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Designing a CUSUM Control Chart for Monitoring the Weibull Scale Parameter in Two-Stage Processes

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Abstract

In this paper, we propose a CUSUM control chart to monitor the Weibull scale parameter of type II censored reliability data in two-stage processes. A cumulative sum (CUSUM) control chart is devised to detect decreases in the mean level of reliability-related quality characteristic. The proposed control scheme is based on standard smallest extreme value (SSEV) distributions derived from Weibull processes to effectively account for the cascade property which is the main characteristic of multistage processes. Subsequently, simulation study is conducted to evaluate the performance of the control charts using average run length (ARL) criterion. Moreover sensitivity analysis is done to study the impact of failure number in the sample size on the performance of proposed control chart.

Keywords: cascade property, multistage process, CUSUM control chart, type II censoring, weibull distribution, smallest extrem value distribution

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1.Introduction

Customers anticipate purchasing products to be reliable. Reliability is often defined as the probability that a system will perform its expected function under stated conditions for a specified period of time(Meeker and Scobar,1998) Thus, monitoring reliability data with the aid of control charts has been turned into a critical issue.
Reliability data have some features requiring the use of special statistical methods. They are typically censored because it is adequate for them to reach a predetermined limit to ensure the reliability of products. (Asadzadeh and Aghaei,2012) Moreover, these reliability data often come from parametric distributions, namely, location-scale and log-location-scale.(Meeker and Scobar,1998) Extreme value, Weibull, and lognormal distributions are the most widely used for modeling purposes. However, Censoring is often a result of constraints on time and cost. Among different types of censoring mechanisms, right censoring is the most common which has two forms; type I or time and type II or failure censoring.
Many researchers studied the monitoring of censored reliability data in processes where the interrelationship among stages is not pronounced. Olteanu(2010) presented a CUSUM control chart to monitor both parameters of the Weibull distribution for processes with right-censored data. A control chart based on the sample ranges of smallest extreme value distribution was constructed by Pascal and Li(2012) to monitor the Weibull shape parameter under type II censoring. Moreover, the ARL-unbiased control charts based on the new statistic were considered by Guo and Wang(2014) to monitor the Weibull shape parameter when the sample is type II censored. Finally, Raza et al.(2015) investigated a CEV EWMA control chart for poisson-exponantial distributed samples with type-I right censoring.
However, most products are produced over several different process-stages where the stages are not independent. This means that a change in an incoming quality characteristic impacts the in-control state of some or all output quality variables. The mentioned property is called cascade property, which is related to multistage processes.(Asadzadeh and Aghaei,2012) Furthermore, the presence of a censoring mechanism makes the situation more complicated. Thus the mentioned obstacles must be thoroughly overcome to propose a novel monitoring scheme in this area. Survival analysis regression models, including the accelerated failure time (AFT) and the cox proportional hazard (PH) models are applied in the field of reliability or lifetime data to account for the effect of influential quality covariate. Steiner and Jones(2009) proposed an updating exponentially weighted moving average (EWMA) control chart to monitor risk-adjusted survival time. A risk-adjusted shewhart chart on the basis of a likelihood ratio test statistic was proposed by Zhang at el.(2012) for monitoring surgical performances in phase I. Asadzadeh at el.(2015) developed CEV-EWMA and CUSUM control charts using modified proportional hazard (PH) model for autcorrelated observations. They considered frailty models in their research to account for unobserved values of quality characteristics. To our best knowledge, monitoring type II censored reliability data in multistage processes has not been studied in the literature. Thus the purpose of this paper is to propose a monitoring procedure in multistage processes for type II censored data .
This paper has been organized as follows. In section 2, the two-stage process, distributions of variables and the likelihood function of type II censored data are introduced. Monitoring approach and the development of the proposed monitoring statistics for Standard smallest extreme value distribution are given in section 3. The results of simulation study and sensitivity analysis are presented in section 4. Concluding remarks are given in section 5.

2. Process description and assuptions

Consider a two-stage manufacturing process in which the incoming quality characteristic related to the first stage (denoted by *x*) affects the output quality characteristic in the second stage (denoted by *y*). In order to monitor such processes, the effect of input variables must be considered on the output variable and the relationship between them should be modeled. Moreover, the presence of type II censoring is pronounced while recording the values related to the second quality characteristic. Thus, the application of survival analysis regression models is indeed needed in the field of reliability data. AFT models express one or more parameters of the intended distribution as a function of influential covariate. In this paper, the AFT model is applied to probability density function (pdf) and cumulative distribution function (cdf) of response variable to account for heterogenety imposed by incoming quality characteristic. Hence, $\boldsymbol{\omega_k}$ is a vector of parameters including $\boldsymbol{\alpha}$ and $\boldsymbol{x_k}$, where $\boldsymbol{\alpha}$ is a vector of regression coefficients and $\boldsymbol{x_k}$ is a vector of influential quality variables impacting the *k*th value of the output quality characteristic. $\boldsymbol{\omega_k}$ is given by

$$\boldsymbol{\omega_k} = \mathrm{q}(\boldsymbol{\alpha}, \boldsymbol{x_k}) \tag{1}$$

Suppose the incoming quality variable *x* follows a normal distribution with mean and standard deviation denoted by μ_x and σ_x respectively, and output quality variable *y* follows Weibull distribution with scale and shape parameters $\eta>0$ and $\beta>0$.
We have 1,2,…,*k* simple random sample of size *n* from the reliability data following a Weibull distribution. Let $y_{k1}<\ldots<y_{kn}$ be the ordered random sample from the mentioned process. Similarly, $\log(y_{k1})<\ldots<\log(y_{kn})$ can be considered the ordered random sample from a smallest extreme value distribution with location and scale parameters $u=\log(\eta)$ and $b=1/\beta$, respectively. Thus, $z_{k1}<\ldots<z_{kn}$ are the ordered random sample from standard smallest extreme value distribution where *z* is obtained as follows:

$$z_{kj} = \frac{\mathrm{Log}(y_{kj}) - u}{b} \tag{2}$$

The probability density function (pdf) and cumulative distribution function (cdf) of standard smallest extreme value distribution denoted by ϕ_{SEV} and Φ_{SEV}, are given by

$$\phi_{SEV}(z) = exp[z - exp(z)] \tag{3}$$

$$\Phi_{SEV}(z) = 1 - \exp[-\exp(z)] \tag{4}$$

Finalyy, type II censoring refers to the situation where only *r* ($r \leq n$) smallest values ($z_{k1}<\ldots<z_{kr}$) in a random sample of size *n* are observed, whereas $z_{k,r+1},\ldots,z_{kn}$ are right censored at z_{kr} and are equal to the value recorded for z_{kr} value. Hence, the likelihood function(L) is given by

$$L = \prod_{j=1}^{r} \phi_{SEV}(z_{kj}) \times (1 - \Phi_{SEV}(z_{kr}))^{n-r} \tag{5}$$

3. Concsturctiong the CUSUM control chart for Weibull distributed reliability data

In this section, a one-sided lower CUSUM control chart is proposed to detect decreasing shifts in mean levels of Weibull distributed reliability data with type II censoring. In order to design the CUSUM control chart, the likelihood function can be summarized as follows:

$$L = \prod_{j=1}^{r} \exp[z_{kj} - \exp(z_{kj})] \times (\exp[-\exp(z_{kr})])^{n-r} \quad (6)$$

The CUSUM statistic is given by

$$s_k = \min(0, s_{k-1} - w_k)$$
$$s_0 = 0 \quad (7)$$

in which, w_k is the CUSUM score defined as follows:

$$w_k = \log\left(\frac{L(z_k \mid \omega_{k1})}{L(z_k \mid \omega_{k0})}\right) \quad (8)$$

where, ω_{k0} and ω_{k1} are predefined in-control and out-of-control values of the parameter associated with the response quality variable. The CUSUM chart signals an alarm as soon as its statistic falls below the lower control limit denoted by lcl_1($s_k < lcl_1$) . The lower control limit(lcl_1) has been selected in a way to achieve the desirable in-control average run length(ARL).

As mentioned, a variety of distributions can be used for modeling reliability data. In our study, Weibull distribution has been employed which is transformed into standard smallest extreme value (SSEV) to account for the cascade property in multistage processes. In case the scale parameter (η) depends on the values of the quality variable in first stage, it can be revised in the following manner:

$$\eta = \exp(\alpha_0 + \alpha_1 x) \quad (9)$$

where α_0 and α_1 are the coefficients of the regression model. Thus, the parameters used in the in-control likelihood function are $\omega_{k0} = (\eta , \beta)$ and the parameters employed in out-of-control likelihood function (optimally designed to find a specifics shift) are $\omega_{k1} = (\theta\eta , \beta)$, where θ is a coefficient used to shift the target in-control value of the scale parameter. The SEV distribution has a mean equal

$$\mu_{SEV} = u - \gamma b = \log(\eta) - \gamma\left(\frac{1}{\beta}\right) \quad (10)$$

Hence, considering that β is constant, changing the scale parameter in Weibull distribution has the same interpretation as changing the mean of SEV distribution. It is noteworthy that $\gamma = 0.5772$ is known as Euler's constant.(Lawless,2003)
Having formed the likelihood function introduced in equation (6), the log-likelihood score w_k(CUSUM score) is obtained as follows

$$w_k = \log\left(\frac{\exp\left[\frac{y_{k1}-\log(\delta\eta)}{\frac{1}{\beta}} - \exp\left(\frac{y_{k1}-\log(\delta\eta)}{\frac{1}{\beta}}\right)\right]\times\dots\times\exp\left[\frac{y_{kr}-\log(\delta\eta)}{\frac{1}{\beta}} - \exp\left(\frac{y_{kr}-\log(\delta\eta)}{\frac{1}{\beta}}\right)\right]\times\left(\exp\left[-\exp\left(\frac{y_{kr}-\log(\delta\eta)}{\frac{1}{\beta}}\right)\right]\right)^{n-r}}{\exp\left[\frac{y_{k1}-\log(\eta)}{\frac{1}{\beta}} - \exp\left(\frac{y_{k1}-\log(\eta)}{\frac{1}{\beta}}\right)\right]\times\dots\times\exp\left[\frac{y_{kr}-\log(\eta)}{\frac{1}{\beta}} - \exp\left(\frac{y_{kr}-\log(\eta)}{\frac{1}{\beta}}\right)\right]\times\left(\exp\left[-\exp\left(\frac{y_{kr}-\log(\eta)}{\frac{1}{\beta}}\right)\right]\right)^{n-r}}\right) \quad (11)$$

4. Performance of the proposed CUSUM chart

Having designed the proposed control chart for monitoring type II censored data in the described two-stage process, the performance of the monitoring scheme should be addressed. Simulations are performed to investigate the performance of the proposed one-sided CUSUM. In addition, without loss of generality, the sample size is considered to be 5 (n=5) and failure numbers are r = 2,3,4,5. The ARL is considered as an evaluation criterion.

Throughout the simulation study, the lower control limits of control chart has been determined so that the in-control ARL value is approximately 200. In order to minimize the errors, the simulation is repeated 10,000 times and the ARLs in line with the standard error (SE) are recorded. Moreover, decreasing mean shifts (δ) size 2.5%,5%,10%,20% and 30% are considered. The results of simulation are illustrated in table 1 for r = 2,3,4 and 5.

Table 1. Comparison of the CUSUM control chart with sample size n=5

δ	CUSUM control chart							
	r=2		r=3		r=4		r=5	
	ARL	SE	ARL	SE	ARL	SE	ARL	SE
1	200.5	1.70	200.6	1.67	200.3	1.69	200.2	1.67
0.975	129.9	0.97	119.1	0.88	112.2	0.83	104.1	0.78
0.95	91.1	0.65	79.3	0.55	71.1	0.51	64.2	0.45
0.9	51.9	0.34	42.3	0.27	36	0.22	31.3	0.19
0.8	23.5	0.13	18.1	0.10	14.7	0.08	12.6	0.07
0.7	13.1	0.07	9.9	0.05	7.8	0.04	6.7	0.03

According to the simulation results, it is remarkable that the CUSUM control chart has appropriate performance for detecting out-of-control states. It is also observed that by increasing r (reducing the censoring) the out of control ARLs of the proposed control chart decreases as expected.(Figure1)

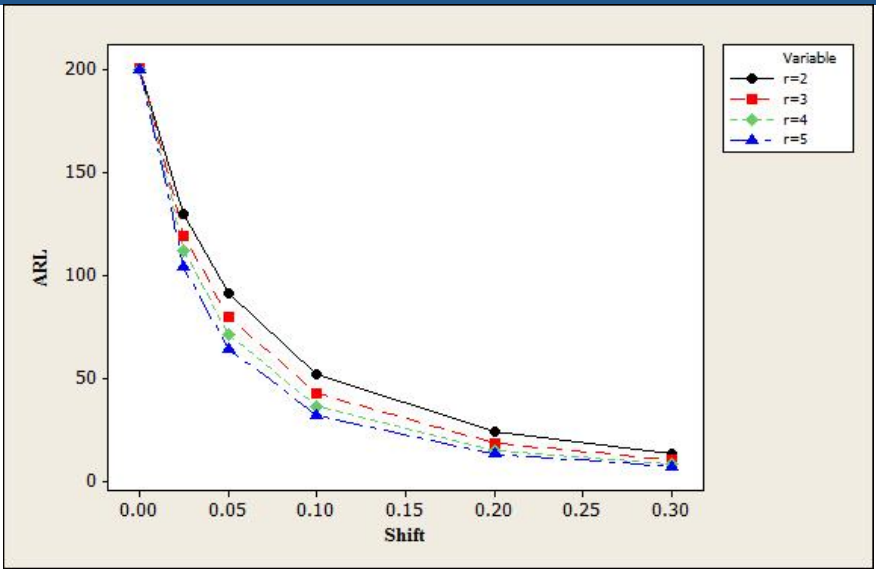

Figure1. The ARLs for the one-sided CUSUM control chart with *n*=5, *r* = 2,3,4,5 considering different shift values

5. Conclusion

A CUSUM control chart was proposed to monitor type II censored reliability data in two-stage processes in which the output quality characteristic is influenced by the input one. The successfully account for the cascade property in the described process, AFT regression models were used for the purpose of modeling the relationship between stages and a transformation of real data from Weibull distribution to the standard smallest extreme value distribution were applied to remove the effect of upstream quality variable. Subsequently, constructing the likelihood function for the type II censored data corresponding to the output quality characteristic was concerned to devise the proposed monitoring scheme. Simulation study and sensitivity analysis were conducted to assess the performance of the control procedure. The results showed the detrimental effect of censoring on the performance of the proposed CUSUM chart, in that, increasing the censoring rate reduces the detect-ability of the monitoring procedure.

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