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Chaotically Tuples of Unilateral Weighted Backward Shifts Acting On Hilbert Spaces

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Abstract
We investigate characterize the Chaotically Conditions for the Tuples of unilateral weighted backward shifts on some Hilbert Spaces. The Tuple $T=(T_1,T_2,T_3,...,T_n)$ is chaotic if and only if T is Hypercyclic and has a non-trivial periodic point if and only if T has a non-trivial periodic point if and only if the series $\sum_{m=1}^{\infty}\left(\prod_{k=1}^{m}(e_{k,\lambda})^{-1}e_m\right), \lambda=1,2,...,n$ are convergence.
Keywords: Hypercyclicity, periodic point, Chaotically Tuples, Infinity Tuples.

Introduction

Let B be an Ordered Banach space and $T_1, T_2, T_3, \ldots$ are commutative bounded linear mapping on B, the infinity Tuple T is an infinity components $T = (T_1, T_2, T_3, \ldots)$, for every $x \in B$ defined

$$T(x) = T_1T_2T_3 \ldots (x) = Sup_n \{T_1T_2T_3 \ldots T_n(x) | n \in \mathbb{N}, n = 1,2,3,\ldots\}$$

Infinity-Tuple $T = (T_1, T_2, T_3, \ldots)$ is said to be hypercyclic infinity-tuple if $Orb(T, x)$ is dense in B, that is $\overline{Orb(T, x)} = B$.

Definition 1.1 The Tuple $T = (T_1, T_2, T_3, \ldots)$ is called chaotic tuple, if we have tree below conditions together,

1. It is topologically transitive, that is, for any given open sets U and V, there exist sequence of positive integers $\{\delta_j\}_{j=1}^{n}$ such that $T_1^{\delta_1} T_2^{\delta_2} \ldots T_n^{\delta_n} (U) \bigcap (V) \neq \phi$.
2. It has a dense set of periodic points, that is, there is a set P such that for each x in P, we can find $\{\lambda_j\}_{j=1}^{n}$ such that $T_1^{\lambda_1} T_2^{\lambda_2} \ldots T_n^{\lambda_n} (x) = x$.

It has a certain property called sensitive dependence on initial conditions.

Equations

If the Tuple satisfy the bellow theorem, we say that Tuple satisfy The Hypercyclic Criterion.

Theorem 1.1 [The Hypercyclicity Criterion] Let B be a separable Banach space and $T = (T_1, T_2, T_3, \ldots)$ is an infinity tuple of commutative continuous linear mappings on B. If there exist two dense subsets Y and Z in B and strictly increasing sequences $\{m_{k,2}\}_{k=1}^{\infty}$, $\{m_{k,3}\}_{k=1}^{\infty}$, … such that:

1. $T_1^{k_{1,j}} T_2^{k_{2,j}} T_3^{k_{3,j}} \ldots \to 0$ on Y as $m_{i,j} \to \infty$ for $i = 1,2,3,\ldots$,
2. There exist function $\{S_k | S_k : Z \to B\}$ such that for every $z \in Z$, $S_k z \to 0$ and $T_1^{k_{1,j}} T_2^{k_{2,j}} T_3^{k_{3,j}} \ldots S_k z \to z$

Then $T = (T_1, T_2, T_3, \ldots, T_n)$ is a Hypercyclic Tuple.

Theorem 2.2 Suppose X be an F-sequence space whit the unconditional basis $\{e_k\}_{k \in N}$ and let $T_1, T_2, T_3, \ldots T_n$ are unilateral weighted backward shifts with weight sequence $\{e_{1,k}\}_{k\in N}, \{e_{2,k}\}_{k\in N}, \ldots, \{e_{n,k}\}_{k\in N}$ and $T=(T_1,T_2,T_3,...,T_n)$ be a tuple of operators $T_1,T_2,T_3,...T_n$. Then the following assertions are equivalent:

1. T is chaotic,
2. T is Hypercyclic and has a non-trivial periodic point,
3. T has a non-trivial periodic point,
4. The series $\sum_{m=1}^{\infty}\left(\prod_{k=1}^{m}(e_{k,i})^{-1}e_m\right)$ are convergence in X for $i=1,2,...,n$.

Proof. Proof of the cases $1\to 2$ and $2\to 3$ are trivial, so we just proof $3\to 4$ and $4\to 1$.

First we proof $3\to 4$, for this, Suppose that T has a non-trivial periodic point, and $x=\{x_n\}_{x_n\in X}$ be a non-trivial periodic point for T, that is there are positive integers μ_1, μ_2, …, μ_n such that $T_1^{\mu_1}T_2^{\mu_2}...T_n^{\mu_n}(x)=x$. Comparing the entries at positions, $k\in N\bigcup\{0\}$, of x and $T_1^{\mu_1}T_2^{\mu_2}...T_n^{\mu_n}(x)$, we will find that

$$x_{j+kM_1}=\prod_{t=1}^{M_1}\left(a_{j+kN+t}.x_{j+k+1}\right), x_{j+kM_2}=\prod_{t=1}^{M_2}\left(a_{j+kN+t}.x_{j+k+1}\right),\ldots, x_{j+kM_n}=\prod_{t=1}^{M_n}\left(a_{j+kN+t}.x_{j+k+1}\right)$$

so for $k\in N\bigcup\{0\}$ and $\lambda=1,2,...,n$, we have

$$x_{j+kM_\lambda}=\left(\prod_{t=j+1}^{j+kM_\lambda}a_t\right)^{-1}.x_j=c_\lambda.\left(\prod_{t=1}^{j+kM_{\lambda 1}}a_t\right)^{-1},\lambda=1,2,...,n$$

Where $c_\lambda=\prod_{t=1}^{j}\left(m_{j,\lambda}.x_j\right),\lambda=1,2,...,n$. Since $\{e_\lambda\}_{\lambda=1}^{\infty}$ is an unconditional basis foe X and $x\in X$ it follows that

$$\sum_{k=0}^{\infty}\left(\frac{1}{\prod_{t=1}^{(j+kM_\lambda)}m_{j,\lambda}}.e_{j+M_\lambda}\right)=\frac{1}{c}\sum_{k=0}^{\infty}\left(x_{j+M_\lambda}.e_{j+M_\lambda}\right),\lambda=1,2,...,n$$

convergence sequences in X. Without loss of generality we may assume that $j\geq N$. Applying the operators $T,T_2,T_3,...,T_{k-1}$, where $k=Min\{M_1,M_2,...,M_n\}$, to this series and note that

$$T_1(e_t)=a_{1,t}e_{t-1},T_2(e_t)=a_{2,t}e_{t-1},\ldots,T_n(e_t)=a_{n,t}e_{t-1}$$

we deduce that

$$\sum_{k=0}^{\infty}\left(\frac{1}{\prod_{t=1}^{(j+kM_1-\varepsilon_1)} m_{1,j}}\right).e_{(j+kM_1-\varepsilon_1)},\sum_{k=0}^{\infty}\left(\frac{1}{\prod_{t=1}^{(j+kM_2-\varepsilon_2)} m_{2,j}}\right).e_{(j+kM_2-\varepsilon_2)},\dots,\sum_{k=0}^{\infty}\left(\frac{1}{\prod_{t=1}^{(j+kM_n-\varepsilon_n)} m_{n,j}}\right).e_{(j+kM_n-\varepsilon_n)}$$

convergence sequences in X. By adding these series, we see that condition 4 holds.

Proof of $4 \to 1$. It follows from theorem (2.1), so under condition 4 the operator T is Hypercyclic. Hence it remains to show that T has a dense set of periodic points. Since $\{e_\alpha\}$ is an unconditional basis, condition 4 with proposition 2.3 implies that for each $M_l \in N$ consider the series

$$\varphi_\lambda(j,M_\lambda)=\sum_{k=0}^{\infty}\frac{1}{\prod_{t=1}^{j+kM_\lambda} m_{k,\lambda}.e_{j+kM_\lambda}}=\prod_{t=1}^{j} m_{k,\lambda}.\sum_{k=0}^{\infty}\frac{1}{\prod_{t=1}^{j+kM_\lambda} m_{k,\lambda}.e_{j+kM_\lambda}},\lambda=1,2,\dots,n$$

All the series converges and define n elements in X. Moreover, if $Mj \geq 0$ then

$$T_1^{m_{j,1}}T_2^{m_{j,2}}\dots T_3^{m_{j,n}}=\varphi_1(j,M_1).T_1^{M_1}T_2^{M_2}\dots T_3^{M_n}.\varphi_1(j,M_1)=\varpi(j,M_1)$$

if $Nj \geq 0$ then

$$x_{j+kM_1}=\prod_{t=1}^{M_1}\left(a_{j+kN+t}.x_{j+k+1}\right),x_{j+kM_2}=\prod_{t=1}^{M_2}\left(a_{j+kN+t}.x_{j+k+1}\right),\dots,x_{j+kM_n}=\prod_{t=1}^{M_n}\left(a_{j+kN+t}.x_{j+k+1}\right)$$

when $m_{i,j} \geq M$, $i=1,2,\dots,n$. So that each $\varphi(i,N)$ for $i=1,2,\dots,n$ is a periodic point for T. We shall show that T has a dense set of periodic points. Since $\{e_\lambda\}_{\lambda=1}^{\infty}$ is a basis, it suffices to show that for every element $x \in span\{e_\lambda : \lambda \in N\}$ there is a periodic point y arbitrarily close to it. For this, let $x=\sum_{j=1}^{m} x_j.e_j$ and $\varepsilon > 0$. We can assume without lost of generality that

$$\left|x_1.\prod_{t=1}^{1} a_{1,t}\right| \leq 1,\left|x_2.\prod_{t=1}^{2} a_{2,t}\right| \leq 1,\dots,\left|x_n.\prod_{t=1}^{n} a_{n,t}\right| \leq 1$$

Since $\{e_n\}_{n=1}^{\infty}$ is an unconditional basis, then condition 4 implies that there are $M_j \geq m, j=1,2,\dots,n$ such that

$$\left\|\sum_{n=M_1+1}^{\infty}\left(\varepsilon_{1,n}.\frac{1}{\prod_{t=1}^{\alpha} a_{t,1}e_t}.e_k\right)\right\|<\frac{\varepsilon}{m_1},\left\|\sum_{n=M_2+1}^{\infty}\left(\varepsilon_{2,n}.\frac{1}{\prod_{t=1}^{\alpha} a_{t,2}e_t}.e_k\right)\right\|<\frac{\varepsilon}{m_2},\dots,\left\|\sum_{n=M_n+1}^{\infty}\left(\varepsilon_{k,n}.\frac{1}{\prod_{t=1}^{\alpha} a_{t,n}e_t}.e_k\right)\right\|<\frac{\varepsilon}{m_n}$$

for every $k = 1,2,...,n$ sequences $\{\varphi_{\alpha,i}\}$ taking values 0 or 1. By conditions 1 and 2, for $l = 1,2,...,n$ the elements $y_l = \sum_{i=1}^{m_l} x_i$ of X is a periodic point for T, and we have

$$\|y_\lambda - x\| = \left\|\sum_{i=1}^{m_\lambda}(x_i.\psi(i, M_\varphi) - e_i)_i\right\| = \left\|\sum_{i=1}^{m_\lambda}\left(x_i.\prod_{t=1}^{i} d_t, M_\lambda\right)\sum_{k=1}^{\infty}\left(\frac{1}{\prod_{t=1}^{i+kM_\lambda} a_t, M_\lambda} + e_i + M_\lambda\right)\right\|$$

$$\leq \sum_{i=1}^{m_\lambda}\left\|\left(x_i.\prod_{t=1}^{i} d_t, M_\lambda\right)\sum_{k=1}^{\infty}\left(\frac{1}{\prod_{t=1}^{i+kM_\lambda} a_t, M_\lambda} + e_i + M_\lambda\right)\right\| \leq \sum_{i=1}^{m_\lambda}\left\|\left(\sum_{k=1}^{\infty}\left(\frac{1}{\prod_{t=1}^{i+kM_\lambda} a_t, M_\lambda} + e_i + M_\lambda\right)\right)\right\| \leq \varepsilon$$

So we find $|y_\lambda - x| < \varepsilon$. By this, the proof is complete.

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