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Quasi-Two-Dimensional Discrete Vapour Cavity Model

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ABSTRACT

In the simulation of transient flow in hydraulic piping systems, vapourous cavitation occurs when the calculated pressure head falls to the liquid vapour pressure head. Rapid valve closure or pump shutdown causes fast transient flow which results in large pressure variations, local cavity formation and distributed cavitation which potentially cause problems such as pipe failure, hydraulic equipment damage and corrosion. This paper presents a numerical study of water hammer with column separation in a simple reservoir-pipeline-valve system. In this study, as a new approach, water hammer with column separation has been modelled in a quasi-two-dimensional form. The governing equations for transient flow in pipes are solved based on the method of characteristics using a quasi-two-dimensional discrete vapour cavity model (quasi-two-dimensional DVCM) and a one-dimensional discrete vapour cavity model (one-dimensional DVCM). It was found that Quasi-two-dimensional DVCM correlates better with the experimental data than one-dimensional DVCM in terms of pressure magnitude. The simulation results clearly show that proper selection of the number of computational reaches and the number of computational grids in radial direction in quasi-two-dimensional DVCM significantly improves the computational results.

Keywords: Pipelines, Water hammer, Column separation, Transient cavitating flow, Quasi-two-dimensional discrete vapour cavity model.

1. Introduction

Pipes are used to carry liquid reliably in water supply systems, irrigation networks and nuclear power plants. Hydraulic systems have a broad change of flow velocity which produces a pressure change. The rapid valve closure or pump failure causes fluid transients which produce large pressure change and cavitation. Cavitation can have a serious effect on the pumps, valves and other components performance. Therefore, it would be significant to predict the commencement and amount of cavitation occurring in order to improve the performance and reliability of systems. This will allow improving both pump and circuit design.

Vapourous cavitation occurs when the liquid pressure drops to the liquid vapour pressure. It may happen in two different types of cavitating flow: (1) a localized vapour cavity and (2) distributed vapourous cavitation. The first type has a large void fraction and happens at a boundary like closed valve or at a high point along the pipeline. The second type extends over long sections of the pipe. The void fraction for this type is small and close to zero and it happens when the pressure drops to the liquid vapour pressure over an extended region of the pipe. The collapse of large vapour cavity and the propagation of the shock wave through the vapourous cavitation zone cause the vapour change back to liquid. When vapour cavities change to liquid, large pressures with steep wave fronts may happen. As an outcome, fluid transients may lead to severe damages.

Various types of vapourous cavitation models have been introduced [1] including discrete cavity and interface models. The discrete vapour cavity model (DVCM) is the most popular model for column separation and distributed cavitation in recent years [1]. One positive point of the DVCM is that it is easily implemented and that it reproduces many features of column separation in pipelines [1]. The DVCM may produce unrealistic pressure pulses (spikes) due to the collapse of multi-cavities [2], but there are some methods which reduce the unrealistic pressure pulses. Using quasi-two-dimensional DVCM or using unsteady friction model in one-dimensional DVCM reduces unrealistic pressure pulses.

In order to consider the DVCM in connection with transient flows, first it is needed to model the transient flow. Water hammer has been modelled by many researchers as either one-dimensional models or two-dimensional models. Although one-dimensional models are more popular, some important assumptions are ignored. For example, the velocity profile is assumed to be uniform in the cross section, but according to the no slip condition near the wall, the velocity should be considered zero. Therefore, two-dimensional models can be helpful to study some features which are not seen in one-dimensional models.

Quasi-two dimensional numerical model for turbulent water hammer flows has the attributes of being robust, consistent with the physics of wave motion and turbulent diffusion, and free from the inconsistency associated with the enforcement of the no slip condition while neglecting the radial velocity at boundary elements, such as valves and reservoirs [3].

Quasi-two-dimensional governing equations form a system of hyperbolic–parabolic partial differential equations which cannot, in general, be solved analytically [3]. The numerical solution of Vardy and Hwang (1991) solves the hyperbolic part of the governing equations by the method of characteristics and the parabolic part by finite differences in a quasi-two-dimensional form [4]. The solutions by Eichinger and Lein (1992) and Silva-Araya and Chaudhry (1997) solve the hyperbolic part of the governing equations by the method of characteristics in one-dimensional form and the parabolic part of the equations by finite differences in quasi-two-dimensional form [5, 6]. The solution by Pezzinga (1999) uses finite difference-based techniques to solve both hyperbolic part and parabolic part of the governing equations of turbulent flow in water hammer [7].

In this study, as a new approach, water hammer with column separation has been modelled in a quasi-two-dimensional form. The results and the efficiency of quasi-two-dimensional modeling have been compared with one-dimensional modeling. First, the structure of one-dimensional DVCM and quasi-two-dimensional DVCM are explained and then the DVCM is considered to study the column separation in a reservoir-pipeline-valve system. The numerical model of Vardy and Hwang (1991) is chosen for quasi-two-dimensional DVCM. They used the method of characteristics in longitudinal direction and finite-difference discretization in radial direction which makes it suitable to study the physics of the flow.

2. One-dimensional discrete vapour cavity model

When the pressure is more than the liquid vapour pressure, transient flow in pipelines is described by one-dimensional equations of continuity and motion [8]:

$$\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{fQ\left|Q\right|}{2DA}=0 \qquad (1)$$

$$c^2\frac{\partial Q}{\partial x}+gA\frac{\partial H}{\partial t}=0 \qquad (2)$$

where x = distance along the pipe, ρ = density of liquid, c = liquid (elastic) wave speed, t = time, f = Darcy-Weisbach friction factor, D = internal pipe diameter, g = gravitational acceleration, Q = discharge, H = pressure head and A = cross-sectional flow area.

The method of characteristics is a standard method for solving the unsteady-flow equations. The method of characteristics transformation of the equations gives the compatibility equations which are valid along the characteristics lines [8]. The compatibility equations, written in a finite-difference form for the *i*-th computational section within the staggered (diamond) grid are [8].

- along the C^+ characteristic line ($\Delta x/\Delta t = c$):

$$H_i^n-H_{i-1}^{n-1}+\frac{c}{gA}\left(Q_i^{n,u}-Q_{i-1}^{n-1,d}\right)+\frac{f\,\Delta x}{2gDA^2}Q_i^{n,u}\left|Q_{i-1}^{n-1,d}\right|=0 \qquad (3)$$

- along the C^- characteristic line ($\Delta x/\Delta t = -c$):

$$H_i^n-H_{i+1}^{n-1}-\frac{c}{gA}\left(Q_i^{n,d}-Q_{i+1}^{n-1,u}\right)-\frac{f\,\Delta x}{2gDA^2}Q_i^{n,d}\left|Q_{i+1}^{n-1,u}\right|=0 \qquad (4)$$

where Q^u and Q^d are the upstream and downstream discharge respectively which are introduced to accommodate the DVCM. They are equal for the case of no column separation. If the pressure falls to the liquid vapour pressure, column separation happens either as a discrete cavity or a vapourous cavitation zone in the liquid.
In the DVCM, cavities are allowed to form at the computational sections if the computed pressure becomes less than the liquid vapour pressure. However, the DVCM does not differentiate between localized vapour cavities and distributed vaporous cavitation [9]. The classical water hammer solution is no longer valid at a vapour pressure section. To solve the compatibility equations at a vapour pressure section, the head is set equal to the vapour pressure head H_{vap}. Pure liquid with a constant wave speed c is assumed to occupy between computational sections. Then the continuity equation for cavity volume $\forall_{vap}$ is expressed as [10]:

$$\left(\forall_{vap}\right)_i^n = \left(\forall_{vap}\right)_i^{n-2} + \left(\left(1-\psi\right)\left(Q_i^{n-2,d} - Q_i^{n-2,u}\right) + \psi\left(Q_i^{n,d} - Q_i^{n,u}\right)\right)2\Delta t \quad (5)$$

where ψ = weighting factor and takes values between 0 and 1.0. The cavity collapses when its calculated volume becomes negative and the one-phase liquid flow is re-established so that the water hammer solution using Eqs. (3) and (4) (with ($Q_i^{n,u} \equiv Q_i^{n,d}$) is valid again.

3. Friction Model

The friction term in one-dimensional transient flow is expressed as the sum of the unsteady part f_u and the quasi-steady part f_q:

$$f = f_q + f_u \quad (6)$$

There are several friction models which have been introduced by many researchers to consider the unsteady part. The Brunone model [11], which is based on mean flow velocity and local acceleration, has special popularity because of its simplicity and accuracy.
This model was improved by Vitkovsky in 1998 to predict a correct sign of the convective term in the case of closure of the upstream end valve in a simple pipeline system with the initial flow is in positive x direction [12]:

$$f_u = \frac{kD}{V\left|V\right|}\left[\frac{\partial V}{\partial t} + c\,sign\left(V\right)\left|\frac{\partial V}{\partial x}\right|\right] \quad (7)$$

where, k is the Brunone's friction coefficient and x is the distance along the pipe and sign(V) = (+1 for $V \geq 0$ or -1 for $V < 0$). Vardy and Brown proposed the following empirical relationship to derive the Brunone coefficient analytically:

$$k = \frac{\sqrt{C^*}}{2} \quad (8)$$

The Vardy shear decay coefficient C^* from Vardy and Brown (1996) is:
- laminar flow:

$$C^* = 0.00476 \quad (9)$$

- turbulent flow:

$$C^* = \frac{7.41}{\mathrm{Re}^{\log\left(14.3/\mathrm{R}^{0.05}\right)}} \quad (10)$$

in which Re = Reynolds number (Re = VD/υ).

4. Quasi-two-dimensional discrete vapour cavity model

Quasi-two-dimensional continuity and motion equations for an elastic pipe with circular cross section are defined as [4]:

$$\frac{g}{c^2}\frac{\partial H}{\partial t}+\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial rv}{\partial r}=0 \quad (11)$$

$$\frac{\partial u}{\partial t}+g\frac{\partial H}{\partial x}=\frac{1}{r\rho}\frac{\partial r\tau}{\partial r} \quad (12)$$

where x = distance along the pipe, r = distance from the axis in radial direction, t = time, H = pressure head, u = local longitudinal velocity, v = local radial velocity, g = gravitational acceleration, c = liquid (elastic) wave speed; ρ = density of liquid and τ = shear stress.

Numerical solutions are used to approximate the hyperbolic–parabolic partial differential equations. The numerical solution of Vardy and Hwang (1991) solves the hyperbolic part of the governing equations by the method of characteristics and the parabolic part by finite differences in a quasi-two-dimensional form.

The shear stress τ can be expressed as:

$$t = ru\frac{\partial u}{\partial r} - r\overline{u'v'} \quad (13)$$

where υ = kinematic viscosity, u , v = turbulence fluctuations corresponding to longitudinal velocity u, and radial velocity v respectively. Turbulence models are needed to solve the Reynolds stress term $\rho\overline{u'v'}$. According to the Boussinesq approximation, the turbulent shear stress is given by:

$$-\rho\overline{u'v'}=\rho\upsilon_t\frac{\partial u}{\partial r} \quad (14)$$

where, υ_t is eddy viscosity.

The grid system is shown in Fig. 1. The pipe is discretized into N_r cylinders with constant area ΔA in radial direction. The wall thickness of the mth cylinder is denoted by Δr_m, where $m = 1,\ldots,j,\ldots,N_r$ and $\Delta r_m = r_m - r_{m-1}$. The pipe length, L, is divided into N_x equal reaches such that $\Delta x = L/N_x$. The time step is determined by $\Delta t = \Delta x/c$ (i.e., Courant number $C_r = 1.0$).

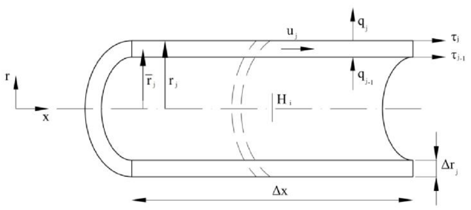

Fig. 1. Grid system for numerical solution

Using the grid shown in Fig. 1, integrating along the positive and negative characteristics gives:

$$H_i^{n+1}-\theta C_{q1}(j)q_{i,j-1}^{n+1}+\theta C_{q2}(j)q_{i,j}^{n+1}-\varepsilon C_{u1}(j)u_{i,j-1}^{n+1,u}+\left[\frac{c}{g}+\varepsilon C_{u2}(j)\right]u_{i,j}^{n+1,u}$$
$$-\varepsilon C_{u3}(j)u_{i,j+1}^{n+1,u}=H_{i-1}^{n}+(1-\theta)C_{q1}(j)q_{i-1,j-1}^{n}-(1-\theta)C_{q2}(j)q_{i-1,j}^{n}$$
$$+(1-\varepsilon)C_{u1}(j)u_{i-1,j-1}^{n,d}+\left[\frac{c}{g}-(1-\varepsilon)C_{u2}(j)\right]u_{i-1,j}^{n,d}+(1-\varepsilon)C_{u3}(j)u_{i-1,j+1}^{n,d} \quad (15)$$

$$H_i^{n+1}-\theta C_{q1}(j)q_{i,j-1}^{n+1}+\theta C_{q2}(j)q_{i,j}^{n+1}+\varepsilon C_{u1}(j)u_{i,j-1}^{n+1,d}-\left[\frac{c}{g}+\varepsilon C_{u2}(j)\right]u_{i,j}^{n+1,d}$$
$$+\varepsilon C_{u3}(j)u_{i,j+1}^{n+1,d}=H_{i+1}^{n}+(1-\theta)C_{q1}(j)q_{i+1,j-1}^{n}-(1-\theta)C_{q2}(j)q_{i+1,j}^{n}$$
$$-(1-\varepsilon)C_{u1}(j)u_{i+1,j-1}^{n,u}-\left[\frac{c}{g}-(1-\varepsilon)C_{u2}(j)\right]u_{i+1,j}^{n,u}-(1-\varepsilon)C_{u3}(j)u_{i+1,j+1}^{n,u} \quad (16)$$

where u_i^u and u_i^d are the upstream and downstream longitudinal velocity, respectively which have been introduced to accommodate the DVCM. They are equivalent for the classical water hammer. θ, ε = weighting coefficients; subscript i, j and superscript n indicate the spatial and temporal locations, respectively, of the grid point with coordinate ($i\Delta x$, r_j, $n\Delta t$); Δt = time step; $r_j=\sum_{m=1}^{j}\Delta r_m$; and $\overline{r_j}=(r_{j-1}+r_j)/2$ such that $r_0=0$. The coefficients in Eqs. (15) and (16) are as follows:

$$C_{q1}(j)=C_{q2}(j)=\frac{c^2\Delta t}{g}\frac{1}{\overline{r_j}\Delta r_j}\quad C_{u1}(j)=\frac{c\Delta t\upsilon_{T_{j-1}}}{g}\frac{1}{\overline{r_j}\Delta r_j}\frac{r_{j-1}}{(\overline{r_j}-\overline{r_{j-1}})}\quad C_{u3}(j)=\frac{c\Delta t\upsilon_{T_j}}{g}\frac{1}{\overline{r_j}\Delta r_j}\frac{r_j}{(\overline{r_{j+1}}-\overline{r_j})}$$
$$C_{u2}(j)=C_{u1}(j)+C_{u3}(j)$$

where υ_T = total viscosity.

When the computed head is more than the liquid vapour pressure at a given location along the pipe i, there are two equations for each cylinder, namely Eqs. (15) and (16). Since there are N_r cylinders in total, the number of equations is $2N_r$.

When computed head falls to the liquid vapour pressure, the classical water hammer solution is no longer valid at a vapour pressure section. The head at this section is set equal to the liquid vapour pressure head and it is needed to solve the compatibility equations separately and local radial velocity is neglected. It should be noted that in quasi-two-dimensional DVCM, the continuity equation for cavity volume is similar to one-dimensional DVCM (Eq. (5)).

5. Comparison of numerical models

The computational results are compared with the results of experimental studies conducted by Bergant and Simpson which were carried out using a long horizontal pipe with length of 37.20 m and inner diameter of 0.0221 m that connects upstream and downstream reservoirs . The water hammer wave speed was experimentally determined as c = 1319 m/s. A transient event is initiated by a rapid closure of the ball valve.

Five pressure transducers are mounted at equidistant points along the pipeline including as close as possible to the reservoirs. Pressures measured at the valve (H_v) and at the midpoint (H_{mp}) are presented in this paper. The uncertainties in the measurements are fully described by Bergant and Simpson [13].

In order to investigate the performance of quasi-two-dimensional DVCM and one-dimensional DVCM and the effects of mesh size on accuracy of the results, the numerical and experimental results are compared in three runs. Computational runs are performed for a rapid closure of the valve positioned at the downstream end of the horizontal pipe at the downstream reservoir (see **Error! Reference source not found.**). The initial velocity is V_0 = 0.3 m/s and the constant static head in the upstream reservoir and the vapour pressure head are H_{ur} = 22 m and H_{vap} = -10.25 m. The initial Reynolds number is 5970 and the rapid valve closure begins at time t = 0 s. The weighting factor ψ in Equation (5) was chosen 1.0 in all three runs. To study the effects of mesh size, various numbers of reaches were selected, N_x = {32, 128, 202} for quasi-two-dimensional DVCM and one-dimensional DVCM and different numbers of computational grids in radial direction, N_r = {20, 40, 50} were selected for quasi-two-dimensional DVCM.

Rapid valve closure for the discussed low-initial flow velocity case generates a water hammer event with moderate cavitation. The location and intensity of discrete vapour cavities is governed by the type of transient regime, layout of the piping system and hydraulic characteristics (Bergant and Simpson 1999). The maximum head at the valve which has been measured in the lab, is 96.6 m and it occurred 0.18 s after valve closure.

The computational results for the first run with the number of computational reaches N_x = 32 for quasi-two-dimensional DVCM and one-dimensional DVCM and the number of computational grids in the radial direction N_r = 20 for quasi-two-dimensional DVCM are presented in Fig. 2. Fig. 2 agrees well till 0.22 s. The discrepancies between the results are magnified later times. The maximum computed heads predicted by quasi-two-dimensional DVCM and one-dimensional DVCM are:

(1) Quasi-two-dimensional DVCM: $H_{v,max} = \{N_x = 32, N_r = 20, \psi = 1$, 111.588 m at t = 0.169 s}

(2) One-dimensional DVCM: $H_{v,max} = \{N_x = 32, \psi = 1$, 110.53 m at t = 0.173 s}

Both quasi-two-dimensional DVCM and one-dimensional DVCM slightly overestimate the maximum heads. According to Fig. 2, one-dimensional DVCM yields better conformance with the experimental data while quasi-two-dimensional DVCM yields poor results, and gives a better timing of the transient event than quasi-two-dimensional DVCM.

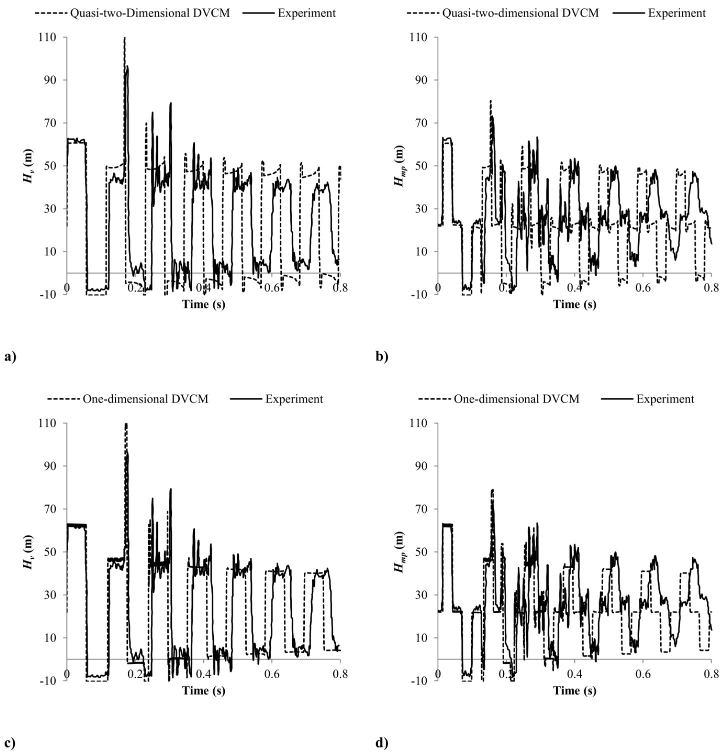

Fig. 2. Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}): V_0 = 0.3 m/s, ψ = 1, N_x = 32 and N_r = 20

The results of the second and third runs are presented in Figs. 4 and 5 respectively. It should be noted that in the second run the number of longitudinal reaches is 128 and the radial ones is 40, and in the third run these parameters are considered 202 and 50 respectively.

Figs. 4 and 5 agree well till 0.22 s, and the discrepancies between the numerical and experimental results are magnified as time increases. The maximum computed heads predicted by quasi-two-dimensional DVCM and one-dimensional DVCM are:

In the second run:

(1) Quasi-two-dimensional DVCM: $H_{v,max} = \{N_x = 128, N_r = 40, \psi = 1$, 109.30 m at t = 0.172 s}

(2) One-dimensional DVCM: $H_{v,max}$ = {N_x = 128, ψ = 1, 110.31 m at t = 0.175 s}

In the third run:

(1) Quasi-two-dimensional DVCM: $H_{v,max}$ = {N_x = 202, N_r = 50, ψ = 1, 108.55 m at t = 0.172 s}

(2) One-dimensional DVCM: $H_{v,max}$ = {N_x = 202, ψ = 1, 110.19 m at t = 0.175 s}

In both runs, both quasi-two-dimensional DVCM and one-dimensional DVCM slightly overestimate the maximum heads. In the second run (Fig. 3), the quasi-two-dimensional DVCM has become more successful to predict the maximum head, the same result is also seen in the third run (Fig. 4), but still one-dimensional DVCM has better agreement in terms of simulating the time of the maximum head in both second and third runs.

In these two models, unrealistic pressure pulses do not exist and generally, quasi-two-dimensional DVCM exhibits a capability to reproduce the experimental oscillations while one-dimensional DVCM disregards them and just reproduces them with sufficient accuracy in a short time immediately after closing the valve (Figs. 3, 4 and 5). Results obtained by one-dimensional DVCM show strong attenuation of the main pressure pulses at later times (see Figs. 3, 4 and 5). It is worth noting that one-dimensional DVCM produces less phase shift than quasi-two-dimensional DVCM even in the finest mesh.

The influence of different numbers of reaches (N_x) for quasi-two-dimensional DVCM and one-dimensional DVCM and the influence of different numbers of computational grids in radial direction (N_r) for quasi-two-dimensional DVCM were investigated. Examination of computational results reveals numerically stable behavior of the models.

Apparently, when the number of reaches (N_x) and the number of computational grids in radial direction (N_r) are larger, quasi-two-dimensional DVCM gives better results when it comes to the maximum heads and timing of the transient event and it gives a better prediction of the oscillations which exist in the experimental results.

In the first run with the coarsest mesh, the maximum volume of the cavity at the valve predicted by one-dimensional DVCM and quasi-two-dimensional DVCM is about 1.05×10^{-6} m^3 and 0.53×10^{-6} m^3 respectively (0.24 % of the reach volume for one-dimensional DVCM and 0.12 % of the reach volume for quasi-two-dimensional DVCM). In the second run, the maximum volume of the cavity at the valve predicted by one-dimensional DVCM and quasi-two-dimensional DVCM equals to about 0.96×10^{-6} m^3 and 0.54×10^{-6} m^3 respectively (0.87 % of the reach volume for one-dimensional DVCM and 0.49 % of the reach volume for quasi-two-dimensional DVCM). Finally, the values are about 0.93×10^{-6} m^3 in quasi-two-dimensional DVCM and 0.54×10^{-6} m^3 in one-dimensional DVCM (1.33 % of the reach volume for one-dimensional DVCM and 0.7 % of the reach volume for quasi-two-dimensional DVCM).

Careful examination of quasi-two-dimensional DVCM and one-dimensional DVCM reveals that one-dimensional DVCM produces more intense cavitation along the pipe than quasi-two-dimensional DVCM. The discrepancies between the computed results found by time-history comparisons may be attributed to the intensity of cavitation along the pipeline (distributed vapourous cavitation regions, actual number and position of intermediate cavities) resulting in a slightly different timing of cavity collapse and consequently a different superposition of waves.

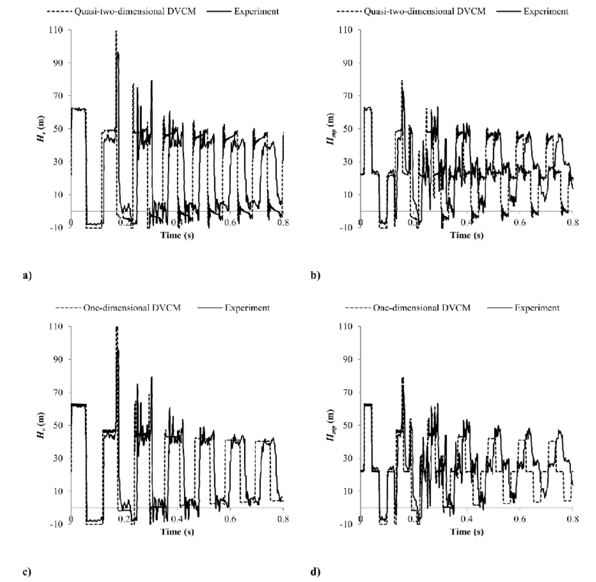

Fig. 3. Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}): $V_0 = 0.3$ m/s, $\psi = 1$, $N_x = 128$ and $N_r = 40$

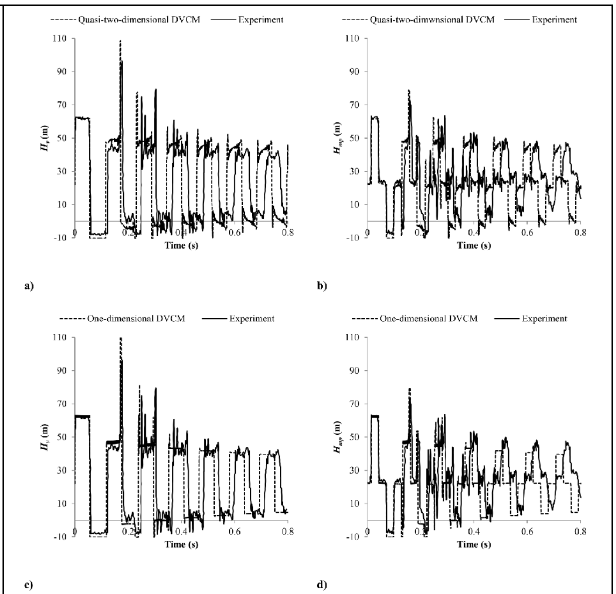

Fig. 4. Comparison of heads at the valve (H_v) and at the midpoint (H_{mp}): $V_0 = 0.3$ m/s, $\psi = 1$, $N_x = 202$ and $N_r = 50$

6. Conclusion

Column separation occurs when the liquid pressure decreases to the liquid vapour pressure. When vapour cavities change to liquid, large pressures with steep wave fronts may happen. The DVCM is the most popular model for column separation and distributed cavitation in recent years. Unrealistic pressure pulses (spikes) in the DVCM due to the collapse of multi-cavities can be reduced by using quasi-two-dimensional DVCM or using unsteady friction model in one-dimensional DVCM.

In this study, as a new approach, transient flow with column separation has been modelled in quasi-two-dimensional form. In comparison of quasi-two-dimensional with one-dimensional models, the following results were deduced:

- Quasi-two-dimensional DVCM is better at simulating the oscillations which exist in the experimental results while one-dimensional DVCM does not show these oscillations.
- Quasi-two-dimensional DVCM corresponds with sufficient accuracy to the experimental data and predicts the maximum heads very well for large number of reaches (N_x) and computational grids in radial direction (N_r).
- There are not unrealistic pressure pulses in quasi-two-dimensional DVCM and one-dimensional DVCM which are one of the drawbacks of the DVCM.
- One-dimensional DVCM estimates the cavity volume larger than quasi-two-dimensional DVCM.
- One-dimensional DVCM produces less phase shift than two-dimensional DVCM even in the finest mesh.

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