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Numerical computing of axial potential distribution in accelerator lens immersed in the field of two electrodes and 300 keV electron accelerator column designing

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Abstract

Electrons electrostatic accelerator column is one of essential parts of electrostatic accelerators. Geometric shape designing must be such as applying a voltage to the electrodes, the potential gradient and potential levels in addition to accelerating the particle beam to the desired energy, focus the beam and energy distribution of accelerated particles may be broadening. Electrodes immersed in the field geometry around the central page, are perpendicular to the optical axis. Different models, such as linear model, analytical model, two cylindrical lenses model, polynomial lenses model have been introduced for axial potential distribution of this system. In this article series expansions in terms of Bessel functions are used to obtain axial potential distribution of electrodes of accelerator immersed in the two electrodes field and by solving the final equation in the least squares method, compared with the above models. Finally, by using the Studio CST software and information that have gained from central potential distribution, an electron accelerator Column that has an optimal energy distribution and output radius is designed and simulated.

Keywords: Accelerator tube, Lenses immersed in the field, axial potential distribution, electrode.

Introduction

One of the major topics discussed in nuclear engineering, is electrostatic lenses, because these lenses are used in the construction of accelerator tube. After many years of research in laboratories in the field of designing, constructing and completing of the accelerator tube, still it is the ultimate factor in the construction of the electrostatic accelerator (Lee, 2004). Electrostatic lenses often are used to form an image in a fixed location with a wide range of beam energy. This lenses usually are classified in term of relationship between potential of their electrodes (potential of the image to the object), which contains the main groups of Einzel lens (Syms etal, 2003; Toivanen etal, 2013; Gillespie and Brownb 1997) that have same potential in both the object and its image, are lenses immersed in the field with two different constant potential on both sides and single-span lenses with a inductional homogeneous field on one side and Foil lens (Moore etal, 2009). in the case that the potential is not equal on both sides of lens, the lens operates in the state of accelerator or accelerated(speed reducer). Lenses mentioned above can contain two or more electrodes (Szilagyi, 1998). Two electrodes lenses are used to focus the electron or proton beams (Read, 1969). It is also possible to characterize electrostatic lenses despite their profiles, by their axial potential distribution, because further focusing instruments that in the ionic and electronic optics include axial symmetrical electric field, can be shown by scalar potential function (Reiser, 2008).

Profiles of lens immersed in the field

The simplest electrostatic lenses are two electrodes if the potential on both sides are different that called immersed in the field. Electrodes parameters changes cannot be made practical and systematic review. For this reason, we start with an analysis of potential distribution. The axial potential distribution of a immersed in two-piece field U(z), have a very simple approximate shape, as shown in Figure (1) which is a monotonic function of z coordinate. Assume that potential of the first electrode is V_1 and the second isV_2. In the range of z=a to z=b, it will be created an accelerator lens if $|V_2| > |V_1|$, and an accelerated (speed reducer) if $|V_2| < |V_1|$. In these lenses, potential distribution does not include constant potential area, because to keep such areas, it needs additional electrodes.
In the potential distribution of this system, there is a blending point in Z_m where the axial component of field $(|U'|_{max})$ reaches to its highest net value. This point can be located in the geometric center or at another point and hence two types of symmetrical lenses can be defined (Szilagyi, 1987; Szilagyi, 1998).

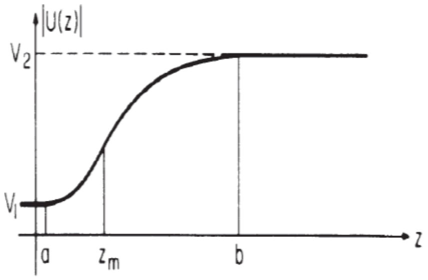

Figure 1. Axial potential distribution of a lens immersed in field of two electrodes

Studying of lenses immersed in the field's models

For lenses immersed in the field that are perpendicular to the symmetric optical axis around central page geometrically, where Z_m exactly is coincident to the average value of a and b, has been introduced models that specifies the potential distribution, that is anti-symmetric around Z_m. Several models like linear model, analytical model, two cylindrical lenses model and polynomial lenses model are presented for axial potential distribution of this system that these results are shown in Figure (2) (Szilagyi, 1998).

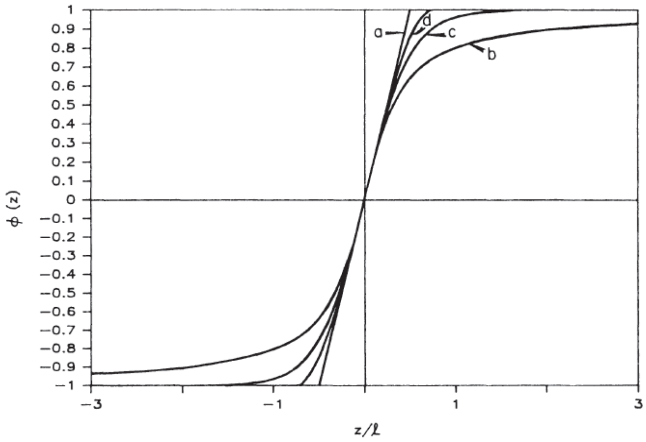

Figure 2. Potential distribution of lenses immersed in a symmetric binary field:
(a) Linear model, (b) analytical model, (c) two cylinder lenses model, (d) cubic polynomial lenses model

Numerical calculation of axial potential of the lens immersed in the field by using of series expansion of the Bessel functions

For numerical calculation of axial potential of lenses immersed in the field, two flat leading plate, each with a hole which have diameter D and are separated by distance a, are considered. The applied voltage to the plates (electrodes), are V_1 and V_2 that the kinetic potential of electrons are measured zero. In the next calculation, thickness of the plates will be ignored (when the thickness is much less than A and D that dependence of lenses parameters on the thickness of plates will be very weak). And also, it is needed to consider three assumptive cylinder with a diameter D', that are on potential V_2 , V_1 in region I and III, while middle cylinder has a linear potential that changed from V_1 to V_2 in its length.
It is possible to argue that the middle cylinder in a uniform potential V_m(e.g., equal to V_1,V_2 or their average) is more consistent to the actual lens, but this is not done because of two reasons: the first, if $D' \lesssim 2A$, this middle cylinder changes axial potential distribution between cavities and it needs to lens parameters for different values of D' and V_m, be calculated; second, if it is $D' \gtrsim 2A$, lens parameters are almost the same as the potential of the middle cylinder changed linearly, that it made longer the calculation add less the accuracy.

Figure (3) shows paraxial and nonparaxial beams and Q and P distances of object and image, both measured from the midpoint of the lens (Read, 1969). In computing of electrostatic lenses by digital computers, used known method, is always Laplace equation (Lanczos, 1957). There is another way which potential can be expanded in term of Bessel function.

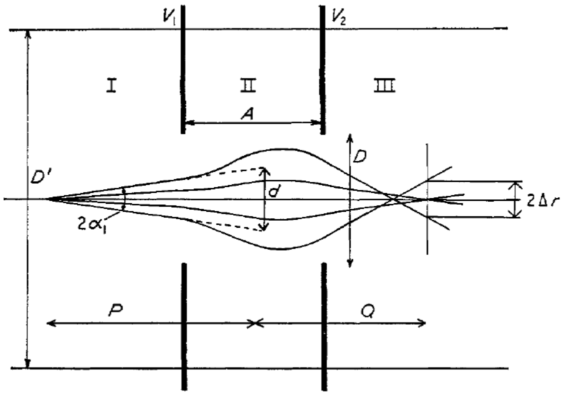

Figure 3. Geometry of mentioned lenses

Although the method of solving the Laplace equation is a simple way, accurate and reliable results can be achieved faster and more decisive by improvement of expansion method. It seems that used method in calculating probably is more accurate and powerful for the whole range of issues which there is a cylindrical symmetry. Series for three regions shown in Figure (3) can be written as follow:

$$V_I(r,z) = V_1 + \sum_n A_n \exp(k_n z) J_0(k_n r) \quad (1)$$
$$V_{II}(r,z) = \frac{V_1+V_2}{2} + \frac{V_2-V_1}{A} z + \sum_n \{B_n \exp(-k_n z) + B_n{}' \exp(k_n z)\} J_0(k_n r) \quad (2)$$
$$V_{III}(r,z) = V_2 + \sum_n C_n \exp(-k_n z) J_0(k_n r) \quad (3)$$

The equations are written in cylindrical coordinates (r, θ, z) and z is measured from the midpoint of lens. $J_0(x)$ is zero-order of Bessel function. Every sentence of the expansions is a solution of the Laplace equation with undefined boundary conditions. Exponential variables marks are selected in such ways that are satisfied boundary conditions at $z = \pm\infty$ automatically. The boundary conditions at $r = \frac{1}{2}D'$ are fulfilled if value of K_n is limited to such values which we have:

$$J_0\left(\frac{1}{2}k_n D'\right) = 0 \quad (4)$$

Now, satisfying the boundary conditions at $z = \pm\frac{1}{2}A$ and ensuring that the potential over the two holes is continuous, remains.
From orthogonally relation for Bessel function:

$$V_I\left(r, -\frac{1}{2}z\right) = V_{II}\left(r, -\frac{1}{2}z\right) \qquad \text{for} \quad 0 \le r \le \frac{1}{2}D' \quad (5)$$

For all values of N, we have:

$$A_n \exp\left(-\frac{1}{2}k_n A\right) = B_n \exp\left(\frac{1}{2}k_n A\right) + B_n{}' \exp\left(-\frac{1}{2}k_n A\right) \quad (6)$$

This equation also ensures that radial component of field in the first cavity is continuous. Clearly boundary condition of left electrode's potential for $\frac{1}{2}D \le r_2 \le \frac{1}{2}D'$ is:

$$\sum_n \left\{B_n \exp\left(\frac{1}{2}k_n A\right) + B_n{}' \exp\left(-\frac{1}{2}k_n A\right)\right\} J_0(k_n r_2) = 0 \quad (7)$$

And for $0 \le r_1 \le \frac{1}{2}D'$, it will be:

$$\sum_n B_n k_n \exp\left(\frac{1}{2}k_n A\right) J_0(k_n r_1) = \frac{V_2 - V_1}{2A} \quad (8)$$

Similar equations are also applied to second cavity. Because of the symmetry around a central point, $B_n{}' = -B_n$, $C_n = -A_n$ will be established. Although equations (7) and (8) are fulfilled only when exactly an infinite number of sentences are considered, it can be hopeful that a finite number of N, tends the difference between the right and left side of the equation to zero. Therefore, N value of R in the range of 0 to D, are selected and N number of (7), (8) obtained equations and solved to obtain N number of B_n. This process does not lead to convergence by increasing N, but instead of that increases in highly oscillating values for the factor of B_n. For r values between those who have been selected, equation (7), (8) are poorly met.

To overcome this problem, the method can be useful but numerical calculations shows that the least square method is very convenient and why it is used. In this case, the least square method includes forming quantity:

$$S = \sum_{r_2}\left[\sum_{n=1}^{N} B_n\left\{\exp\left(\frac{1}{2}k_nA\right) - \exp\left(-\frac{1}{2}k_nA\right)\right\}J_0(k_nr_2)\right]^2 + \sum_{r_1}\left\{\sum_{n=1}^{N} B_nk_n\exp\left(\frac{1}{2}k_nA\right)J_0(k_nr_1) - \frac{V_2-V_1}{2A}\right\}^2 \quad (9)$$

And then is obtaining N equation of $\frac{\partial S}{\partial B_n} = 0$ for $1 \le n \le N$:

$$\sum_{r_2}\left[\sum_{j=1}^{N} 2B_j\left\{2\sinh\left(\frac{1}{2}k_jA\right)\right\}J_0(k_jr_2)\right]\left\{\left\{2\sinh\left(\frac{1}{2}k_iA\right)\right\}J_0(k_ir_2)\right\} + \sum_{r_1} 2\left\{\sum_{j=1}^{N} B_jk_j\exp\left(\frac{1}{2}k_jA\right)J_0(k_jr_1) - \frac{V_2-V_1}{2A}\right\}k_i\exp\left(\frac{1}{2}k_iA\right)J_0(k_ir_1) = 0 \quad (10)$$

Then the equations can be solved to obtain the coefficients of B_n. By writing FORTRAN program that can be generalized to a longer number of electrodes and selecting $V_1 = 100$ V, $V_2 = 60$ V, $d = 3$cm, $N = 20$ unknown coefficients obtained from the above equations numerically and by placing in equation (1) to (3) can be revealed potential in all space for two electrodes electrostatic system that the result are shown in Figure (4). Axial potential distribution will be achieved when $r = 0$.

Note that the number of values $r_1\left(0 - \frac{1}{2}D\right)$, $r_2 = \left(\frac{1}{2}D - \frac{1}{2}D'\right)$ must be greater than N (in practice about 3N). This process will result converged solution for equations (7), (8) and axial potential $V(0, z)$ that the potential will be converged by increasing N.

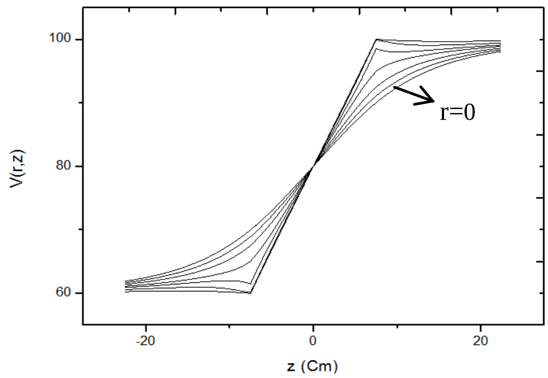

Figure 4. The potential in the term of z for constant radius. Potential of electrodes are$V_1 = 100$ V, $V_2 = 60$ V

Designing and simulating of 300 keV electron accelerator column

Numerical calculations show that for the crater lens best achieved when electrodes diameter are the same and the rate of distance between electrodes to diameter is 0.1 to 0.3. To simulate the accelerator column, the external radius of electrodes is 15cm considered; therefore the distance between the electrodes will be 5cm as mentioned. Also by observing the distance between the electrodes and defining ceramic based material between the electrodes, electrical discharge can be prevented. The software used to simulate is CST. This application is one of the most powerful software in the field of numerical simulation of electromagnetic fields, design of antennas, electrical circuit design and high-frequency circuits that is the result of several years of effort and research in the field of analysis and design of electromagnetic. In this software observations are three-dimensional that is needed for electromagnetic circuits. This software is using the Monte Carlo method to draw and show the path of the particle beam. So we consider accelerator column design according to our experience in the construction in Figure (5):

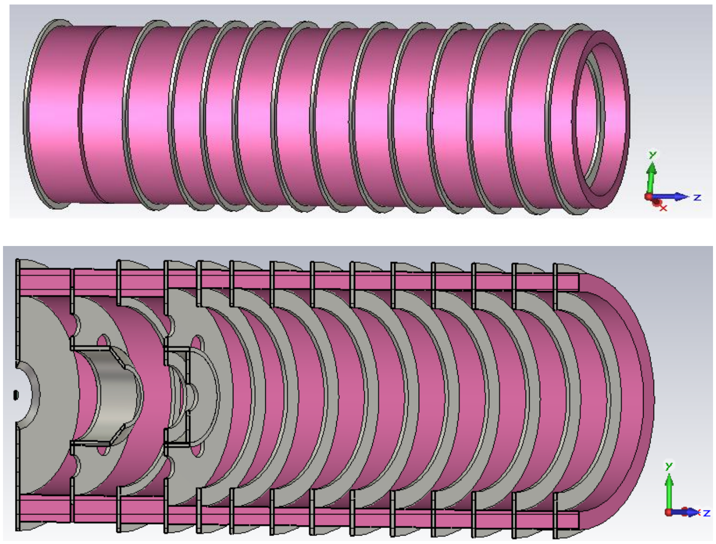

Figure 5. Designing of electrostatic electron accelerator tube with accelerator electrodes

Potential are usually divided equally between the end electrodes. -300kV potential relative to the earth potential between the electrodes with stairs 25 kV (the potential of this step is entirely optional and depending on the geometry of the accelerator system can be chose another potential step) other than the second electrode is divided. In other words, the first electrode, third, fourth and so on until the end of the electrode arrangement has the potential to equal -300kV, -275kV, -250kV... 0kV toward the ground. The second electrode’s potential that is focusing lens has a variable voltage +10kV than the first electrode. In total we have 14 electrodes made of aluminum. To Figure (5) by applying the above potential, potential distribution and axial potential distribution is obtained such as Figure (6):

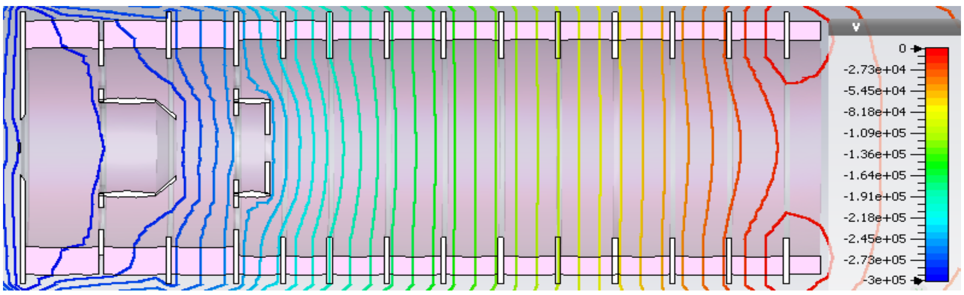

Figure 6. Potential line along the column accelerator

The path of the particles and electron energy in Figure (7-a) are given. Electron beam once between 5 to 10 cm and again between 20 and 25 cm will be focal that shows fine focusing electrodes are designed.
As vertical axis of Figure (7-b) shows radius of the output electron beam will be less than 8 mm (Electrons are emitted in distance 72cm from accelerator column).

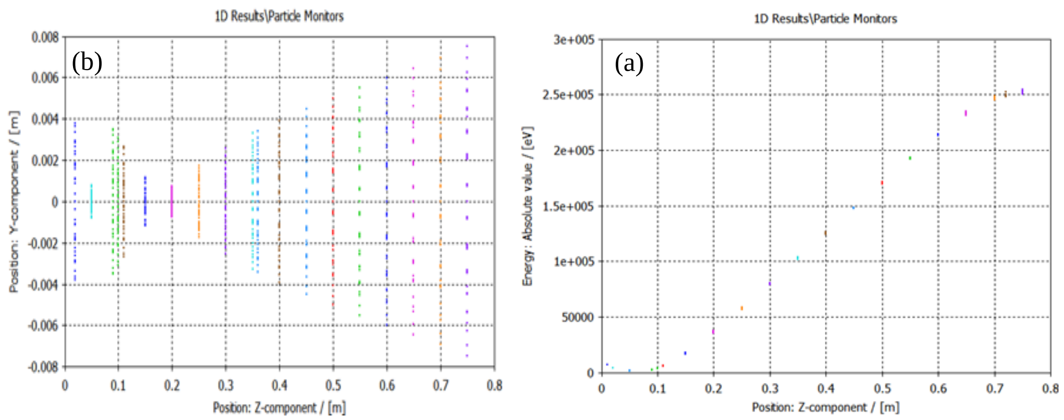

Figure 7. a) Energy of electron beam passing along to accelerator tube $V_2 = -290$ kV **b) View the path of the electron beam**

According to the figure (7-a) for the first geometry, electron beam output energy will be in the range of 250keV. As shown in Figure (8) to be seen if the second electrode voltage is -300kV, electron beam output energy is in the range of 300keV.

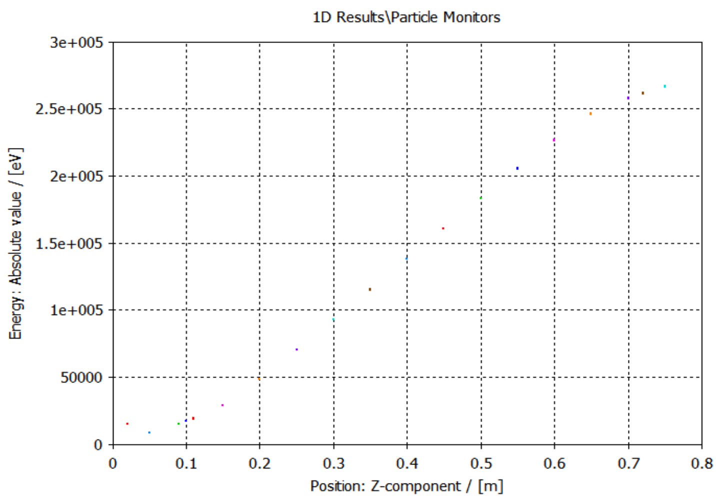

Figure 8. Energy of electron beam passing along to accelerator tube $V_2 = -300$ kV

If you need an electron beam with a radius of less than 8 mm can be achieved by putting an electrical quadrupole lens in the direction of output electron beam, the output beam diameter can be controlled.
In this software the path of beam is considered vacuum that is not possible in reality, so it is better to apply net voltage more than -300keV to accelerator column.

Conclusion

To design and build the accelerator tube should have complete information of the electrostatic lens for electrons, protons and ions under the influence of an electrical potential gradient that these lenses create, will be accelerated and reaches to final desired speed. As well as focusing the beam of charged particles is also the responsibility of the lens. In the case of low-energy beam is a need to control, two electrodes electrostatic lens used. With axial potential distribution which can be characterized by an electrostatic lens can predict how charged particle beams moves.

By improvement of Bessel functions with the least square method can be seen that the results are much closer to the model provided for the electrostatic lens and at best an axial potential distribution makes output electron beam energy is maximum and focal when electrodes diameter are the same and the rate of distance between electrodes to diameter is 0.1 to 0.3.

Simulations show that if we have more electrodes, level potential and its gradient will be more uniform. Similarly, if the potentials rare is closer to one, focal length will be longer aberration coefficients is less. Potential distribution and axial potential distribution are shown in Figure (6) that this distribution in designed accelerator column makes a best state. Results are obvious in Figures (7-a), (7-b) and (8).

Reference

Gillespie, Brownb. (1997). Optics elements for modeling electrostatic lens and accelerator components I. Einzel lenses. Particle Accelerator Conference. Vol 2. 2559 – 2561.

Lanczos. (1957). Applied Analysis. London: Pitman.

Lee. (2004). Accelerator Physics. 2nd edition, Singapore: World Scientific Publishing Co. Pte. Ltd, 575P.

Moore, Davis, Coplan and Greer. (2009). Bulding Scientific Apparatus. 4th edition, New York: Camdridge University Press, 629P.

Read. (1969). Accurate Calculations of double- Aperture Electostatic Immersion Lenses .Journal of Physics E. Vol 2. 165-169.

Read. (1969). Calculated properties of electrostatic einzel lenses of three apertures .Journal of Physics E. Vol 2. 679-684.

Reiser. (2008). Theory and Design of Charged Particle Beams. Weinhein: Wiley- Vclt.

Syms, Michelutti and Ahmad. (2003). Two-dimensional microfabricated electrostatic einzel lens. Sensors and Actuators A 107. 285–295.

Szilagyi. (1998). Electron and Ion optics. New York: Plenum press.

Szilagyi. (1987). Electrostatic Spline Lenses. Journal of Vacuum Science & Technology A5. 273- 278.

Toivanen, Kalvas, Koivisto, Komppula and Tarvainen. (2013). Double einzel lens extraction for the JYFL 14GHzECR ion source designed with IBSimu. Journal of Instrumentation. Volume 8.