A Multi-item Capacitated Lot-sizing Problem with backlogging, safety stocks and outsourcing in a closed loop supply chain

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Abstract
Capacitated Lot-sizing Problem (CLSP) has received an increased attention from the research community due to its inherent applicability to real-world problems. This paper proposes a new mixed integer programming model for Multi-item Capacitated Lot-sizing Problem (MCLSP) with backlogging, lost sales, safety stocks, demand shortages and outsourcing in a closed loop supply chain. The aim of this paper is to highlight, once more, the powerfulness of how the mathematical formulation of the Capacitated Lot Sizing Problem can be easily adapted to solve further practical closed loop supply chain applications especially related to manufacturing and production environment. Mathematical formulations and computational experiences of solving the proposed model with Lingo solver will be provided to support these statements.

Keywords: Lot sizing, Mathematical modeling, Production planning, Closed-loop supply chain

Introduction

The production planning problems encountered in real life situations are generally intractable due to a number of practical constraints. The decision maker has to find a good feasible solution in a reasonable execution time rather than an optimal one. The lot-sizing problem (LSP) is a crucial step and well-known optimization problem in production planning which involves time-varying demand for set of *N* items over *T* periods. It is a class of production planning problems in which the availability amounts of the production plan are always considered as a decision variable. Two versions of the lot-sizing problems are capacitated and uncapacitated lot-sizing problem. In industrial applications, several factors may sophisticate making the best decisions. For this reason, the capacitated lot-sizing problem and its variations have received a lot of attention from academic researchers. On the other hand, backlogging, safety stocks, limited outsourcing and returned products are four complicating constraints to reach the desired solutions in lot-sizing problem. Chen and Thizy proved that the multi-item capacitated lot-sizing problem with setup times is strongly NP-hard (Chen and Thizy, 1990). There are many references dealing with the capacitated lot-sizing problem and explanation of why it is one of the most popular among exact and approximate solution methods using Lagrangian relaxation of the capacity constraint and comparing this approach with every alternate relaxation of the classical formulation of the problem. Absi and Kedad-Sidhoum addressed a multi-item capacitated lot-sizing problem with setup times, safety stock and demand shortages (Absi and Kedad-Sidhoum, 2009). Süral et al. considered a lot-sizing problem with setup times where the objective is to minimize the total inventory carrying cost only (Süral et al., 2009). Wu et al. proposed two new mixed integer programming models for capacitated multi-level lot-sizing problems with backlogging. They proposed a new and effective optimization framework that achieves high quality solutions in reasonable computational time (Wu et al., 2011). Kirca and kökten proposed a new heuristic approach for solving the single level multi-item capacitated dynamic lot- sizing problem. Their approach used an iterative item-by-item strategy for generating solutions to the problem Kirca and kökten. Özdamar and Barbarosoglu proposed a heuristic approach for the solution of the dynamic multi-level multi-item capacitated lot-sizing problem with general product structures (Özdamar and Barbarosoglu , 2000). Rizk et al. studied a class of multi-item lot-sizing problems with dynamic demands, as well as lower and upper bounds on a shared resource with a piecewise linear cost. The problem was formulated as a mixed-integer program (Rizk et al., 2006). Absi et al. studied the multi-item capacitated lot sizing problem with setup times and lost sales. Because of lost sales, demands can be partially or totally lost. They proposed a non-myopic heuristic based on a probing strategy and a refining procedure. They also proposed a metaheuristic based on the adaptive large neighborhood search principle to improve solutions (Absi et al., 2013). Gutierrez et al. investigated the dynamic lot-sizing problem considering multiple items and storage capacity. They proposed a heuristic procedure based on the smoothing technique (Gutierrez et al., 2013). Governmental and social pressures as well as economic opportunities have motivated many firms to become involved with the return of used products for recovery (Gungor and Gupta, 1999). The demands of a certain product for each period can be satisfied by items which have been either remanufactured from used products arriving at the beginning of every period, or have been newly manufactured. Golany et al. studied a production planning problem with remanufacturing. They proved the problem is NP-complete and obtained an O(T3) algorithm for solving the problem (Golany et al., 2001). Teunter and Pelin Bayındır addressed the dynamic lot-sizing problem for systems with product returns. They presented an exact, polynomial time dynamic programming algorithm (Teunter and Pelin Bayındır, 2006). Li et al. analyzed a version of the capacitated dynamic lot-sizing problem with substitutions and return products. They first applied a genetic algorithm to determine all periods requiring setups for batch manufacturing and batch remanufacturing, and then developed a dynamic programming approach to provide the optimal solution to determine how many new products are manufactured or return products are remanufactured in each of these periods (Li et al., 2007). Pan et al. addressed the capacitated dynamic lot-sizing problem arising in closed-loop supply chain where returned products are collected from customers. They assumed that the capacities of production, disposal and remanufacturing are limited, and backlogging is not allowed. Moreover, they proposed a pseudo-polynomial algorithm for solving the problem with both capacitated disposal and remanufacturing (Pan et al., 2009). Pin˜eyro and Viera investigated a lot-sizing problem with different demand streams for new and remanufactured items, in which the demand for remanufactured items can also be satisfied by new products, but not vice versa. They provided a mathematical model for the problem and demonstrated that it is NP-hard, even under particular cost structures (Pin˜eyro and Viera, 2010). Zhang et al. investigated the capacitated lot-sizing problem in closed-loop supply chain considering setup costs, product returns, and remanufacturing. They formulated the problem as a mixed integer program and proposed a Lagrangian relaxation-based solution approach (Zhang et al. 2012). Returned products are collected from customers. These returned products can either be disposed or be remanufactured to be sold as new ones again; hence, the market demands can be satisfied by either newly produced products or remanufactured ones. Due to the variety of products in the current manner under review, each product might be produced through different manners, and the costs of each unit and the value of resources used depend on the selected manner of production. In most wide industrial implications, one of the most important questions is to identify the best value of production. In this research, an integer linear programming model is developed for the multi-item capacitated lot-size by taking into consideration many industrial limitations. The goal is to minimize the costs of production, inventory costs, shortage costs, safety stock deficit costs, out-sourcing costs, disposing returned products costs, and remanufacturing returned products costs. The rest of this paper is organized as follows: Section 2 describes an MIP (mixed integer programing) formulation of the multi-item capacitated lot-sizing problem with safety stocks in closed-loop supply chain. Computational experiments are presented in Section 3. The conclusions and suggestions for future studies are included in Section 4.

2. The mathematical model

In this section, we present a mixed integer programming model for Multi-item Capacitated Lot-sizing Problem.

2.1. Model Parameters

T: Number of Periods, t=1, 2,…, T.
N: number of items, i=1, 2,…, N.
J : number of production manners, j=1, 2,…,J.
d_{it}: demand for item i at period t.
φ_{it}: backlogging unit cost for item i at period t.
α: risk of backlog.
π_{it}: lost sale unit cost for item i at period t.
I_{it}: safety stock of item i at period t.
a_{ijt}: production unit cost for item i produced through manner j at period t.
B_{ijt}: setup cost for item i produced through manner j at period t.
h^{+}_{it} : the unit inventory holding cost of item i at period t.
h^{-}_{it} : unitary safety stock deficit cost of item i at period t.
C_t : available capacity at period t.
V_{ij}: unitary resource consumption for item i produced through manner j.
Q_i: unitary resource consumption for repairing item i.
f_{ij}: the quantity of wasted resource for itemt i produced through manner j.
M: a large number. M could be set to the minimum between the total demand requirements for item *i* on section [*t*, *T*] of the horizon and the highest quantity of item *i* that could be produced regarding the capacity constraints. M is equal to $M = \min\left\{\frac{C_t - f_{ij}}{V_{ij}}, \sum_{t=1}^{T} d_{it}\right\}$.
CD_{it}: the cost of disposing returned products for each unit of item i at period t.
CR_{it}: the cost of remanufacturing returned products for each unit of item i at period t.
CRI_{it}: the unit holding cost of item i of returned products at period t.

XDmax: the maximum number of returned item i that could be disposed at period t.
XRmax: the maximum number of returned item i that could be remanufactured at period t.
CS_{it}: Unit outsourcing cost of item i at period t.
E_{it}: total number of returned item i at period t.

2.2 Decision Variables

x_{ijt}: the quantity of item i produced through manner j at period t.
y_{ijt}: binary setup variable, equal to 1 if item i is produced through manner j at period t (i.e. $x_{ijt}>0$), and 0 otherwise.
r_{it}: the quantity of shortage of item i at period t.
S^{+}_{it}: the quantity of overstock deficit of item i at period t.
S^{-}_{it}: the quantity of safety stock deficit of item i at period t.
XD_{it}: number of returned products of item i that disposed at period t.
XR_{it}: number of returned products of item i that remanufactured at period t.
OS_{it}: outsourcing level of item i at period t.
XRI_{it}: number of returned products of item i that held in inventory at the end of period t.

2.3 The proposed model

$$\begin{aligned} Min\ Z=&\sum_{i=1}^{n}\sum_{t=1}^{T}[(\sum_{j=1}^{J}(\mathrm{a}_{ijt}.x_{ijt}+B_{ijt}.y_{ijt}))]+\sum_{i=1}^{N}\sum_{t=1}^{T}\ \alpha.\varphi_{it}.r_{it}+\sum_{i=1}^{N}\sum_{t=1}^{T}\ (1-\alpha).\pi_{it}.r_{it} \\ &+\sum_{i=1}^{n}\sum_{t=1}^{T}(\mathrm{h}^{+}_{it}\mathrm{S}^{+}_{it}+\mathrm{h}^{-}_{it}\mathrm{S}^{-}_{it})+\sum_{i=1}^{n}\sum_{t=1}^{T}\ CS_{it}OS_{it}+\sum_{i=1}^{N}\sum_{t=1}^{T}\ CD_{it}XD_{it} \\ &+\sum_{i=1}^{N}\sum_{t=1}^{T}\ CR_{it}XR_{it}+\sum_{i=1}^{n}\sum_{t=1}^{T}\ \mathrm{CRI}_{it}\mathrm{XRI}_{it} \end{aligned} \tag{1}$$

Subject to:

$$s^{+}_{i,t-1}-s^{-}_{i,t-1}-\alpha.r_{i,t-1}+r_{it}+\sum_{j=1}^{J}x_{ijt}+OS_{it}+XR_{it}=d_{it}+\mathrm{I}_{it}-\mathrm{I}_{i,t-1}+s^{+}_{it}-s^{-}_{it},\ \forall\ i=1,\ 2,\ ...,n\ ;\ t=1,\ 2,\ ...,T-1 \tag{2}$$

$$s^{+}_{i,t-1}-s^{-}_{i,t-1}-\alpha.r_{i,t-1}+\sum_{j=1}^{J}x_{ijt}+OS_{it}+XR_{it}=d_{it}+\mathrm{I}_{it}-\mathrm{I}_{i,t-1}+s^{+}_{it}-s^{-}_{it},\ \forall\ i=1,\ 2,\ ...,\mathrm{n}\ ;\ t=T \tag{3}$$

$$\sum_{i=1}^{n}(\sum_{j=1}^{J}(V_{ij}.x_{ijt}+f_{ij}.y_{ijt})+\mathrm{Q}_{i}.\mathrm{X}R_{it})\le C_{t},\ \forall\ t=1,\ 2,\ ...,T \tag{4}$$

$$x_{ijt}\le M.y_{ijt},\ \ \forall\ i=1,\ 2,\ ...,\ n\ \ ;\ j=1,\ 2,\ ...,\ J\ ;\ t=1,\ 2,\ ...,T \tag{5}$$

$$r_{it}\le d_{it},\ \ \forall\ i=1,\ 2,\ ...,\ \mathrm{n}\ ;\ t=1,\ 2,\ ...,\ T \tag{6}$$

$$s^{-}_{it}\le I_{it},\ \ \ \forall\ i=1,\ 2,\ ...,\ n\ \ ;\ t=1,\ 2,\ ...,\ T \tag{7}$$

$$XD_{it}\le XD\max,\ \ \ \forall\ i=1,\ 2,\ ...,\ \mathrm{n}\ ;\ t=1,\ 2,\ ...,\ T \tag{8}$$

$$XR_{it} \le XR\max, \quad \forall\, i=1, 2, ..., n \; ; \; t=1, 2, ..., T \tag{9}$$

$$r_{i0}=0 \tag{10}$$

$$r_{iT}=0 \tag{11}$$

$$0 \le OS_{it} \le r_{i,t-1} + S^{-}_{i,t-1} + d_{it} + I_{it}, \quad \forall \text{ i=1, 2, ..., n ; t=1, 2, ..., T} \tag{12}$$

$$x_{ijt}, \mathrm{XD}_{it}, XR_{it}, XRI_{it}, r_{it}, s^{+}_{it}, s^{-}_{it} \ge 0, \quad \forall\, i=1, 2, ..., n \; ; \; j=1, 2, ..., J \; ; \; t=1, 2, ..., T \tag{13}$$

$$y_{ijt} \in \{0,1\}, \quad \forall\, i=1, 2, ..., n \; ; \; j=1, 2, ..., J \; ; \; t=1, 2, ..., T \tag{14}$$

The objective function (1) minimizes the total cost induced by the production plan that is production costs, setup costs, shortage costs, inventory costs, safety stock deficit costs, outsourcing costs, disposing costs, remanufacturing costs and the holding costs of returned products. Constraints (2) are the inventory flow conservation equations through the planning horizon. Constraints (3) are the inventory flow conservation equations through the last period that shortage is banned in it. Constraints (4) are the capacity constraints; the overall resource consumption must remain lower than or equal to the available capacity. If we produce an item *i* through the manner j at period *t*, then constraints (5) impose that the quantity produced must not exceed a maximum production level M. M could be set to the minimum between the total demand requirements for item *i* on section [*t*, *T*] of the horizon and the highest quantity of item *i* that could be produced regarding the capacity constraints. Constraint (6) and (7) define upper bounds on, respectively, the demand shortage and the safety stock deficit for item *i* in period *t*. Constraints (8) and (9) are the capacity constraints of disposal and remanufacturing. Constraints (10) and (11) show that the demand shortage is equal to 0 in period 0 and period T. Constraints (12) ensure that outsourcing level OS_{it} at period t is non-negative and cannot exceed the sum of the demand, safety stock of period t, the quantity backlogged and safety stock deficit from previous periods. Constraints (13) and (14) characterize the variable's domains: x_{ijt}, r_{it}, s_{it}^{+}, s_{it}^{-}, XD_{it}, XR_{it} and XRI_{it} are non-negative and y_{ijt} is a binary variable.

3. Results

In order to evaluate the performance of the model, 10 problems with different sizes are randomly generated. The proposed model coded with LINGO 15 software for solving the instances. All tests are conducted on a note book with Intel Core i7 Processor clocked at 2.2 GHz and 6 GB of RAM.Table 1 shows details of computational results for all test problems.

Table 1 Details of computational results

Test problem number	Items (i)	Production manners (j)	Periods (t)	Lingo	Time (s)
1	2	2	3	14939900	1
2	2	2	5	26062100	1
3	2	3	5	25952870	1
4	5	2	6	126088300	2
5	5	3	5	315735500	2
6	5	2	6	140212100	16

7	6	3	6	164030900	46
8	7	3	5	187679700	452
9	8	4	5	206378700	3432
10	10	4	5	211920400	5248

4. Conclusion

The purpose of this paper is to formulate and solve the Multi-item Capacitated Lot-sizing Problem with backlogging, lost sales, safety stocks, demand shortages and outsourcing in a closed loop supply chain. We develop a mixed integer programming model that can be used to compute optimal solution for the problems by an operation research solver. Table 1 shows that, increasing the number of products and manufacturing manners have a significant impact on increasing the CPU time. One straightforward opportunity for future research is extending the assumption of the proposed model for including real conditions of production systems such as fuzzy demands and etc. Also, developing a new heuristic or metaheuristic to construct feasible solutions. Ultimately, Adding inflation to proposed model is suggested to consider real world conditions.

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