2nd INTERNATIONAL CONFERENCE ON RESEARCH IN SCIENCE AND TECHNOLOGY
Istanbul-Turkey 14 March 2016
RSTCONF
دومین کنفرانس بین المللی پژوهش در علوم و تکنولوژی
ترکیه – استانبول ۲۴ اسفند ۱۳۹۴

On Quadratic BRK-algebra

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Abstract
In this paper we introduce the notion of quadratic BRK-algebra which is a medial quasigroup, and obtain that every quadratic BRK-algebra on a field X with$|X| \geq 3$, is a BCI-algebra.
Keywords: BCI-algebra, BRK-algebra, quadratic BRK-algebra.

1. Introduction

By an algebra $G = (G,*,0)$ we main a non-empty set G together with a binary multiplication $(*)$ and a some distinguished element 0.
In 1996, Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK- algebras and BCI-algebras[1,2]. It is known that the class of BCK- algebras is a proper subclass of the class of BCI-algebras. Q. P. Hu and X. Li introduced a wide class of abstract algebras: BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. An implication in each BCK- algebra can be define by $y \to x = x * y$. So $(*)$ can be seen as the dual implication of BCK-logic. J. Neggers and H. S. Kim introduced the notion of d-algebras, which is another useful generalization of BCK-algebras. Y. B. Jun, E. H. Roh and H. S. Kim introduced a new notion, called an BH-algebra, which is a generalization of BCH/BCI/BCK-algebras. J. Neggers, S. S. Ahn and H. S. Kim introduced the notion of a Q-algebra, and generalized some theorems discussed in BCI-algebras. Recently, J. Neggers and H. S. Kim introduced and investigated a class of algebras, called a B-algebra which is related to several interest classes of algebras such as BCH/BCI /BCK-algebras. and which seems to have rather nice. In 2002 Ravi Kumar Bandaru introduced a new notion, called a BRK-algebra, which is a generalization of BCK/BCI/BCH/Q/QS/BM-algebras [1,2,3,4,5].

2. Preliminaries

Definition 2.1. A BCI-algebra is an algebra $(G,*,0)$ of type $(2,0)$ satisfying the following conditions:
(BCI1) $\big((x*y)*(x*z)\big)*(z*y)=0$,
(BCI2) $\big(x*(x*y)\big)*y=0$,
(BCI3) $x*x=0$,
(BCI4) $x*y=y*x=0$ implies $= y$,
For all $x,y,z \in G$.

If a BCI-algebra G satisfies $(BCI5)$ $0*x=0$ for all $x \in G$, then we say that G is BCK-algebra. It is known that every BCK-algebra is BCI-algebra but not conversely. On any BCI-algebra $(G,*,0)$ we can define the partial order putting $x \leq y$ if and only if $x*y=0$. A BCI-algebra G has the following properties:
$(x*y)*z=(x*z)*y$

$x * 0 = x$
$x \le y \to x * z \le y * z$ and $z * y \le z * x$
$x * 0 = 0 \to x = 0$

For elements x and y of a BCK-algebra G we denote ; $x \wedge y = y * (y * x)$. A BCI-algebra is said to be commutative if it satisfies $x \wedge y = y \wedge x$ for all $x, y \in G$. A non-empty subset S of a BCI-algebra G is called a BCI-subalgebra of G, if $x * y \in S$ whenever $x, y \in S$. A non-empty subset I of a BCI-algebra G is called an BCI-ideal of G if it satisfies (i) $0 \in I$, $(ii) x * y \in I$ and $y \in I$ imply that $x \in I$, for all $x, y \in G$. The set $B = \{x \in G: 0 * x = 0\}$ is called the BCK-part of G.

A B-algebra is a non-empty set G with a constant e and a binary operation $*$ satisfying the following axioms:

$(i) x * x = e$,
$(ii) x * e = x$,
(iii) $(x * y) * z = x * (z * (e * y))$

For all $x, y, z \in G$.

Definition 2.2. A BRK-algebra is a nonempty set G with a constant 0 and a binary operation $*$ Satisfying axioms:

$(BRK1) x * x = 0$,
$(BRK2) (x * y) * x = 0 * y$,

for any $x, y \in G$.

Example 2.3. Let G be the set of all real numbers except for a negative integer n. Define a binary operation $*$ on G by

$x * y = \frac{n(x-y)}{y(n+y)}$

Then $(G,*,0)$ a BRK-algebra with a constant 0.

We know that every BCK-algebra is a BCI-algebra and every BCI-algebra is a BCH-algebra and every BCH-algebra is a Q-algebra. We can observe that every Q-algebra is a BRK-algebra but converse needs not be true. Also we know that every QS-algebra is a BM-algebra and we can observe that every BM-algebra is a BRK-algebra but converses need not be true.J. Neggers, S. S. Ahn and H. S. Kim introduced the notion of Q-algebra, as an algebra $(G,*,0)$ satisfying (BCI3) and (BCI6) $x * 0 = x$ And (BCI7) $(x * y) * z = (x * z) * y$ for all $x, y, z \in G$.

2.1 Quadratic BRK-algebras

Definition 2.4. Let G be a field with $|X| \ge 3$. An algebra $(G,*,0)$ is said to be quadratic if $x * y$ is defined by $x * y = a_1 x^2 + a_2 xy + a_3 y^2 + a_4 x + a_5 y + a_6$; where $a_1, \dots, a_6 \in G$ are fixed. A quadratic algebra $(G,*,0)$ is said to be a quadratic BRK-algebra if for some fixed $0 \in G$ it satisfies the conditions (BRK1), (BRK2).

Similarly, a quadratic algebra $(G,*,0)$ is said to be a quadratic Q-algebra if for some fixed $e \in G$ it satisfies the conditions (i), (ii) and (iii). It is proved that in every quadratic Q-algebra $(G,*,e)$ the operation $*$ has the form $x * y = x - y + e$. We prove that the similar result is true for quadratic BRK-algebras.

Theorem 2.5. Let G be a field with $|X| \ge 3$. Then every quadratic BRK- algebra $(G,*,e)$, $e \in G$ has the form $x * y = x - y + e$; where $x, y \in G$.

Proof. Let $x * y = Ax^2 + Bxy + Cy^2 + Dx + Ey + F$ (1) for some fixed $A, \dots, F \in G$ .By (i) we have $e = x * x = (A + B + C)x^2 + (D + E)x + F$ (2)

Let $x = 0$ in equation (2) . Then we obtain $= e$. Hence (1) turns out to be $x * y = Ax^2 + Bxy + Cy^2 + Dx + Ey + e$ (3). If $x = y$ in (3) , then $e = x * x = (A + B + C)x^2 + (D + E)x + e$; for any $x \in G$, and hence we obtain $A + B + C = D + E = 0$, i.e., $E = -D$ and $B = -A - C$. Hence (3) turns out to be $x * y = (x - y)(Ax - Cy + D) + e$ (4), now let $y = e$ in (4). Then by (ii) we have $x = x * e = (x - e)(Ax - Ce + D) + e$. By this equation we have $(Ax - Ce + D - 1)(x - e) = 0$. Since G is a field, either $x - e = 0$ or $Ax - Ce + D - 1 = 0$. Since $|X| \ge 3$, we have $Ax - Ce + D - 1 = 0$; for any $x \in G$. This means that $A = 0$ and $1 - D + Ce = 0$. Thus (4) turns out to be $x * y = (x - y) +$

$C(x-y)(e-y)+e$ (5). To satisfy the condition (iii) we need to determine the constant C, but its computation is so complicated that we use (iii) instead. If we replace e by x and x by y respectively in (5), then $e*x=(e-x)+C(e-x)(e-x)+e$ (6). It follows that $e*(e*x)=e*[(e-x)+C(e-x)^2+e]=x-C(e-x)^2+C(e-x)\{1+C(e-x)\}^2=x+C^3(e-x)^4+2C^2(e-x)^3$. Since , $x=e*(e*x)$; we obtain $C^2(e-x)^3-Cx+2+Ce=0$. Since G is a field with $|X|\geq 3$, we obtain $C=0$. This means that every quadratic BRK-algebra $(G,*,e)$ has the form $x*y=x-y+e$; where $x,y\in G$, completing the proof.

It follows from Theorem 2.5 that the quadratic BRK-algebras are medial quasigroups.

Example 2.6. Let $K=GaloisF(p^n)$ be a Galois field. Define $x*y=x-y+e$, $e\in K$. Then $(K,*,e)$ is a quadratic BRK-algebra.

Theorem 2.7. Let G be a field with $|X|\geq 3$, then every quadratic B-algebra on G is a BCI-algebra.

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