Measuring Productivity Changes in Banking System

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Abstract

The purpose of this paper is to develop an output oriented methodology for measuring productivity changes by using data envelopment analysis (DEA) and malmquist productivity index (MPI), and applies it to five numbers of Iranian commercial banks over the 5-years period (2009-2013). In this regard, measuring the MPI with DEA for single process units has been researched by large number of authors up to now. In this paper and for extending the above measurements to the two-stage processes, we have proposed a model for calculating of the overall efficiency of two-stage processes via both CRS and VRS reference technologies, and combined them with MPI's distance functions for access to the two-stage DEA-based MPI conclusions. Finally, we have examined the results to banking system in Iran.

Keywords: DEA, MPI, Productivity Changes, Bank.

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1. Introduction

DEA is a linear programming and non-parametric based methodology to measure the relative efficiency which can measurement of homogeneous multiple inputs and outputs and can also evaluate decision-making units (DMU) both qualitatively and quantitatively. DEA was proposed by Charnes, Cooper and Rhodes in order to apply linear programming to estimate an empirical production technology frontier for first time which later known as the CCR model from their acronyms (Charnes et al, 1978). The evolutionary form of CCR model was suggested by Banker, Charnes and Cooper which later known as the BCC model from their acronyms (Banker et al, 1984). Since that, there have been several books and papers written on DEA models or its applying was developed by a large number of researchers. Orientation (Input/Output), returns to scale (Constant return to scale; CRS/Variable returns to scale; VRS), disposability and are different aspects that can be seen in these models.

Given that many production processes and services in real issues are interdependent and have several complexities, moreover traditional DEA models considering the DMUs as black boxes which are not considered of internal communication processes, therefore it is necessary that we adopt models compatible with these situations for a more detailed evaluation of the DMUs under discussion. One issue that widespread debated in recent years to solve this matter, is DMU with two-stage network structure. Consider the fundamental two stage process shown schematically in Fig.1, and whereas there exist n DMUs to be evaluated each of DMU_j (j=1,…, n) which has two sub processes with m inputs x_{ij} (i=1,…,m) in the first stage, D intermediate measures z_{dj} (d=1,…,D) as outputs of the first stage and inputs of the second stage, s outputs y_{rj} (r=1,…,s) in the second stage. According to the above mentioned matters, the efficiencies of the first and the second processes can be calculated as a single process by using the conventional DEA methodology. Ray and also Fried, Lovell and Eeckaut have raised the two stage DEA models so that using the standard model with desirable factors in the first stage and analysis regression in the second stage, however, undesirable factors as independent variables were considered (Ray, 1991 & Fried et al, 1993) . Kao is shown that overall efficiency under linear production frontiers is a weighted arithmetic mean of the efficiencies of the outputs (Kao, 1995). Similarly, decomposed the overall efficiency with respect to input factors as well, and some results are derived. Seiford and Zhu examined the performance of the 55 U.S. commercial banks via a two stage process that separates profitability and marketability as results of the first and second stage, respectively (Seiford and Zhu, 1999). Fare and Grosskopf proposed a method for decompose the black boxes of the traditional DEA to evaluate organizational performance and its components performance (Fare and Grosskopf, 2000). The proposed general structure of network DEA model can be applied to variety situations. Sexton and Lewis and Chilingerian and Sherman used standard two stage DEA models for evaluate of process performance on the major league baseball and health care applications, respectively (Sexton and Lewis, 2003 & Chilingerian and Sherman, 2011). Kao and Hwang modified the conventional DEA model by propose a relational two stage DEA model and tested in 24 non-life insurance companies (Kao and Hwang, 2008). Chen, Cook, Li and Zhu developed an additive efficiency decomposition approach wherein the overall efficiency is expressed as weighted sum of the efficiencies of the individual stages (Chen et al, 2009). Furthermore, the two stage DEA models has also been applied by a large number of researchers to measure the performance of information technology (Chen and zhu, 2004 & Chen et al, 2006), supply chain (Liang et al, 2006), Bank Industry (Fukuyama and Weber, 2010 & Zha and Liang, 2010 & Paradi et al, 2011), R&D (Li et al, 2012), aviation industry (Wanke, 2013), etc. It must be noted that major limitation of above models which is not applicable for measuring efficiency over time periods.

In recent years, among the researchers who are analyzed the performance of units, measured productivity changes over time is very important problem. Malmquist productivity index (MPI) was originally defined by Professor Sten Malmquist as a quality index for analyze the consumption of production resources (Malmquist, 1953). MPI is based on the concept of the production function and makes use of distance functions to measure productivity changes and also it can defined using input and output oriented distance functions. Hence, MPI as a concept is compatible and coincident with the DEA methodology. MPI approach was proposed and entered for first time in productivity literature with introduced by Caves, Christensen and Diewert (Caves et al, 1982). Afterwards, Fare, Grosskopf, Lindgren and Roos had developed an input oriented based non-parametric methodology for calculating productivity changes and applied it to some swedish pharmacies and were later named the FGLR decomposition from their acronyms (Fare et al, 1992). In this connection, they had combined ideas from the efficiency measurement by Farrell (Farrell, 1957) and the productivity measurement by Caves and his colleagues (Caves et al, 1982) and finally constructed the DEA-based MPI to decompose it into efficiency changes and technology changes (frontier shifts) over time. In this regard, Fare, Grosskopf, Lindgren and Roos developed FGLR decomposition to output oriented for measuring productivity growth analyzing in Swedish hospitals (Fare et al, 1994a). Afterwards, they had developed an output oriented based non-parametric methodology for calculating productivity changes and applied it to industrialized countries and was later named the FGNZ decomposition from their acronyms (Fare et al, 1994b). In this regard, we developed and compared FGLR and FGNZ decompositions to output oriented for measuring productivity growth analyzing in five Iranian commercial banks. Chen and Ali had introduced a new insight of input oriented DEA based MPI and applied it to computer industry (Chen and Ali, 2004). For this purpose, they provided an extension to the DEA-based Malmquist approach by analyzing the two aforementioned Malmquist components (efficiency and technology changes). Maniadakis and Thanassoulis had proposed a cost MPI that applicable when producers are cost minimizers and input prices are known (Maniadakis and Thanassoulis, 2004). Tohidi and razavian had introduced a circular global profit MPI in DEA (Tohidi and razavian, 2013). This index is applicable when the input costs and output prices are known and when manufacturers seek to maximize the total profit of their DMUs. Grifell-Tatje and Lovell had used the traditional BCC model to measuring the productivity growth of Spanish banking system with a single process (Grifell-Tatje and Lovell, 1997). Ray and Desli had taken a comment on the FGNZ approach and they had applied it to the same countries which had been observed adjacent years (Ray and Desli, 1997). Chen and Yeh had extended the output oriented DEA-based MPI with reference technology exhibiting VRS frontier and applied it to 34 commercial banks in Taiwan (Chen and Yeh, 2000). Balk had developed generic measure of scale efficiency for a multiple-input and multiple-output firm, and also he combined measures of technological change, technical efficiency change and scale efficiency change into a primal measure of productivity change (Balk, 2001). Mukherjee, Ray and Miller had isolated the contributions of technical change, technical efficiency change and scale efficiency change to productivity growth using DEA method with VRS technology for measuring the MPI's input distance functions and applied them to 201 large US commercial banks over the initial post-deregulation period during 1984-1990 (Mukherjee et al, 2001). Furthermore, the DEA based MPI models has also been applied by a large number of researchers in various industries and situations such as aviation industry (Piers and Fernandes, 2012), non-life insurance industry (Kao and Hwang, 2014), power industry (Arabi et al, 2014), banking system (Krishnasamy et al, 2003 & Portela and Thanassoulis, 2006), industrialized countries (Lovell, 2003) and etc.

In this paper, we propose an overall efficiency for two stage output oriented DEA models with both constant and variable returns to scale for production technology which can be used over time and apply it to measuring the productivity growth of under review banks.

The rest of the paper is organized as follows: Section 2 proposes the relational two-stage DEA based MPI models. Section 3 defines the two-stage DEA based MPI and measurement models. In the section 4, the proposed models and MPIs are applied to the productivity analysis of the Iranian commercial banks. Paper conclusions presented in the last Section.

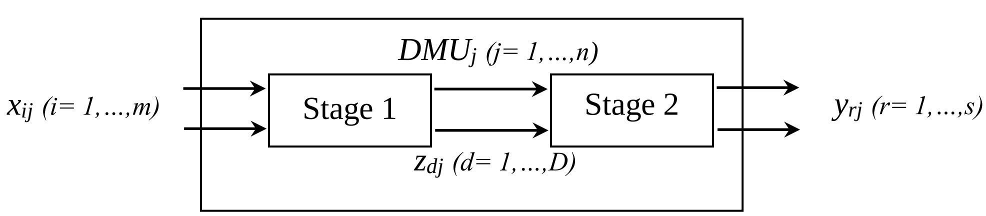

Fig.1. Fundamental Two Stage Process

2. Relational two-stage DEA Model based MPI

Consider n DMUs (DMU$_j$: j=1,…,n) using m inputs x_{ij} (i=1,…,m) to generate D outputs z_{dj} (d=1,…,D) in the first stage and D inputs z_{dj} (d=1,…,D) as intermediate measures to generate s outputs y_{rj} (r=1,…,s) in the second stage. Let x_{ij}, z_{dj} and y_{rj} be the ith input, Dth intermediate measure and sth output of the jth DMU, respectively. The efficiency of the DMU$_j$ through the conventional output-oriented DEA model under assumption of constant return to scale (CCR model) for DMU$_p$ is measuring as below:

$$Min \sum_{i=1}^{m} v_i x_{ip} \,/ \sum_{r=1}^{s} u_r y_{rp} \tag{1}$$

$$s.t. \sum_{r=1}^{s} u_r y_{rp} \,/ \sum_{i=1}^{m} v_i x_{ij} \le 1 \quad , j = 1, \dots, n$$

$$v_i, u_r \ge \varepsilon, \; i = 1, \dots, m, \; r = 1, \dots, s$$

Regarding to the above mentioned model for measuring the efficiency of DMU$_p$ with single process and considering the fundamental two-stage process shown in Fig. 1, we can be used it for measuring the efficiencies of DMU$_p$ in the two individual stages (first and second stages) as follows:

$$E_p^1 = Min \sum_{i=1}^{m} v_i x_{ip} \,/ \sum_{d=1}^{D} w_d z_{dp} \tag{2}$$

$$E_p^2 = Min \sum_{d=1}^{D} w_d z_{dp} \,/ \sum_{r=1}^{s} u_r y_{rp}$$

Based on the above efficiencies about the first and second stages of DMU$_j$, the overall efficiency of DMU$_j$ in the entire two-stage process defined in a number of methods such as below items:

- Kao and Hwang had defined the overall efficiency of DMU$_j$ as the product of the efficiencies of the two sub-processes that is to say $E_p = E_p^1 \times E_p^2$ (Kao and Hwang, 2008). It necessary be noted that aforesaid relational efficiency (E_p) only can be applied for efficiency measurement in single period and not practical for performance evaluation in the multiple period, but they had modified it later (Kao and Hwang, 2014).
- Chen and his colleagues had proposed the overall efficiency as weighted summarize of the two individual stages, namely, $Max\; w_1(\sum_{d=1}^{D} \eta_d z_{dp} \,/ \sum_{i=1}^{m} v_i x_{ip}) + w_2(\sum_{r=1}^{s} u_r y_{rp} \,/ \sum_{d=1}^{D} \eta_d z_{dp})$, where w_1 and w_2 are user-specified weights such that $w_1 + w_2 = 1$(Chen et al, 2009).
- Wang and Chin had defined the overall efficiency where the intermediate measures z_{dj} (d=1,…,D) serve as both inputs and outputs of DMU$_p$ at the same time (Wang and Chin, 2010). They introduced the $\lambda_1, \lambda_2 > 0$ as set of relative importance weighs of the two-stages such that $\lambda_1 + \lambda_2 = 1$. Then, the total input and output of DMUj could be measured as $\lambda_1 \sum_{d=1}^{D} \eta_d z_{dj} + \lambda_2 \sum_{r=1}^{s} u_r y_{rj}$ and $\lambda_1 \sum_{i=1}^{m} v_i x_{ij} + \lambda_2 \sum_{r=1}^{s} \eta_d z_{dj}$, respectively. Then, the overall efficiency of DMU$_p$ could be measured as Max $(\lambda_2 \sum_{r=1}^{s} u_r y_{rp} + \lambda_1 \sum_{d=1}^{D} \eta_d z_{dp}) / (\lambda_1 \sum_{i=1}^{m} v_i x_{ip} + \lambda_2 \sum_{d=1}^{D} \eta_d z_{dp})$.
- Saleh, Hosseinzadeh Lotfi and Toloie Eshlaghy had defined the overall efficiency where the intermediate measures z_{dj} (d=1,…,D) serve as outputs of DMU$_p$ (Saleh et al, 2011). Consequently, they proposed the below model for measuring the overall efficiency of the two-stage processes:

$$Min\; \theta_p \tag{3}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{ip} \qquad , i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j z_{dj} \geq z_{dp} \qquad ,d=1,\dots,D$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp} \qquad ,r=1,\dots,s$$
$$\lambda_j \geq 0,\ \varphi_p\ free,\ j=1,\dots,n$$

According to the above and consider the Fig. 2, we define the overall efficiency of DMU$_j$ such that the intermediate measures z_{dj} (d=1,…,D) serve as inputs of DMU$_p$. Thereupon, the propose output oriented two-stage DEA model under CRS assumption can be constructed as bellow model:

$$\begin{aligned} & Min \ (\sum_{i=1}^{m} v_i x_{ip} + \sum_{d=1}^{D} w_d z_{dp}) / \sum_{r=1}^{s} u_r y_{rp} \\ & s.t. \quad \sum_{r=1}^{s} u_r y_{rj} / (\sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D} w_d z_{dj}) \leq 1 \quad ,j=1,\dots,n \\ & u_r, v_i, w_d \geq \varepsilon,\ j=1,\dots,n \end{aligned} \tag{4}$$

Now, the dual model (4) is:

$$\begin{aligned} & Max \ \varphi_p \\ & s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ip} \qquad ,i=1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j z_{dj} \leq z_{dp} \qquad ,d=1,\dots,D \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi_p y_{rp} \qquad ,r=1,\dots,s \\ & \lambda_j \geq 0,\ \varphi_p\ free,\ j=1,\dots,n \end{aligned} \tag{5}$$

Considering the above models (4 & 5), we can add a free variable such as ω to the model no.4 and $\sum_{j=1}^{n} \lambda_j = 1$ to the model no.5 for modify the aforesaid models to the traditional BCC model. Further to the above relational efficiency, we can apply it to evaluating of the performance changes for DMUs between two periods for multi-period problems. For this purpose, we use the Malmquist productivity index (MPI), because that is an index which has been broadly used by researchers for measuring the performance changes and it can be combined with the DEA models.

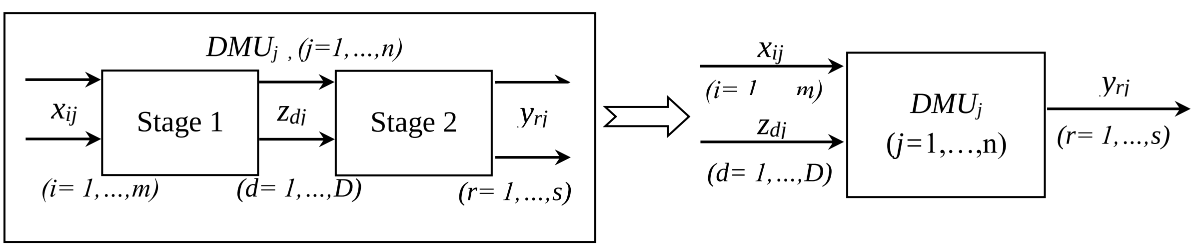

Fig.2. Transformation from two-stage Process to a Single Process

3. The Two-stage DEA-based MPI

According to the stipulated considerations in the above section, concerning inputs, intermediate measures and outputs of the two-stage processes, Denote x_{ij}^{t}, z_{dj}^{t} and y_{rj}^{t} as the process data at the time period t and x_{ij}^{t+1}, z_{dj}^{t+1} and y_{rj}^{t+1} at the time period $t+1$, respectively. For measuring the two-stage DEA based MPI under CRS reference technology, we should solve the following linear programming problems for two single period and two mixed period measures. For measuring the two-stage DEA based MPI under VRS reference technology, we can add a constraint as $\sum_{j=1}^{n} \lambda_j = 1$ to all following linear programming problems:

1) The first single period measures the efficiencies of DMUp in time period t

$$D_1^t(x_p^t \text{،} z_p^t) = Max\ \alpha_p^t(t) \qquad (6)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \mu_j^t x_{ij}^t \le x_{ip}^t \qquad ,i = 1, \dots, m$$

$$\sum_{j=1}^{n} \mu_j^t z_{dj}^t \ge \alpha_p^t(t) z_{dp}^t \qquad ,d = 1, \dots, D$$

$$\mu_j^t \ge 0,\ \alpha_p^t(t)\ free,\ j=1,\dots,n$$

$$D_2^t(z_p^t \text{،} y_p^t) = Max\ \beta_p^t(t) \qquad (7)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \delta_j^t z_{dj}^t \le z_{dp}^t \qquad ,d = 1, \dots, D$$

$$\sum_{j=1}^{n} \delta_j^t y_{rj}^t \ge \beta_p^t(t) y_{rp}^t \qquad ,r = 1, \dots, s$$

$$\delta_j^t \ge 0,\ \beta_p^t(t)\ free,\ j=1,\dots,n$$

$$D_O^t(x_p^t \text{،} y_p^t) = Max\ \varphi_p^t(t) \qquad (8)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le x_{ip}^t \qquad ,i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j^t z_{dj}^t \le z_{dp}^t \qquad ,d = 1, \dots, D$$

$$\sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge \varphi_p^t(t) y_{rp}^t \qquad ,r = 1, \dots, s$$

$$\lambda_j^t \ge 0,\ \varphi_p^t(t)\ free,\ j=1,\dots,n$$

2) The second single period measures the efficiencies of DMUp in time period $t+1$

$$D_1^{t+1}(x_p^{t+1} \text{،} z_p^{t+1}) = Max\ \alpha_p^{t+1}(t+1) \qquad (9)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \mu_j^{t+1} x_{ij}^{t+1} \le x_{ip}^{t+1} \qquad ,j = 1, \dots, n$$

$$\sum_{j=1}^{n} \mu_j^{t+1} z_{dj}^{t+1} \ge \alpha_p^{t+1}(t+1) z_{dp}^{t+1} \qquad ,d = 1, \dots, D$$

$$\mu_j^{t+1} \ge 0,\ \alpha_p^{t+1}(t+1)\ free,\ j=1,\dots,n$$

$$D_2^{t+1}(z_p^{t+1}, y_p^{t+1}) = Max\ \beta_p^{t+1}(t+1) \tag{10}$$

s.t.

$$\sum_{j=1}^{n} \delta_j^{t+1} z_{dj}^{t+1} \le z_{dp}^{t+1} \qquad ,d = 1, \dots, D$$

$$\sum_{j=1}^{n} \delta_j^{t+1} y_{rj}^{t+1} \ge \beta_p^{t+1}(t+1) y_{rp}^{t+1} \qquad ,r = 1, \dots, s$$

$$\delta_j^{t+1} \ge 0,\ \beta_p^{t+1}(t+1)\ free,\ j=1,\dots,n$$

$$D_O^{t+1}(x_p^{t+1}, y_p^{t+1}) = Max\ \varphi_p^{t+1}(t+1) \tag{11}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j^{t+1} x_{ij}^{t+1} \le x_{ip}^{t+1} \qquad ,i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^{t+1} \le z_{dp}^{t+1} \qquad ,d = 1, \dots, D$$

$$\sum_{j=1}^{n} \lambda_j^{t+1} y_{rj}^{t+1} \ge \varphi_p^{t+1}(t+1) y_{rp}^{t+1} \qquad ,r = 1, \dots, s$$

$$\lambda_j^{t+1} \ge 0,\ \varphi_p^{t+1}(t+1)\ free,\ j=1,\dots,n$$

3) The first mixed period measures the efficiencies of DMUp in time period t with using the frontier of the time period t+1 instead of t

$$D_1^{t+1}(x_p^{t}, z_p^{t}) = Max\ \alpha_p^{t+1}(t) \tag{12}$$

s.t.

$$\sum_{j=1}^{n} \mu_j^{t+1} x_{ij}^{t+1} \le x_{ip}^{t} \qquad ,i = 1, \dots, m$$

$$\sum_{j=1}^{n} \mu_j^{t+1} z_{dj}^{t+1} \ge \alpha_p^{t+1}(t) z_{dp}^{t} \qquad ,d = 1, \dots, D$$

$$\mu_j^{t+1} \ge 0,\ \alpha_p^{t+1}(t)\ free,\ j=1,\dots,n$$

$$D_2^{t+1}(z_p^{t}, y_p^{t}) = Max\ \beta_p^{t+1}(t) \tag{13}$$

s.t.

$$\sum_{j=1}^{n} \delta_j^{t+1} z_{dj}^{t+1} \le z_{dp}^{t} \qquad ,d = 1, \dots, D$$

$$\sum_{j=1}^{n} \delta_j^{t+1} y_{rj}^{t+1} \ge \beta_p^{t+1}(t) y_{rp}^{t} \qquad ,r = 1, \dots, s$$

$$\delta_j^{t+1} \ge 0,\ \beta_p^{t+1}(t)\ free,\ j=1,\dots,n$$

$$D_O^{t+1}(x_p^{t}, y_p^{t}) = Max\ \varphi_p^{t+1}(t) \tag{14}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j^{t+1} x_{ij}^{t+1} \le x_{ip}^{t} \qquad ,i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^{t+1} \le z_{dp}^{t} \qquad ,d = 1, \dots, D$$

$$\sum_{j=1}^{n} \lambda_j^{t+1} y_{rj}^{t+1} \geq \varphi_p^{t+1}(t) y_{rp}^{t} \qquad , r = 1, \dots, s$$
$$\lambda_j^{t+1} \geq 0, \varphi_p^{t+1}(t) \text{ free}, \ j=1,\dots,n$$

4) The second mixed period measures the efficiencies of DMU_p in time period *t*+1 with using the frontier of the time period *t* instead of *t*+1

$$D_1^t(x_p^{t+1}, z_p^{t+1}) = Max\ \alpha_p^t(t+1) \tag{15}$$
s.t.
$$\sum_{j=1}^{n} \mu_j^t x_{ij}^t \leq x_{ip}^{t+1} \qquad , j = 1, \dots, n$$
$$\sum_{j=1}^{n} \mu_j^t z_{dj}^t \geq \alpha_p^t(t+1) z_{dp}^{t+1} \qquad , d = 1, \dots, D$$
$$\mu_j^t \geq 0, \alpha_p^t(t+1) \text{ free}, \ j=1,\dots,n$$

$$D_2^t(z_p^{t+1}, y_p^{t+1}) = Max\ \beta_p^t(t+1) \tag{16}$$
s.t.
$$\sum_{j=1}^{n} \delta_j^t z_{dj}^t \leq z_{dp}^{t+1} \qquad , d = 1, \dots, D$$
$$\sum_{j=1}^{n} \delta_j^t y_{rj}^t \geq \beta_p^t(t+1) y_{rp}^{t+1} \qquad , r = 1, \dots, s$$
$$\delta_j^t \geq 0, \beta_p^t(t+1) \text{ free}, \ j=1,\dots,n$$

$$D_O^t(x_p^{t+1}, y_p^{t+1}) = Max\ \varphi_p^t(t+1) \tag{17}$$
s.t.
$$\sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq x_{ip}^{t+1} \qquad , i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j^t z_{dj}^t \leq z_{dp}^{t+1} \qquad , d = 1, \dots, D$$
$$\sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq \varphi_p^t(t+1) y_{rp}^{t+1} \qquad , r = 1, \dots, s$$
$$\lambda_j^t \geq 0, \varphi_p^t(t+1) \text{ free}, \ j=1,\dots,n$$

Then, we can use the FGLR method for measuring the MPI (under CRS assumption) and its components for first stage, second stage and whole process as bellows:

$$MPI_{1C} = \frac{D_1^{t+1}(x^{t+1}, z^{t+1})}{D_1^t(x^t, z^t)} \left[\frac{D_1^t(x^{t+1}, z^{t+1})}{D_1^{t+1}(x^{t+1}, z^{t+1})} \, \frac{D_1^t(x^t, z^t)}{D_1^{t+1}(x^t, z^t)} \right]^{1/2} \tag{18}$$

$$MPI_{2C} = \frac{D_2^{t+1}(z^{t+1}, y^{t+1})}{D_2^t(z^t, y^t)} \left[\frac{D_2^t(z^{t+1}, y^{t+1})}{D_2^{t+1}(z^{t+1}, y^{t+1})} \, \frac{D_2^t(z^t, y^t)}{D_2^{t+1}(z^t, y^t)} \right]^{1/2} \tag{19}$$

$$MPI_{OC} = \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} \left[\frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^{t+1}, y^{t+1})} \, \frac{D_O^t(x^t, y^t)}{D_O^{t+1}(x^t, y^t)} \right]^{1/2}$$

(20)

Regarding to the MPI_C in (18, 19 & 20), $MPI_C>1$ demonstrates productivity progress, $MPI_C=1$ indicates productivity constant and $MPI_C<1$ represents productivity decline.

The first component of the MPI_C in (18, 19 & 20) is as bellows:

$$Efficiency\ Change\ (EC_1) = \frac{D_1^{t+1}(x^{t+1},z^{t+1})}{D_1^t(x^t,z^t)} \tag{21}$$

$$Efficiency\ Change\ (EC_2) = \frac{D_2^{t+1}(z^{t+1},y^{t+1})}{D_2^t(z^t,y^t)} \tag{22}$$

$$Efficiency\ Change\ (EC_O) = \frac{D_O^{t+1}(x^{t+1},y^{t+1})}{D_O^t(x^t,y^t)} \tag{23}$$

According to the EC in (21, 22 & 23), EC>1 indicates efficiency improve, EC=1 represents efficiency constant and EC<1 means that efficiency decline. The second component that measures the technological change is as follows:

$$Technological\ Change\ (TC_1) = \left[\frac{D_1^t(x^{t+1},z^{t+1})}{D_1^{t+1}(x^{t+1},z^{t+1})}\ \frac{D_1^t(x^t,z^t)}{D_1^{t+1}(x^t,z^t)}\right]^{1/2} \tag{24}$$

$$Technological\ Change\ (TC_2) = \left[\frac{D_2^t(z^{t+1},y^{t+1})}{D_2^{t+1}(z^{t+1},y^{t+1})}\ \frac{D_2^t(z^t,y^t)}{D_2^{t+1}(z^t,y^t)}\right]^{1/2} \tag{25}$$

$$Technological\ Change\ (TC_O) = \left[\frac{D_O^t(x^{t+1},y^{t+1})}{D_O^{t+1}(x^{t+1},y^{t+1})}\ \frac{D_O^t(x^t,y^t)}{D_O^{t+1}(x^t,y^t)}\right]^{1/2} \tag{26}$$

Chen and Ali had illustrated that analyzing and make conclusion from TC, needed more discussion because of the efficiency frontiers can have a downward or an upward shift (Chen and Ali, 2004).

Moreover, we can use the FGNZ method for measuring the MPI (under VRS assumption) and its components for first stage, second stage and whole process as bellows:

$$MPI_{1V} = \frac{D_1^{t+1}(x^{t+1},z^{t+1})}{D_1^t(x^t,z^t)}\left[\frac{D_1^t(x^{t+1},z^{t+1})}{D_1^{t+1}(x^{t+1},z^{t+1})}\ \frac{D_1^t(x^t,z^t)}{D_1^{t+1}(x^t,z^t)}\right]^{1/2} \tag{27}$$

$$MPI_{2V} = \frac{D_2^{t+1}(z^{t+1},y^{t+1})}{D_2^t(z^t,y^t)}\left[\frac{D_2^t(z^{t+1},y^{t+1})}{D_2^{t+1}(z^{t+1},y^{t+1})}\ \frac{D_2^t(z^t,y^t)}{D_2^{t+1}(z^t,y^t)}\right]^{1/2} \tag{28}$$

$$MPI_{OV} = \frac{D_O^{t+1}(x^{t+1},y^{t+1})}{D_O^t(x^t,y^t)}\left[\frac{D_O^t(x^{t+1},y^{t+1})}{D_O^{t+1}(x^{t+1},y^{t+1})}\ \frac{D_O^t(x^t,y^t)}{D_O^{t+1}(x^t,y^t)}\right]^{1/2} \tag{29}$$

According to the above mentioned equations (27, 28 & 29), the first factor on the write hand side (efficiency change) can be decomposed to the pure efficiency change (PEC) and scale efficiency change (SEC), furthermore, the technology change (TEC) as the second factor should not be decomposed. Therefore, MPI_V can be decomposed as bellow:

$$MPI_V = (PEC \times SEC) \times TEC \tag{30}$$

Regarding to the above MPI_V decomposition, we can be wrote PEC as bellows:

$$PEC_1 = \frac{D_1^{t+1}(x^{t+1},z^{t+1})}{D_1^t(x^t,z^t)} \tag{31}$$

$$PEC_2 = \frac{D_2^{t+1}(z^{t+1},y^{t+1})}{D_2^t(z^t,y^t)} \tag{32}$$

$$PEC_O = \frac{D_O^{t+1}(x^{t+1},y^{t+1})}{D_O^t(x^t,y^t)} \tag{33}$$

The SEC component should be wrote both CRS and VRS technologies as follows:

CRS: $\left[\frac{D_c^t(x^{t+1},y^{t+1})}{D_c^{t+1}(x^t,y^t)}\ \frac{D_c^{t+1}(x^{t+1},y^{t+1})}{D_c^t(x^t,y^t)}\right]^{1/2}$ (34)

VRS: $\left[\frac{D_v^t(x^t,y^t)}{D_v^{t+1}(x^{t+1},y^{t+1})}\ \frac{D_v^{t+1}(x^t,y^t)}{D_v^t(x^{t+1},y^{t+1})}\right]^{1/2}$ (35)

Hence, the SEC component for two-stage process can be written as bellows:

$$SEC_1=\left[\frac{D_{1c}^t(x^{t+1},z^{t+1})}{D_{1c}^{t+1}(x^t,z^t)}\ \frac{D_{1c}^{t+1}(x^{t+1},z^{t+1})}{D_{1c}^t(x^t,z^t)}\right]^{1/2}\times\left[\frac{D_{1v}^t(x^t,z^t)}{D_{1v}^{t+1}(x^{t+1},z^{t+1})}\ \frac{D_{1v}^{t+1}(x^t,z^t)}{D_{1v}^t(x^{t+1},z^{t+1})}\right]^{1/2} \quad (36)$$

$$SEC_2=\left[\frac{D_{2c}^t(z^{t+1},y^{t+1})}{D_{2c}^{t+1}(z^t,y^t)}\ \frac{D_{2c}^{t+1}(z^{t+1},y^{t+1})}{D_{2c}^t(z^t,y^t)}\right]^{1/2}\times\left[\frac{D_{2v}^t(z^t,y^t)}{D_{2v}^{t+1}(z^{t+1},y^{t+1})}\ \frac{D_{2v}^{t+1}(z^t,y^t)}{D_{2v}^t(z^{t+1},y^{t+1})}\right]^{1/2} \quad (37)$$

$$SEC_O=\left[\frac{D_{Oc}^t(x^{t+1},y^{t+1})}{D_{Oc}^{t+1}(x^t,y^t)}\ \frac{D_{Oc}^{t+1}(x^{t+1},y^{t+1})}{D_{Oc}^t(x^t,y^t)}\right]^{1/2}\times\left[\frac{D_{Ov}^t(x^t,y^t)}{D_{Ov}^{t+1}(x^{t+1},y^{t+1})}\ \frac{D_{Ov}^{t+1}(x^t,y^t)}{D_{Ov}^t(x^{t+1},y^{t+1})}\right]^{1/2} \quad (38)$$

Regarding to the above MPIs in 27, 28 & 29 equations, $MPI_V>1$ demonstrates productivity progress, $MPI_V=1$ indicates productivity constant and $MPI_V<1$ represents productivity decline.

The second components that measure the technological change are as follows:

$$TEC_1=\left[\frac{D_1^t(x^{t+1},z^{t+1})}{D_1^{t+1}(x^{t+1},z^{t+1})}\ \frac{D_1^t(x^t,z^t)}{D_1^{t+1}(x^t,z^t)}\right]^{1/2} \quad (39)$$

$$TEC_2=\left[\frac{D_2^t(z^{t+1},y^{t+1})}{D_2^{t+1}(z^{t+1},y^{t+1})}\ \frac{D_2^t(z^t,y^t)}{D_2^{t+1}(z^t,y^t)}\right]^{1/2} \quad (40)$$

$$TEC_O=\left[\frac{D_O^t(x^{t+1},y^{t+1})}{D_O^{t+1}(x^{t+1},y^{t+1})}\ \frac{D_O^t(x^t,y^t)}{D_O^{t+1}(x^t,y^t)}\right]^{1/2} \quad (41)$$

It must be remembrance that MPI_V, PEC and TEC factors has been constructed under VRS technology, but the SEC factor has been combined of CRS and VRS assumptions.

4. Application of the MPI_C and MPI_V to the Banking System

In this section, we apply the proposed method to measure and analyze the productivity changes of the five numbers of Iranian commercial banks over the five years period (2009-2013).

The inputs (x_{ij}), intermediate measures (z_{dj}) and outputs (y_{rj}) data are provided in Tables 1 to 5, where five commercial banks with name of Mellat, Saderat, Sina, Pasargad and Eghtesad Novin as the DMUs are evaluated. In this connection, physical assets (PA), number of employees (NE), deposits value (DV) and operational costs (OC) are the four factors which considered as the whole process inputs, also received commissions (RC), loans payments (LP) and investment amount (IA), are the three intermediate measures in two-stage process and finally whole process output is the net revenue (NR). It should be noted that all data all data which presented in below tables are based on the published reports from the independent auditor and legal inspector of the Banks, furthermore, all digits (except NE) are billion Rials.

Table 1: Data set for five DMUs with four inputs, three intermediate measures and one output in 2009

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	13,979	24737	386,262	17,827	3,505	1,234	4,563	3,770
Saderat	21,819	29218	324,713	22,083	3,125	6,430	14,654	3,813
Sina	745	1561	30,315	1,063	59	2,112	3,148	700
Pasargad	2,986	4067	105,121	19,415	1,389	400	1,116	3,109
Eghtesad N.	2,640	2693	96,417	2,875	771	276	646	2,150

Table 2: Data set for five DMUs with four inputs, three intermediate measures and one output in 2010

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	16,126	23997	487,596	34,311	4,799	2,735	7,107	6,590
Saderat	23,330	29379	410,007	24,226	3,875	14,009	26,621	7,391
Sina	969	1721	41,848	1,543	186	1,109	4,399	1,118
Pasargad	5,769	4531	136,769	30,638	1,910	1,473	2,954	5,924
Eghtesad N.	2,753	2970	115,640	3,286	767	276	1,200	3,003

Table 3: Data set for five DMUs with four inputs, three intermediate measures and one output in 2011

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	22,293	23014	558,787	34,153	6,578	2,118	20,852	8,067
Saderat	25,458	33856	570,490	30,309	5,512	3,808	21,487	5,111
Sina	1,687	2264	55,928	1,826	406	1,462	7,594	1,706
Pasargad	10,872	5708	166,091	37,674	2,429	2,086	6,298	9,522
Eghtesad N.	2,990	3907	152,071	3,167	989	474	2,773	4,490

Table 4: Data set for five DMUs with four inputs, three intermediate measures and one output in 2012

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	37,815	22495	826,116	37,747	5,221	2,190	26,945	15,159
Saderat	64,766	33079	523,476	34,391	4,843	1,147	21,863	7,888
Sina	1,847	2238	76,531	1,845	548	4,476	7,621	4,840
Pasargad	22,584	6720	227,412	38,818	4,996	593	4,987	13,558
Eghtesad N.	4,324	3861	194,576	4,524	1,207	1882	3,293	4,401

Table 5: Data set for five DMUs with four inputs, three intermediate measures and one output in 2013

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	43,025	22157	926,408	48,854	13,778	3,321	32,391	21,978
Saderat	69,991	32713	637,692	38,565	5,223	1,271	30,299	9,888
Sina	2,102	2374	93,866	2,404	733	753	6,922	2,592
Pasargad	51,127	7758	294,406	59,465	6,238	2,129	6,948	18,143
Eghtesad N.	3,967	4096	253,493	4,713	1,652	8,765	7,437	5,396

In this case, first of all and for each DMU, we run CCR model to calculate MPI's distance functions for two individual stages and overall process and then measure the MPI_C values for two-stage process by equations (18)~(20). Afterward, we can measure the EC and TC for two-stage process by equations (21)~(26). Similarly, we run BCC model to calculate MPI's distance functions and subsequently acquire their MPI_V values by equations (27)~(29), then measure the PEC, SEC and TEC for two-stage process by equations (31)~(41).

As it's clear in Table 6 about the results of the CCR model, productivities of all DMUs (except Sina) improved during 2009-2010 and productivity growth rates for the DMUs are 30.2% for Mellat, 56.99% for Saderat, -15.1% for Sina, 22.24% for Pasargad and 2.73% for Eghtesad N., respectively. It is clear that Second DMU (Saderat) achieved the greatest productivity progress with 56.99% increase in productivity, while third DMU (Sina) exhibited the most productivity regress with 15.1% decrease in productivity. Similarly, we can be analyzed the productivity changes of all DMUs during 2010-2013. Moreover, on average, the most annual productivity improvement is related to the Saderat Bank which it is resulted from the annual efficiency change (16.99% progress) and technology change (3.25% progress) during the years that under review.

Similarly, As we can see in Table 7 about the results of the BCC model, productivities of all DMUs (except Sina) improved during 2009-2010 and productivity growth rates for the DMUs are 30.2% for Mellat, 56.99% for Saderat, -15.1% for Sina, 22.24% for Pasargad and 2.73% for Eghtesad N., respectively. It is clear that Second DMU (Saderat) achieved the greatest productivity progress with 56.99% increase in productivity, while third DMU (Sina) exhibited the most productivity regress with 15.1% decrease in productivity. Similarly, we can be analyzed the productivity changes of all DMUs during 2010-2013. Moreover, on average, the most annual productivity improvement is related to the Saderat Bank which it is resulted from the annual pure efficiency change (2.25% regress), scale efficiency change (2.78% regress) and technology change (27.10% progress) during the years that under review.

For make final conclusion, we should be compared CCR and BCC values together. As specific, productivity growth rate of all DMUs during 2009-2011 are equal and therefore we can be made same conclusion about productivity changes for both CCR and BCC models, But, the only deference between these models is in the ingredients. For more clarification, we should be explained that the MPI_C has been involving the two component (EC and TC), but MPI_V has been involving the three component (PEC, SEC and TEC). For more explanation, we can be more detailed in study of the inefficiency causes/factors with using the BCC models than CCR models. In this regard, it must be noted that the mentioned explanations does not mean that the BCC model is better than the CCR model, but each of them can be used in appropriate circumstances and proper conditions.

5. Conclusions

The current paper develops relational models for measuring the total efficiency for whole process of a two-stage process unit in two conventional DEA models (CCR & BCC models). Subsequently, we measured the MPI's distance functions for two individual stages and whole process by aforesaid DEA models and supposed relational models in the case of output oriented attitude.

The achieved results have been shown us that the productivity growth in both CCR and BCC models are equal and their just difference in the required level of the detailed information for study is to discover the causes of inefficiency. Finally, for examine the propose models and also for analyze the productivity changes, we had been applied the models to the five Iranian commercial banks during 2009-2013. The results have distinctly proven our expected outcomes and shown that the MPI values measured from the CCR and BCC models are completely equal and just their decompositions type and internal factors are differ together.

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Table 6: The DEA-based MPI values for the Iranian commercial banks (CCR Model)

DMU	2009-2010			2010-2011			2011-2012			2012-2013			Average		
	MPI_{OC}	EC_O	TC_O	MPI_{OC}	EC_O	TC_O	MPI_{OC}	EC_O	TC_O	MPI_{OC}	EC_O	TC_O	MPI_{OC}	EC_O	TC_O
Mellat	1.3020	1.0758	1.2102	0.9377	1.0065	0.9316	1.7118	1.8254	0.9377	0.9339	1.0369	0.9007	1.1820	1.1965	0.9878
Saderat	1.5699	1.2892	1.2177	0.5076	0.4208	1.2062	2.3358	2.0457	1.1418	1.1436	1.6879	0.6775	1.2079	1.1699	1.0325
Sina	0.8490	1	0.8490	0.9258	0.9801	0.9446	1.7032	1.0203	1.6693	0.6487	1	0.6487	0.9654	1	0.9654
Pasargad	1.2224	1	1.2224	1.0235	1	1.0235	1.4382	1	1.4382	0.6559	1	0.6559	1.0423	1	1.0423
Eghtesad N.	1.0273	1	1.0273	1.0019	1	1.0019	0.8147	1	0.8147	0.9317	1	0.9317	0.9402	1	0.9402

Table 7: The DEA-based MPI values for the Iranian commercial banks (BCC Model)

DMU	2009-2010				2010-2011				2011-2012				2012-2013				Average			
	MPI_{OV}	PEC_O	SEC_O	TEC_O	MPI_{OV}	PEC_O	SEC_O	TEC_O	MPI_{OV}	PEC_O	SEC_O	TEC_O	MPI_{OV}	PEC_O	SEC_O	TEC_O	MPI_{OV}	PEC_O	SEC_O	TEC_O
Mellat	1.3020	1	0.8853	1.4707	0.9377	1	0.7058	1.3285	1.7118	1	1.0799	1.5851	0.9339	1	0.8154	1.1454	1.1820	1	0.8612	1.3724
Saderat	1.5699	1	0.9044	1.7358	0.5076	1	0.5596	0.9071	2.3358	0.6053	1.7586	2.1942	1.1436	1.5081	1.0038	0.7554	1.2079	0.9775	0.9722	1.2710
Sina	0.8490	1	0.9999	0.8491	0.9258	0.9802	1	0.9446	1.6693	1.0202	1	1.6694	0.6487	1	1	0.6487	0.9654	1	1	0.9654
Pasargad	1.2224	1	0.9406	1.2996	1.0235	1	0.9127	1.1213	1.4382	1	0.9317	1.5437	0.6559	1	0.9324	0.7035	1.0423	1	0.9293	1.1216
Eghtesad N.	1.0273	1	1	1.0273	1.0019	1	1	1.0019	0.8147	1	1	0.8147	0.9317	1	1	0.9317	0.9402	1	1	0.9402

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