



Inventory Routing Problem with a Transshipment Option

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Abstract

This paper investigates the Inventory Routing Problem where multiple capacitated vehicles distribute products from multiple suppliers to a single plant, and the final products produced to customers over a finite planning horizon. The demand associated with each product is assumed to be deterministic and time varying. In this supply chain, the products are assumed to be ready for collection at the supplier site when the vehicle arrives. A transshipment option is considered as a possible solution to increase the performance of the supply chain and shows the impact of this solution on the environment. A green logistic issue is also incorporated into the model by considering the interrelationship between the transportation cost and the greenhouse-gas emission level. The proposed model is a mixed-integer linear program that solved by GAMS software and the results is explained. The computational results show that the proposed model generates high quality schedules in a timely fashion.

Keywords: Routing Problem, Supply Chain, Transshipment.



1. Introduction

Supply Chain (SC) can be defined as a wide set of manufacturers, suppliers, warehouses, distribution centers, transportation services, storage facilities, retailers and consumers at markets, in which raw materials are acquired, transformed into final products, and delivered to customers. Therefore, they are complex systems and characterized by numerous activities spread over multiple functions and organizations (Arshinder & Deshmukh, 2008). Supply Chains (SCs) are the backbones of our globalized network economy and provide the infrastructure for the production, storage and distribution final product to customers (Nagurney, 2014). The aims of most of the problems are to minimize the total cost and have received much attention, especially after recognizing the importance of the logistics costs in the cost structure of them (Jamshidi, Fatemi Ghomi, & Karimi, 2012; Fox, Barbuceanu, & Teigen, 2001).

Supply Chain Management (SCM) can be one of the important topics in manufacturing research in the last decade. SCM aims to efficiently control the material flow through the SC in order to improve its performance as a system. An effective SCM helps to substantially reduce operational costs and increase the customer service level. On the downstream side of the SC, distribution involves the transfer of multiple final items from factories to demand points directly or via transshipment facilities (Zegordi & Beheshti Nia, 2009).

In recent years, supply chain and supply chain management problems have been addressed by many researchers (Klose & Drexler, 2005; Sahin & Süral, 2007; Melo, Nickel, & Saldanha-da-Gama, 2009). For example, Ryu (2010) proposed a modeling methodology for supply chain operations with a focus on the relationships of supply chain entities. Geunes, Levi and Romeijn (2011) proposed a generalization of a broad class of traditional supply chain planning and logistics model. They considered a set of markets, each specified by a sequence of demands and associated with revenues. The goal is to minimize the overall lost revenues of rejected markets and the production cost.

Transportation is a significant component of SC operations. Considering transportation costs in inventory replenishment decisions can reduce the total SC cost (Toptal, 2009). Transportation problems are well known problems that were proposed at first by Hitchcock (1941). A general assumption in transportation problem is that the transportation cost is proportional to the number of units transported (Diaby, 1991). Many practical transportation and distribution problems can be modeled as fixed cost transportation problems (Adlakha & Kowalski, 1999; Sun, Aronson, Mckeown, & Drinka, 1998). Verities of assumptions, models and search-based approaches have been proposed by researchers for solutions of supply chain and transportation problems. Molla-Alizadeh-Zavardehi, Hajiaghahi-Keshteli, and Tavakkoli-Moghaddam (2011) present a mathematical model for a capacitated fixed-charge transportation problem in a two-stage supply chain network, in which potential places are candidate to be as distribution centers (DCs) and customers with particular demands. The presented model minimizes the total cost. Then, in order to tackle the problem, they proposed an artificial immune algorithm (AIA) and a genetic algorithm (GA). Tsao and Lu (2012) develop an algorithm to solve supply SCM problems using nonlinear optimization techniques. The problem considered two types of transportation discounts simultaneously: quantity discounts for the inbound transportation cost and distance discounts for outbound transportation cost. Seo, Jeong, S. Lee, D. Lee, and Park (2012) proposed a mathematical formulation for supply chain planning problems emerging in the open business environment, and then proposed a heuristic algorithm based on GA to large scale problems.

Supply chain problems are usually considered as a single objective problem, and can be to use a weighting method on them (Melkote & Daskin, 2001; Santoso, Ahmed, Goetschalckx, & Shapiro, 2005). But, in recent years, multi-objective supply chain optimization has been considered by many researchers. For example, Paksoy, Ozceylan and Weber (2010) modeled a supply chain to minimize total cost, prevent more CO₂ gas emissions and encourage customers to use recyclable products. They proposed different transportation choices between echelons, according to CO₂ emissions. Mincirardi,



Paolucci and Robba (2002) proposed a multi-objective model to minimize solid waste in a supply chain. Alcada-Almeida, Coutinho-Rodrigues and Current (2009) addressed a multi-objective programming approach to identify the locations and capacities of hazardous material incineration facilities, and balance social, economic, and environmental impacts. Wang, Lai, and Shi (2011) studied a multi-objective optimization model that captures the tradeoff between total cost and environmental influence. Jamshidi et al. (2012) addressed the modeling and solving of a supply chain design for annual cost minimization, while considering environmental effects such as the amount of NO₂, CO and volatile organic particles produced by facilities and transportation. In this paper, they propose a multi-objective optimization problem for sale its. Olivares-Benitez, Rios-Mercado and Gonzalez-Velarde (2013) addressed a supply chain design problem based on a two-echelon single-product system. In the first echelon, the plant transports the product to distribution centers. In the second echelon the distribution centers, transport the product to the customers. The problem is modeled as a bi-objective mixed-integer problem. In this paper, the aim is to minimize cost and lead time.

Inventory Routing Problem (IRP) is a well-known topic for logistic and supply chain problems. The IRP determines vehicle scheduling to minimize cost criterion and defines delivery routes and optimal inventory levels (Zachariadis, Tarantilis, & Kiranoudis, 2009; QuariguasiFrotaNeto, Walther, Bloemhof, Van Nunen, & Spengler, 2009; Moin, Salhi, & Aziz, 2011). IRP can be classified in various ways: planning horizons (finite or infinite), period (single or multiple), customer (single or multiple), product (single or multiple), type of demand and vehicles and etc. (Bertazzi, Bosco, Guerriero, & Lagana, 2011; Coelho, Cordeau, & Laporte, 2012; Huang & Lin, 2010). Mirzapour Alehashem and Rekik (2013) presented a multi-product multi-period IRP where multiple capacitated vehicles distribute products from multiple suppliers to a single plant to meet the given demand of each product over a finite planning horizon. Their study attempts a novel approach to reduce GHG emissions in IRP for achieving a balance between economic and environmental objectives. They propose a mixed-integer linear program model and solve by CPLEX. They also provide a numerical study showing the applicability of the model and underlining the impact of the transshipment option on improved supply chain performance.

In this article, we attend to expand the proposed model (as cited in MirzapourAlehashem et al., 2013) by adding supply chain operations, costs and constraints which transship the final product from the plant to customers. In this article, we investigate an integrated SC with two objectives that delivers products from suppliers to customers. In this paper, we have several transportation options that transport products to downstream facilities, plant and finally customers. All facilities and transportation options have capacity constraints and all of the distances between the suppliers, customers are fixed and determined. In some prior papers (Cordeau, & Laporte, 2012; Huang & Lin, 2010), manufacturers and warehouses have capacity constraints, but in this article, we considers that each transportation option also has a fix capacity and cannot use the transportation option with the least cost and pollution effects and GHG emission. Thus, according to the amount of transported products and distance between facilities-plant and plant-customers we need to choose the best transportation options and the best route. Therefore, the model proposed in this paper is multi-objective. The aim of this paper is minimizing total cost, which consists of transportation, raw material, holding, fixed, backorder and variable costs, with considering the least GHG emission, consisting of NO₂, CO and volatile organic particles produced by facilities and transportation means.

Therefore, the rest of this paper is organized as follows. Problem definition and mathematical formulation of the problem is described in Section 2. Some examples have been solved using the mathematical formulation and the results are shown in Section 3. Finally, conclusions and some possible future research directions are given in Section 4.

2. Transshipment-enabled Inventory Routing Problem (TIRP)

2.1. Problem definition



In this section, we present a mixed integer linear programming (MILP) model to formulate the Transshipment-enabled Inventory Routing Problem (TIRP). TIRP is defined as follows. There are one assembly plant, N suppliers and M customers. The assembly plant assembles and produces one final product type and each supplier produces one incomplete product type for the assembly plant and customers can receive the final product from the plant. Each supplier has a space to keep its product and other suppliers. The plant has got a depot that ships the products from the suppliers to the assembly plant, to the other suppliers and to the customers in each period of time (Only final products ship from assembly plant to customers). All of the distances between the suppliers, customers are fixed and determined. The plant has several types of trucks. Each truck is characterized with fixed capacity, fixed and variable transportation castrate and its GHG emission index. The objective is finding the best configuration of the vehicle types, routes, pickups, deliveries and transshipments in each period while minimizing total cost that is included the inventory holding cost and transportation cost, and finally satisfying all constraints and customers.

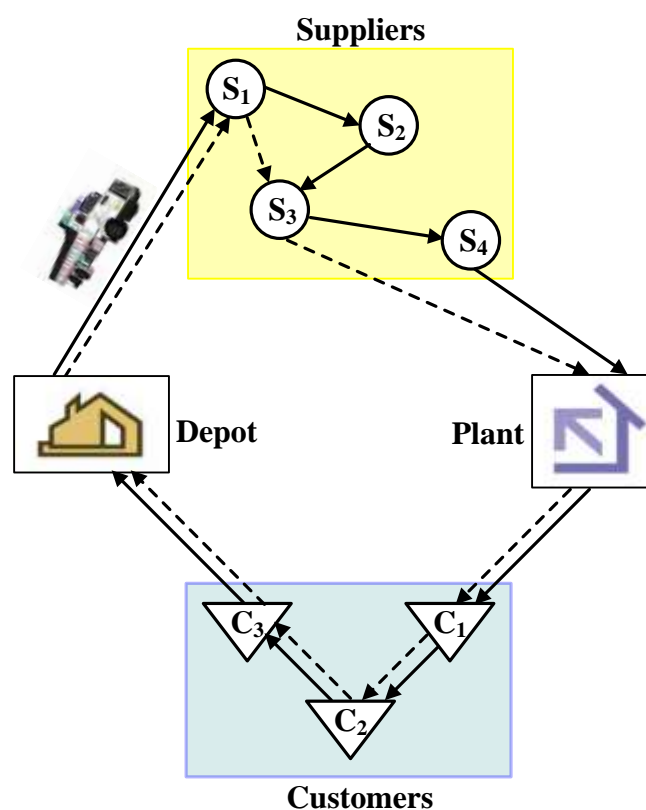


Figure 1. How transshipment can reduce travel distances

As we see in the figure (1), the company consists of several suppliers and one assembly plant. Each supplier produces one incomplete product type for the assembly plant. There is a set of customers that receive final products from the plant. The company also has a contract with a rental truck company (Depot) where the truck types, characterized by their capacity, fixed and variable costs and greenhouse gas emission index. Allowing the vehicles to temporarily store pickups during their trips at a supplier storage area located along their itinerary is known as transshipment-enabled IRP. To illustrate this premise, we depicted the figure. In the depicted figure, we have two periods. In the first period, the truck passes from nodes 1, 2, 3, and 4 (solid arrows), but since the second supplier has no demand for the first period, the vehicle can pick up products from node 2 and store them temporarily at node 3 to reduce the total travel distance for the next period. So the vehicle only visited nodes 1 and 3 (dashed



arrows). This manner has a great impact on reducing the transport and thereby reducing greenhouse gas emissions. It should be noted that there is no temporary storage for customers. The optimization problem must find the best configuration of the vehicle types, routes, pickups, deliveries and transshipments in each period in a manner that minimizes the total cost of the supply chain, including the inventory holding cost and transportation cost, production cost, while satisfying all constraints.

2.2. MILP model

The proposed model uses the following notation:

Sets

$\Omega = \{0, 1, \dots, N, N+1, \dots, N+M+1\}$	Set of all nodes
$W = \{1, 2, \dots, N\}$	Set of suppliers
$O = \{0\}$	Depot
$F = \{N+1\}$	Assembly plant
$Z = \{1, 2, \dots, M\}$	Set of customers

Parameters

Df_{pt}	Demand for product type $p(1, 2, \dots, P)$ in period $t(1, 2, \dots, T)$.
Dc_{it}	Customer's demand for final product i in period $t(1, 2, \dots, T)$.
v_k	Variable transportation cost per unit distance for vehicle type $k(1, 2, \dots, K)$.
f_k	Fixed transportation cost for vehicle type k per trip.
$C_{(N+1)t}$	Produced cost for final product in period $t(1, 2, \dots, T)$.
NT_{kt}	The number of vehicle type k available in period $t(1, 2, \dots, T)$
Cap_k	Capacity of vehicle type k for products that is produced by suppliers.
Cap_{ck}	Capacity of vehicle type k for final product.
h_{ip}	Inventory holding cost in node i for product type p
hc_i	Inventory holding cost for final product in node i
L_{ij}	Length of arc (i, j) . Distance between all of the nodes.
wc_{ip0}	Initial inventory level of product type p in node i .
w_{ip0}	Initial inventory level of product type p in node i .
GHL_t	Allowed level of GHG emission in each period of time.
GHG_k	GHGs produced by vehicle type k per unit distance.

Decision variables

w_{ipt}	The inventory level of produced product for producing final product type p at supplier t in period of t .
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- $wC_{(N+1)pt}$ The inventory level of final product type p in the plant in period of t
- $Pr_{(N+1)t}$ The quantity of final product produced in plant
- X_{ijkt} Binary variable that is 1 if arc (i, j) is visited by vehicle type k in period t
- Y_{ikt} Binary variable that is 1 if supplier i is visited by vehicle type k in period t
- Q_{ijpkt} The quantity of product type p transported by vehicle type k through arc (i, j) in period t .
- R_{ijkt} The quantity of final product transported by vehicle type k through arc (i, j) in period t
- a_{ipt} The quantity of product type p picked up from supplier i in period t
- b_{ipt} The quantity of product type p transshipped to supplier i in period t
- bv_{it} The quantity of final product transshipped to customer i in period t .

2.2. Mathematical formulation

The mixed integer programming for the transshipment-enabled IRP is modeled as follows:

$$\begin{aligned}
 \text{Min } Z = & \sum_{i \in \Omega} \sum_{j \in \Omega} \sum_k \sum_t v_k L_{ij} X_{ijkt} + \sum_{i \in (w \cup F)} \sum_p \sum_t (h_{ip} w_{ipt}) + \sum_{(i \in F)} \sum_t hc_i wC_{it} \\
 & + \sum_{i \in (w \cup F)} \sum_k \sum_t f_k x_{0ikt} + \sum_t c_{(N+1)t} Pr_{(N+1)t}
 \end{aligned} \tag{1}$$

Subject to:

$$w_{ipt} = w_{ip(t-1)} + b_{ipt} - a_{ipt} \quad \forall i \in w, p \neq i, t \tag{2}$$

$$w_{(N+1)pt} = w_{(N+1)p(t-1)} + \sum_{i \in w} \sum_k Q_{i(N+1)pkt} - Df_{pt} \quad \forall p, t$$

$$wC_{(N+1)t} = wC_{(N+1)(t-1)} + Pr_{(N+1)t} - \sum_{i \in z} Dc_{it} \tag{3}$$

$$\sum_{j \in \Omega} x_{ijkt} = \sum_{j \in \Omega} x_{jikt} = y_{ikt} \quad \forall i \in (w \cup z), k, t$$

$$\sum_k y_{ikt} \leq 1 \quad \forall i \in (w \cup z), t \tag{4}$$

$$\sum_{j \in (w \cup O)} \sum_k Q_{jipkt} + a_{ipt} - b_{ipt} = \sum_{j \in (w \cup F)} \sum_k Q_{ijpkt} \quad \forall i \in w, p, t$$

$$\sum_{j \in (F \cup z)} \sum_k R_{jikt} - bv_{it} = \sum_{j \in (z \cup F)} \sum_k R_{ijkt} \quad \forall i \in z, t \tag{5}$$

$$\sum_p Q_{ijpkt} \leq Cap_k \cdot x_{ijkt} \quad \forall (i, j) \in (w \cup F), k, t$$

$$R_{ijkt} \leq Cap_k \cdot x_{ijkt} \quad \forall (i, j) \in z, k, t \tag{6}$$

$$\sum_t Pr_{(N+1)t} \geq \sum_t \sum_{i \in z} Dc_{it}$$

$$a_{ipt} \leq w_{ip(t-1)} \quad \forall i \in w, p \neq i, t \tag{7}$$



$$\sum_{i \in \omega} x_{oikt} \leq NT_{kt} \quad \forall k, t \quad (8)$$

$$\sum_{i \in \omega} \sum_k x_{oikt} \geq 1 \quad \forall t$$

$$\sum_{i \in z} x_{iokt} \geq 1 \quad \forall k, t \quad (9)$$

$$\sum_{i \in \omega} x_{i(N+1)kt} \geq 1 \quad \forall k, t$$

$$\sum_{i \in z} \sum_k x_{(N+1)ikt} \geq 1 \quad \forall t \quad (10)$$

$$\sum_{i \in \Omega} \sum_{j \in \Omega} \sum_k GHG_k d_{ij} x_{ijkt} \leq GHL_t \quad \forall t$$

$$\sum_t Pr_{(N+1)t} \geq \sum_t \sum_{i \in z} Dc_{it} \quad (11)$$

$$\mathcal{R}_{ijkt} \leq x_{ijkt} \quad \forall i, j \in (F \cup Z), k, t$$

$$x_{iokt} = 0 \quad \forall i \in \omega, k, t$$

$$x_{oikt} = 0 \quad \forall i \in z, t$$

$$x_{o(N+1)kt} = 0 \quad \forall k, t \quad (12)$$

$$x_{(N+1)ikt} = 0 \quad \forall i \in \omega, k, t$$

$$x_{i(N+1)kt} = 0 \quad \forall i \in z, k, t \quad (13)$$

$$x_{iikt} = 0 \quad \forall i \in \Omega, k, t$$

$$x_{ijkt} = 0 \quad \forall i \in z, j \in \omega, k, t \quad (14)$$

$$Q_{iopkt} = 0 \quad \forall i \in \omega, p, k, t$$

$$R_{i(N+1)kt} = 0 \quad \forall i \in z, k, t \quad (15)$$

Equation (1) is determined to minimize total costs, including the inventory holding costs and the transportation costs. Constraint (2) creates an inventory balance between suppliers in periods of consecutive, namely, the level of inventory for product p at supplier i in period t is equal to its previous inventory, period $t-1$, and quantity transshipped by the vehicles in period t minus the quantity picked up by the vehicle during period t . Set of constraints (3) ensure balance for inventory of final product at the assembly planting periods of consecutive. Set of Constraints (4) emphasis suppliers and customers must be visited by the vehicles at most once in each period. Set of Constraints (5) is showing a balance for the arc (i, j) related to traffic inventory and final products in each period. Set of Constraints (6) guarantees the vehicle's capacity for transferring inventories and final products. Constraint (7) expresses the number of final products is more than or equal to demand of the product in each period. Constraint (8) expresses the inventories picked up from each supplier in each period should not exceed the suppliers' capacity in the previous period. Constraint (9) ensures maximum number of vehicles available in each period. Set of Constraints (10 and 11) are shown there are at least one trip from the plant to suppliers and customers in each period. Constraint (12) is related to the greenhouse gas emissions. Set of Constraints (13-16) determine the impossible arcs.

3. Experimental results



A proper method to compare and be effective the proposed model is using it for solving a few examples with different sizes. Therefore, to evaluate the suggested model in the continuing part of the article two examples designed and solved that one of them is for the small size, and the other one is for large-size problem.

3.1. Small-sized test problem

Suppose a company that has two vehicles for carrying its products. The characteristics of the vehicles like the fix and variable costs, the number of each type of it, capacity and the affection of greenhouse gas on the environment are presented in the table 1. The planning time horizon is assumed to be two periods.

Table 1. Vehicle characteristics

vehicle type k	v_k	f_k	NT_{kt}		Cap_k	GHG_k
			$t = 1$	$t = 2$		
1	13	1000	3	3	500	1.3
2	11	3000	3	3	1000	5.1

This company has several suppliers, and each of them produces only one product type. In addition, the company uses these incomplete products from the supplier to assemble and produce one product type. These suppliers are established in the specific distances from each other's and the assembly plant which are shown in table 2. The final product of plant purchases by some customers which are in the specific distances (table 2). In addition, the distances between the depot of vehicles with suppliers, plant and customers are clear in the table 2.

Table 2. Travel distances between nodes (L_{ij})

L_{ij}	depot	S_1	S_2	S_3	S_4	S_5	plant	C_1	C_2	C_3
depot	0	30	25	50	60	90	90	240	200	180
S_1		0	35	50	45	70	65	200	150	160
S_2			0	30	60	70	95	120	145	150
S_3				0	50	45	120	205	185	160
S_4					0	40	45	265	110	130
S_5						0	60	165	105	200
plant							0	125	100	110
C_1								0	100	95
C_2									0	85
C_3										0

The other suppositions for this problem are as follows:



The initial inventories of products in all nodes are assumed to be zero.

The unit inventory holding cost per period for the final product is 20 and incomplete products are 5.

In the table 3, we can see the demand of the assembly plant for incomplete products from suppliers and on the other hand, the demand of customer for the final product from the plant.

Table 3. Demand for each product in each period

P	Period t		C	Period t	
	1	2		1	2
1	200	400	1	20	15
2	0	200	2	25	10
3	300	150	3	35	20
4	350	0			
5	150	100			

The aim of this problem is number of final product in

production costs and define optimum routes to minimize the cost of transportation and greenhouse gas emission. Finally, the average authorized GHG emission levels among all periods are assumed to be 950.

determining the optimum each period to decrease

We used a mixed integer linear programming model and GAMS software to solve. All computations were performed on a PC Pentium IV-1.8 GHz i5 with 1GB RAM operating under Windows 7. The results are reported in tables 4-9.

Table 4. Greenhouse gas emission level (comparison)

	Relaxed model	Green model	Δ%
Greenhouse gas emission level	1628	906	-44.34

Table 5. Objective function component (comparison)

	Relaxed model	Green model	Δ%
Inventory holding cost	0	1750	-
Transportation cost	16960	17720	4.28
Production cost	7900	7900	-
Total cost	24860	27370	9.17

Solving the problem by GAMS software last 5 seconds. As seen in tables 4-9, we compared best solution in two cases. By considering GHG emission level limitation and on the other hand, ignoring the GHG constraint. In Table 5, the GHG emission level produced by the vehicles during the planning time horizon is compared for both the models, namely Relaxed and Green. As shown in the Table, a 44.34% savings is obtained by using the GHG limit. As seen in table 6, the total cost increases 9.17%. The increase probability related to an extra charge incurred by using fuel-efficient and expensive vehicles to satisfy the GHG limitations. This increase is also related to inventory holding costs all of products, namely final and suppliers' once. In Table 6 to 9, all of the decision variables (x), pickup values (a), transshipped quantities (b) and transshipped quantities of final product (b_v) is reported for the periods and is compared the value is related to Relaxed model with Green model.

Table 6. The visited arcs (X_{ijkt})

Relax Model	Green Model
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Period 1		Period 2		Period 1		Period 2	
X_{ijkt}	Value	X_{ijkt}	Value	X_{ijkt}	Value	X_{ijkt}	Value
$X_{(0,1,2,1)}$	1	$X_{(0,1,2,2)}$	1	$X_{(0,1,2,1)}$	1	$X_{(0,1,2,2)}$	1
$X_{(1,3,2,1)}$	1	$X_{(1,2,2,2)}$	1	$X_{(0,4,1,1)}$	1	$X_{(1,5,2,2)}$	1
$X_{(3,5,2,1)}$	1	$X_{(2,3,2,2)}$	1	$X_{(1,2,2,1)}$	1	$X_{(5,6,2,2)}$	1
$X_{(5,4,2,1)}$	1	$X_{(3,5,2,2)}$	1	$X_{(2,3,2,1)}$	1		
$X_{(4,6,2,1)}$	1	$X_{(5,6,2,2)}$	1	$X_{(3,5,2,1)}$	1		
				$X_{(4,6,1,1)}$	1		
				$X_{(5,6,2,1)}$	1		

Table 7. Pickups (a_{ipt})

Relax Model				Green Model			
Period 1		Period 2		Period 1		Period 2	
a_{ipt}	Value	a_{ipt}	Value	a_{ipt}	Value	a_{ipt}	Value
$a_{(1,1,1)}$	200	$a_{(1,1,2)}$	400	$a_{(1,1,1)}$	200	$a_{(1,1,2)}$	400
$a_{(3,3,1)}$	300	$a_{(2,2,2)}$	200	$a_{(2,2,1)}$	200	$a_{(5,2,2)}$	200
$a_{(4,4,1)}$	350	$a_{(3,3,2)}$	150	$a_{(3,3,1)}$	450	$a_{(5,3,2)}$	150
$a_{(5,5,1)}$	150	$a_{(5,5,2)}$	100	$a_{(4,4,1)}$	350	$a_{(5,5,2)}$	100
				$a_{(5,5,1)}$	150		

Table 8. Transshipped quantities (b_{ipt})

Relax Model				Green Model			
Period 1		Period 2		Period 1		Period 2	
b_{ipt}	Value	b_{ipt}	Value	b_{ipt}	Value	b_{ipt}	Value
-	-	-	-	$b_{(5,2,1)}$	200	-	-
-	-	-	-	$b_{(5,3,1)}$	150	-	-

Table 9. Transshipped quantities for final product (bv_{it})

Relax Model				Green Model			
Period 1		Period 2		Period 1		Period 2	
bv_{it}	Value	bv_{it}	Value	bv_{it}	Value	bv_{it}	Value
$bv_{(1,1)}$	20	$bv_{(1,2)}$	15	$bv_{(1,1)}$	20	$bv_{(1,2)}$	15
$bv_{(2,1)}$	25	$bv_{(2,2)}$	10	$bv_{(2,1)}$	25	$bv_{(2,2)}$	10
$bv_{(3,1)}$	35	$bv_{(3,2)}$	20	$bv_{(3,1)}$	35	$bv_{(3,2)}$	20
$bv_{(4,1)}$	15	$bv_{(4,2)}$	18	$bv_{(4,1)}$	15	$bv_{(4,2)}$	18



3.2. Large-sized test problem

To highlight the impact of the transshipment option on supply chain performance as well as GHG levels, we generate a set of large-scale multi-period examples and analyze the results. The demands and distances are presented in Tables 10 and 11. Table 12 shows the dimension of each problem and the associated results.

Table 10. Distances between nodes in problems 1 to 5 (one block for each problem)

Node j	Node i														
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	20	30	15	12	14	30	60	50	34	34	45	56	76	36
1		0	21	32	42	53	43	21	65	23	65	23	23	23	12
2			0	11	23	45	55	33	21	90	76	44	16	10	10
3				0	32	24	36	76	54	34	21	33	77	55	26
4					0	86	78	66	77	40	120	10	35	25	65
5						0	20	32	34	54	99	23	45	62	40
6							0	66	77	24	22	20	64	58	60
7								0	94	12	20	60	73	38	27
8									0	11	30	50	23	92	43
9										0	80	40	14	19	54
10											0	23	54	32	65
11												0	34	62	45
12													0	78	67
13														0	11
14															0

Table 11. Demand for problems 1 to 5 (one block for each problem)

Product	Period									
	1	2	3	4	5	6	7	8	9	10
1	30	21	32	42	53	43	21	65	23	65
2	20	5	11	17	45	55	33	21	90	76
3	40	99	23	32	24	36	76	14	34	21
4	45	0	43	15	86	78	66	14	40	20
5	53	77	43	18	12	20	32	34	54	29
6	67	8	23	13	32	20	66	77	24	22
7	23	9	65	10	49	32	19	94	12	20
8	65	5	13	20	10	25	11	50	11	30
9	32	23	15	10	11	27	7	18	12	80
10	12	12	25	32	13	29	32	30	40	20
11	10	11	14	12	14	16	7	45	15	32
12	2	20	11	13	15	23	9	43	13	43
13	5	19	9	16	17	26	15	42	18	76
14	32	54	5	17	19	34	18	15	19	82

As shown in Table 12, considering the GHG constraint in the proposed model can reduce a considerable amount of GHG emissions (26.95% on average), while the total supply chain cost increases by 7.68% on average for the 5 test problems.

Table 12. Comparison between Relaxed and Green model for medium and large size problems

Problem	Period	Number of		Total cost			Average GHG level		
		Vehicle	Arcs	Relaxed	Green	%Δ	Relaxed	Green	-%Δ
1	3	2	21	26250	29730	11.88	1830	960	47.54
2	5	4	32	35544	39200	10.28	1670	963	42.33



3	7	5	44	83693	90789	8.47	2150	1732	19.44
4	8	6	120	162597	168634	3.71	3684	3352	9.01
5	10	8	180	176260	183410	4.05	5050	4220	16.43

3. Conclusion

In this paper, the Inventory Routing Problem, is investigated. So a mixed integer linear programming (MILP) model is developed. We have considered the final customer as the missing component in MirzapourAl-e-hashem and Rekik (2013) presented in their model. So the product is manufactured and assembled in the plant then the final products are delivered to the final customers. The proposed model has two distinct features. First, a transshipment option is considered as a possible solution to decrease travel distances. Under this policy, a vehicle provided a specific product for the assembly plant, either directly from the supplier which manufactured the product or from the temporary storage of the other suppliers resulting from previous trips. Second, various vehicle types with different capacities and GHG emission indices were considered. These features enabled the model to select the appropriate transportation mode (as well as the transportation route) to reduce the total supply chain costs and improve the environmental health criteria (lowering GHG emissions). The best way to compare and evaluate the effectiveness of the proposed model to apply it to solve a number of known issues have been solved previously by the authors is included. Due to we didn't find some known examples to test the performance of this model, we generate some problems in small, medium and large scale to evaluate the quality of the model and solved by GAMS software and the results are compared. The results showed that the mathematical modeling method presented in this paper is useful in practice.

For further research includes applying the proposed model to developing multi-objective models with respect to green logistics, multiproduct for customers, developing models under uncertain conditions.

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