in the production, with K = {1, 2, …, K}, where K is the number of resources considered;

i, j: item or operation index, i, j = 1,..., N;

t: period index, t =1,...,T;

k: resource (machine) index, k = 1, ... , K;

$m_i$: resource (machine) used by item i;

$S_i$: set of immediate successors of item i in the bill of materials;

$P_i$: set of immediate predecessors of item i in the bill of materials;

$N_k$: set of items produced by resource k.

### 2.2. Parameters

$d_{it}$: demand of item i in period t;

$a_{ij}$: number of units of item i required to produce one unit of its immediate successor item j;

$lt_i$: production lead time of item i;

$pt_{ki}$: production time required by resource k to produce each unit of item i;

$st_{ki}$: setup time for resource k used for the production of item i;

$b_{kt}$: available capacity of resource k in period t (in units of time);

$hc_i$: holding cost for one unit of item i in each period;

$sc_i$: setup cost for the resource used to produce item i;

$oc_k$: unit overtime cost of resource k;

$B_{it}$: large positive number;

$\underline{I}_i$: physical initial inventory of item i;

$N_k$: number of items produced by resource k, $N_k = |\mathcal{N}_k|$.

**2.3. Variables**

$I_{it}$: inventory level of item i at the end of period t;

$X_{it}$: production quantity of item i in period t;

$Y_{it}$: binary setup variable; $Y_{it}= 1$ if item i is produced in period t and $Y_{it}= 0$ otherwise;

$O_{kt}$: overtime of resource k in period t;

$Z_{it}$: binary setup carry-over variable for item i at the beginning of period t.

**2.4. Additional parameters**

OT:amount of space occupied by each unit of item;

$OMAX_{kt}$: maximum overtime of resource k in period t;

$FI_t$: total space available in period t;

$BQ_i$: cost of backlogging each item i;

$Q_i$: cost of lost sales each item i;

$\delta$: risk of backlogging;

$C_i$: Subcontracting cost of item i.

**2.5. Additional decision variables**

$R_{it}$: backorder of item i in period t;

$U_{it}$: Subcontracting volume of item i in period t.

## 2.6. The proposed Model

$$Min \sum_{i=1}^{N} \sum_{t=1}^{T} \left[ hc_i I_{it} + sc_i (Y_{it} - Z_{it}) + C_i U_{it} + \delta BQ_i R_{it} + (1-\delta) Q_i R_{it} \right] + \sum_{k=1}^{K} \sum_{t=1}^{T} oc_k O_{kt} \tag{1}$$

s.t:

$$I_{i,t-1} + X_{it} + U_{it} + \delta R_{i,t} - \delta R_{i,t-1} - \sum_{j \in s_i} a_{ij} X_{j,t+1} - I_{i,t} = d_{it} \quad \forall i, t = 1, \dots, T-1 \tag{2}$$

$$I_{i,T-1} + X_{iT} + U_{iT} + \delta R_{i,T} - \delta R_{iT-1} - I_{i,T} = d_{iT} \quad \forall i; \tag{3}$$

$$\underline{I}_i - \sum_{i \in S_i} a_{ij} X_{j,1} - I_{i,0} = 0 \quad \forall i \tag{4}$$

$$\sum_{i \in N_k} [pt_i X_{it} + st_i (Y_{it} - Z_{it})] \le b_{kt} + O_{kt} \quad \forall k, t \tag{5}$$

$$X_{it} \le B_{it} Y_{it} \quad \forall i, t \tag{6}$$

$$\sum_{i \in N_k} Z_{it} \le 1 \quad \forall k, t \tag{7}$$

$$Z_{it} \le Y_{i,t-1} \quad \forall i, t \tag{8}$$

$$Z_{it} \le Y_{it} \quad \forall i, t \tag{9}$$

$$N_k (2 - Z_{it} - Z_{i,t+1}) + 1 \ge \sum_{i' \in N_k} Y_{i't} \quad \forall k, t, i \in N_k \tag{10}$$

$$Z_{i1} = 0 \quad \forall i \tag{11}$$

$$O_{kt} \le OMAX_{kt} \quad \forall k, t; \tag{12}$$

$$R_{it} \le \tilde{d}_{it} \quad \forall i, t; \tag{13}$$

$$OT \left( \sum_{i=1}^{N} (X_{it} + U_{it}) \right) \le FI_t \quad \forall t; \tag{14}$$

$$X_{it} \ge 0, U_{it} \ge 0, R_{it} \ge 0, Y_{it} \in \{0,1\}, Z_{it} \in \{0,1\}, \quad \forall i, t \tag{15}$$

In model MLCLSP-L, The objective function (1) to be minimized is the sum of inventory holding costs, setup costs, overtime costs, subcontracting costs, shortages backlogging and lost sales costs. Constraints (2)-(4) are inventory balance equations, where equations (3) and (4) give the special boundary conditions for the inventory level of each item at the beginning of the first period and the end of the last period. The introduction of the physical initial inventory for each item i, $\underline{I}i$ , is to take account of the existence of one period production lead time of the item. $\underline{I}i$ must cover at least the total demand from all successors of item i in the first period. Constraints (5) give the capacity constraints of all resources. Constraints (6) indicate that each item can be produced in a period only if the required resource is in the setup state. Constraints (7) imply that in each period, the setup carry-over of a resource is possible only for at most one item. Constraints (8) and (9) indicate that the setup carry-over of a resource for item i occurs in period t only if the resource is set up for the item in both periods t-1 and t. Constraints (10) indicate multi-period setup carryovers. Constraints (11) state that no resource is in its setup state at the beginning of the first period. Constraint (12) limits the overtime level. Constraint (13) limits backorder level. Constraint (14) is stock capacity. The nonnegative real or binary nature of each variable in the model is indicated by constraints (15).

## 3. Fuzzy theory

A linguistic variable is a variable whose values are not numbers but words or sentences in a natural or artificial language (Zaddeh, 1975). Fuzzy numbers are introduced to appropriately express linguistic variables. A fuzzy number is a fuzzy set on the real line that satisfies the conditions of normality and convexity (Hadi, 2008). It is a quantity whose value is imprecise, rather than exact as is the case with ‘‘ordinary’’ (single-valued) numbers. A triangular or trapezoidal fuzzy number is usually adopted to express the decision group’s perception of alternatives’ performances with respect to each criteria (Debashree and Debjani, 2011; Soheil and Kaveh, 2010). In fact, a triangular fuzzy number is a special case of a trapezoidal fuzzy number. When the two most promising values are the same number, the trapezoidal fuzzy number becomes a triangular fuzzy number. A trapezoidal fuzzy number can be defined as $\tilde{m} = (a,b,c,d)$, where the membership function $\mu_{\tilde{m}}$ of $\tilde{m}$ is given by

$$\mu_{\tilde{m}}(x) = \begin{cases} \dfrac{x-a}{b-a} & (a \le x \le b) \\ 1 & (b \le x \le c) \\ \dfrac{d-x}{d-c} & (c \le x \le d) \end{cases} \tag{16}$$

where [b, c] is called a mode interval of $\tilde{m}$ , and a and b are called lower and upper limits of $\tilde{m}$ . Let $\tilde{A}$ and $\tilde{B}$ be two positive trapezoidal fuzzy numbers parameterized by (a1, a2, a3, a4) and (b1, b2, b3, b4), then the operational laws of these two trapezoid fuzzy numbers are as follows (Zheng et al., 2012)

$$\tilde{A}(+)\tilde{B} = (a_1,a_2,a_3,a_4)(+)(b_1,b_2,b_3,b_4) = (a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4) \tag{17}$$

$$\tilde{A}(-)\tilde{B} = (a_1,a_2,a_3,a_4)(-)(b_1,b_2,b_3,b_4) = (a_1-b_1,a_2-b_2,a_3-b_3,a_4-b_4) \tag{18}$$

$$\tilde{A}(\otimes)\tilde{B} = (a_1, a_2, a_3, a_4)(\otimes)(b_1, b_2, b_3, b_4) = (a_1b_1, a_2b_2, a_3b_3, a_4b_4) \quad (19)$$

$$\tilde{A}(\phi)\tilde{B} = (a_1, a_2, a_3, a_4)(\phi)(b_1, b_2, b_3, b_4) = (\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}) \quad (20)$$

$$k\tilde{A} = (ka_1, ka_2, ka_3, ka_4) \quad (21)$$

$$(\tilde{A})^{-1} = (\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}) \quad (22)$$

To convert the trapezoid fuzzy number into matching crisp values, a proper defuzzification is needed. Defuzzification is an inverse transformation which maps the output from the fuzzy domain back into the crisp domain. Assume that the trapezoid fuzzy number is $\tilde{A} = (a, b, c, d)$, then the matching crisp value can be obtained by Eq.(23) (Delgado et al., 1998)

$$N = \frac{(b+c)}{2} + \frac{[(d-c)-(b-a)]}{6} = \frac{(a+2b+2c+d)}{6} \quad (23)$$

where N is the defuzzified crisp value, and we used this approach to convert fuzzy demands into crisp demands. For example:

$$\tilde{d} = (100000, 120000, 140000, 160000)$$

$$d = \frac{(100000 + (2*120000) + (2*140000) + 160000)}{6} = 130000$$

## 4. Results

In order to evaluate the performance of the proposed model, 6 problems with different sizes are randomly generated. The proposed model coded with LINGO 15 software using for solving the instances. All tests are conducted on a not book at Intel inside Core i5 Processor 1.7 GHz and 6 GB of RAM. Table 1 shows details of computational results obtained by solution method for all test problems.

**Table 1** Details of computational results

| NO | Item (i) | Item (j) | Resource (k) | Period (t) | Lingo | Time(second) |
|---|---|---|---|---|---|---|
| **1** | 2 | 2 | 2 | 4 | 15280080 | 0.39 |
| **2** | 2 | 2 | 4 | 6 | 31664100 | 6.94 |
| **3** | 4 | 4 | 6 | 8 | 102322300 | 6.17 |
| **4** | 4 | 4 | 2 | 12 | 143000500 | 14.74 |
| **5** | 6 | 4 | 2 | 12 | 159449000 | 100.16 |
| **6** | 6 | 4 | 4 | 12 | 160184600 | 100.17 |

## 5. Conclusion

This paper develops the formulate and solve the multi-level capacitated lot-sizing problem with linked lot sizes, resource and stock capacity constraint, shortages backlogging and lost sales, outsourcing and fuzzy demands. Also, we used Delgado et al.(1998) approach to convert fuzzy demands into crisp demands, and we implement LINGO15 software to solving this model. Table 1 shows that, increasing the number of Item i and period have a significant impact in increasing the CPU time. One straightforward opportunity for future research is extending the assumption of the proposed model for including real conditions of production systems such as time value of money and etc. Also, implement heuristic or meta-heuristic algorithms to solve large-scale model.

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