Calculation Gyration Radius in quantum billiards

Hooman Fetanat
ZAND INSTITUTE OF HIGHER EDUCATION , SHIRAZ, IRAN
fetanat@zand.ac.ir

Mohammad Reza Eslaminejad
ZAND INSTITUTE OF HIGHER EDUCATION , SHIRAZ, IRAN
eslaminejad@zand.ac.ir

Jamal Gholami
ZAND INSTITUTE OF HIGHER EDUCATION , SHIRAZ, IRAN
j.gholami5@gmail.com

Leila Mortazavifar
ZAND INSTITUTE OF HIGHER EDUCATION , SHIRAZ, IRAN
mortazavifar.l@zand.ac.ir

Iman Jamnejad
ZAND INSTITUTE OF HIGHER EDUCATION , SHIRAZ, IRAN
jamnezhad@zand.ac.ir

Reza Dehghani
ZAND INSTITUTE OF HIGHER EDUCATION , SHIRAZ, IRAN
j.gholami5@gmail.com

Abstract
In this paper we present a quantum mechanical study of a circular and stadium billiard. By using the gyration radius we study chaos in circular and stadium billiard. In quantum the coordinate of the collision zeros of wave function on the boundary of billiard are used for calculate the gyration radius. Then we analyze the results obtained in quantum billiards.

Keywords: gyration radius, stadium billiard, circular billiard

Introduction

The first notion of chaos came at the end of the 19th century . Chaos is a ubiquitous phenomenon in everyday life. It is seen in population dynamics, the weather, and many other places . Chaos is defined as the extreme sensitivity to initial condition .. However, many other interesting involve waves, such as quantum mechanics, electro magnetism, and electrical transport properties of nanoscale quantum dots. Clearly, the notion of chaos as it is usually defined is classical, because quantum mechanics is a theory of wave, not trajectory .. One has to consider wave systems that have high-energy properties that display "ray chaos" the extreme sensitivity of system evolution to initial ray directions . A model system often studied in chaos is the two-dimensional billiard. Two dimensional billiards have long been used as paradigm systems for studying classical and quantum dynamics. The dynamical behavior of the billiards is highly dependent on the geometry of the boundary. A billiards can exhibit either fully integrable (regular) dynamic, fully chaotic dynamic, or mixed integrable/chaotic dynamic . Rectangular and circular shapes are examples of integrable systems, while Bunimovich stadium and Sinai billiards are completely chaotic . Billiards are an important class of systems showing a large variety of dynamical behavior ranging from integrable motion, over mixed dynamics to strongly chaotic behavior . This dynamical behavior is directly reflected in properties of corresponding quantum systems, like eigenvalue statistics or the structure of eigenfunction . The quantum dynamics of systems that are classically chaotic has been subject of considerable interest for nearly three decudes .This paper is organized as follows. In section II, we introduce the gyration radius. In section III, we present the numerical computations of the gyration radius in the limit of quantum state.

2. Gyration radius

lets consider the gyration radius $R_s^{\,2}$. It is defined as:

$$R_s^{\,2} = R^2 - r_{cm}^{\,2} \qquad (1)$$

$$R_s^{\,2} = \frac{1}{2N^2}\sum_i \sum_j \left(r_i - r_j\right)^2 \qquad (2)$$

Where N is the number of collision points with boundary and r_i is the position vector.

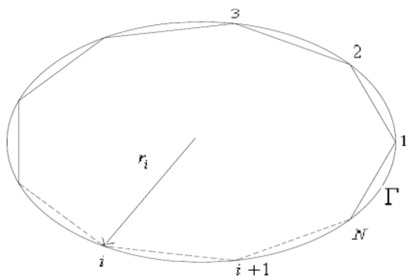


Figure1: The billiard boundary Γ is approximated by a polygon with N vertices

In this paper for calculating the gyration radius in quantum states we can use of the coordinate of the collision of zeros of wave function on the boundary.

3. **Gyration radius in quantum billiard**

Quantum chaos deals with the quantum mechanical properties of classically chaotic system. The single most outstanding feature of the quantum world is its smooth and wavelike nature. This feature leads to the question of how chaos makes itself felt when moving from the classical world to the quantum word. In quantum chaos we have no local formation of the degree of chaoticity, being available in classical chaos via Lyapunov exponents and Poincare sections from phase spase. Exponential separation of trajectories is a purely classical concept, and can not be used as a signature of chaotic quantum mechanics. A classical billiard system is just what one would expect from the name-a particle bouncing around a walled system. The quantum analogue is a wave packet moving around a 2D cavity for example, the cue ball might now be an electron, and it is thus small enough that quantum effects be noticed. The obvious quantities to study for a quantum system are the energy levels and wave functions if the system is closed or scattering states if the system is open.

In this paper we examine the quantum circular and stadium billiards, which is a two-dimensional, billiard shaped region of zero potential (denoted by Ω) enclosed by a boundary $(\partial\Omega)$ outside of that potential is considered infinite along with a function Ψ defined on Ω which satisfies the Halmholtz equation with Dirichlet boundary conditions($\|\Psi\|_{\partial\Omega}=0$).

The circular billiard constitutes an integrable system the number of constants of motion is equal to the number of degrees of freedom. In quantum the plotted eigenstates for the circle show a regular tending of having small values in the middle the corresponds to the classical analogue-aparticle bouncing around inside the circle will tend to have a trajectory that keeps it away from the center. Examples of the eigenstates are shown in Fig2.

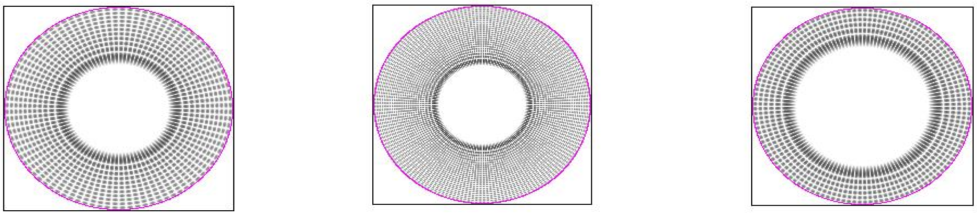


Figure2: The eigenstates of the integrable circular billiard

Ordinary two typical eigenstates exist for stadium, one of the states shows quite clearly the scarring that occurs due to the back and forth bouncing motions that the horizontal walls cause. The other state shows clearly the chaotic nature of the stadium. It is spread out over the whole system, the way a particle would bounce over the whole system. The Fig3 shows example of bouncing ball and irregular states in stadium with $\eta = 0.5$.

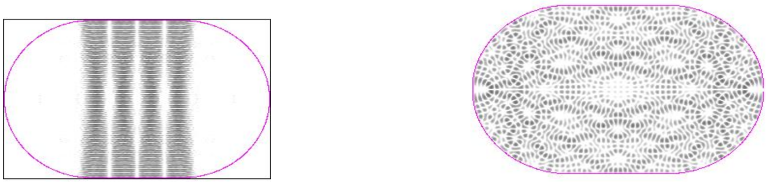


Figure3: The eigenstates of the stadium billiard with $\eta = 0.5$

The other subject to study is the pattern nodal curve on the surface and the distribution of zeros of wave function on the boundary of billiard. This is shows in Fig4.

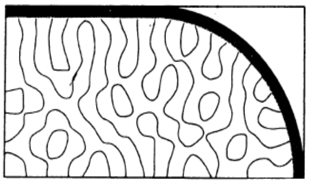


Figure4: the pattern nodal curve distribution of zeros of wave function on the boundary of billiard.

Thus for calculating the gyration radius in quantum states we can use of the coordinate of the collision of zeros of wave function on the boundary. In this case, we use the following equation $R_s^{\ 2} = R^2$ because the center of mass in this collision is at the center of billiard.
In circular billiard distribution of zeros of wave function on the boundary is regular and distance of the collision on billiard boundary until the center of billiard is constant than the quantity gyration radius in circular is equal R^2.But in stadium distribution of zeros of wave function on the boundary is fully chaotic thus we can by using this property and the coordinate collision calculate the gyration radius in different energy levels in billiard.
Fig5 shows examples of the eigenstates in stadium with $\eta = 1$ for different energy levels and Fig6 shows relationship the maximum of gyration radius to bouncing ball states in stadium.

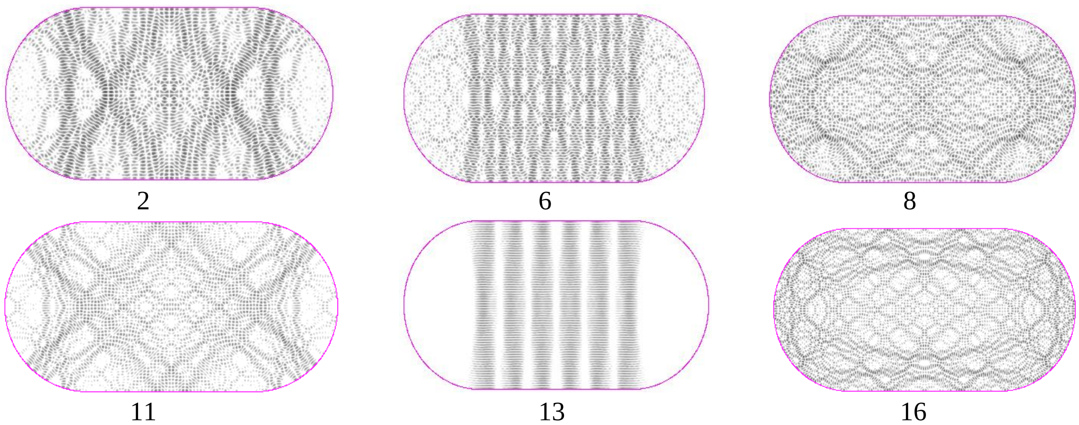


Figure5: Examples of the eigenstates of the stadium billiard with $\eta = 1$

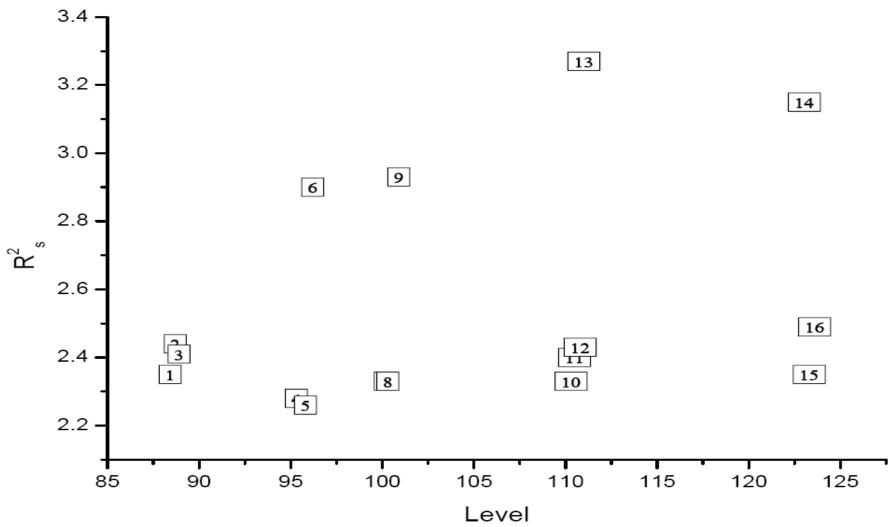
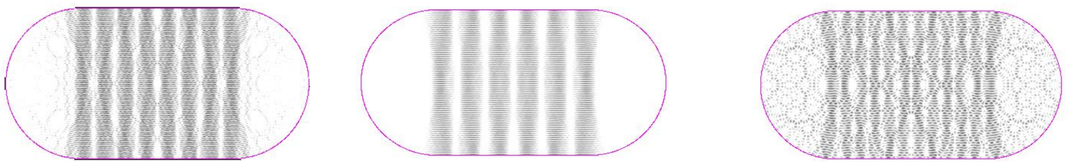


Figure6: The maximum of gyration radius is relation the bouncing motion in stadium with $\eta = 1$

We obvious the maximum of gyration radius is relation the bouncing ball states and with increase regular this states the value gyration radius becomes large. Examples of this states are shown in Fig7:



$R_s^2 = 2.91$ $R_s^2 = 3.3$ $R_s^2 = 3.2$

Figure7:Examples of the bouncing ball states in stadium billiard with $\eta = 1$

Here, we consider the changing gyration radius in stadium with $\eta = 0.5$ for different energy levels. To illustrate this changing, we show in Fig9 for examples are shown in Fig 8 for stadium with $\eta = 0.5$.

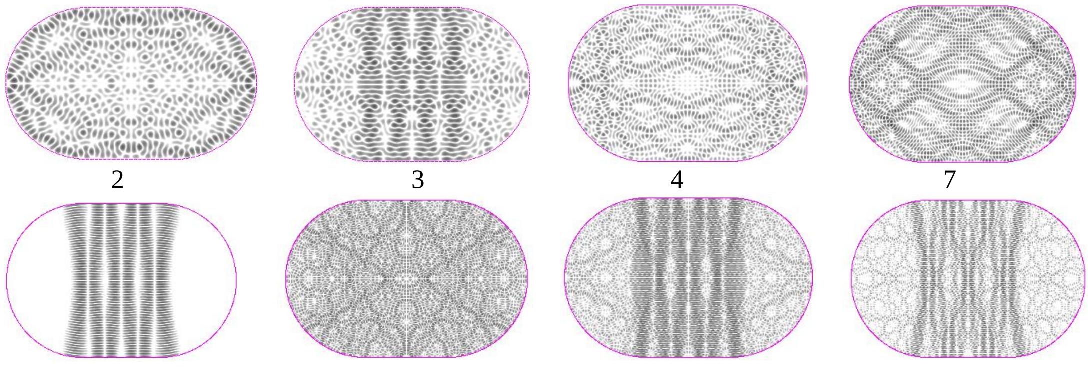


Figure8: Example of the eigenstates of the stadium billiard with $\eta = 0.5$

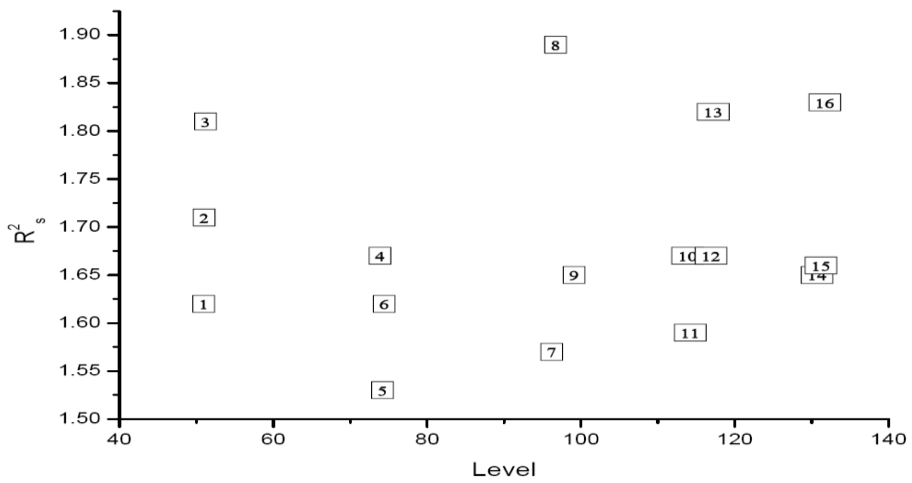
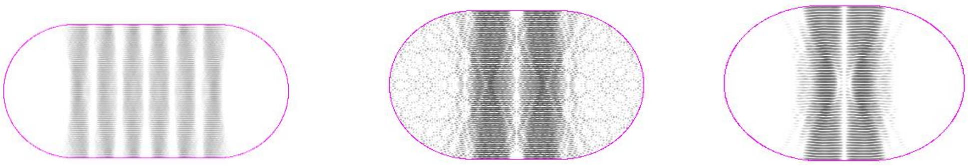


Figure9: The maximum of gyration radius is relation to the bouncing motion in stadium with $\eta = 0.5$

We know most of the eigenstates in stadium are chaotic or bouncing ball than for recognize this states we calculate the gyration radius .In Figs6 and 9 we see that the difference among the gyration radius for various states. In fact the maximum of gyration radius is relation the bouncing ball states because in this states is the ratio number of collision zeros of wave function with circle boundary at number of collision zeros of wave function with straight walls bigger than the other states. Thus we can by using the gyration radius parameter distinct bouncing ball states than the other states. When energy levels become large then the value gyration radius for chaotic states becomes almost equal together.

When discussing chaotic behavior and dynamical systems, we find out regular orbit sideway states in classical and regular bouncing ball in quantum exist in stadium with different parameter and the maximum value gyration radius is relation this states. Examples of this states are shown in Fig10.



$R_s^2 = 1.42$ $R_s^2 = 1.934$ $R_s^2 = 3.272$

Figure26: The maximum of gyration radius is relation to to the bouncing motion in stadium in stadium with different parameter

If we think carefully to this states we find out in classical the particle not contact with the straight walls boundary and in quantum the number of collision zeros of wave function with the straight walls boundary is few thus we observe having something in common for this regular states, first the value gyration radius is maximum, second collision points with the straight walls stadium is minimum.

4. Summary

Thus in section we saw the gyration radius parameter is suitable parameter for consideration chaos in billiards because by using of this parameter we can distinction chaotic billiards (stadium) than regular billiard (circular) and also we can distinction regular than irregular in chaotic billiards..

References

[1]M Henon, "Numerical Exploration of Hamiltonian Systems" North-Holland P.C. (1983).
[2] D. A. Steck; "classical and quantum chaos"; Theoretical Division (T-S). MS B285. Los Alamos National Laboratory, Los Alamos NM 87545 (16 july2002)
[3]Yu.L.Bolotin and V.V.Yanovsky " THE WORLD OF CHAOS" National Science Center "Kharkov Institute of Physics and Technology" , Institute of single crystals, National Academy of Science of Ukraine, 60,Lenin avenue,61001,Kharkov,Ukraine2007
[4]Birkhoff,G,D,"Dynamical system" AMS,Providence,1927
[5] J L Hansen " Lyapunov Exponents for the Stadium Billiards" Report for the spring 95 course: Classical and Quantum Chaos by Predrag Cvitanovic:August 24,1995
[6] Ch.Dellago and H.A.Posch " Lyapunov exponents of systems with elastic hard collisions" Institut for Experimentalphysik, Universitat Wien, Strudlhofgasse 4, A-1090 Wien, Austria
[7] http://en.wikipedia.org/wiki/Lyapunov_exponent
[8]M.Gutzwiller " quantum Chaos " Scientfic American,January 1992
[9] J.P.Bired et al., Rep. Prog. Phys. 66, 583 (2003)
[10]Lorenz,E.N,"Deterministic non-periodic flows",J.Atoms Sci,vol 20,130-141,1963
[11] L.Kouwenhoven and C.Marcus" Quantum dots" PHYSICS WORLD, JUNE 1998
[12] P.Cvitanovic, R. Artuso, R.Mainieri, G. Tanner, G.Vattay,N.Whelan and A.Wirzba " Classical and Quantum Chaos" Chaos Book.org, version 11, Dec 29 2004
[13]N. Saito, H. Hirooka, J. Ford, F. Vivaldi, and G. H. Walker. Numerical study of billiard motion in an annulus bounded by non-concentric circles. Physica D, 5(2-3):273-286, 1982
[14]L.A. Bunimovich "Conditions of stochasticity of two-dimensional billiards "Chaos. 1991, v. 1, p. 187-193
[15] L.A.Bunimovich" On the ergodic properties of certain billiards" Funkt.Anal.Appl.8,254(1974)
[16] Ya. G. Sinai. WHAT IS a billiard. Not. Am. Math. Soc., 51(4):412-413, 2004
[17] http://www.csr.umd.edu/anlage/AnlageHome.htm
[18]A .Porter and L Liboff ,"Chaos on the Quantum Scale", http://www.sigmaxi.org/amsci/articles/01articles/portercape6.htm1
[19] Q-SWITCH Electron Waveguides for Quantum-dased Switching Applications ":http://www.ftf.1th.se/coop/Qswitch.htm1
[20] K. Li and R.Lan " Quantum Dots: Applications in Modern Technology" July 13,2007
[21] R. P. Taylor, " The Role of Surface Gate Technology for AL GaAS/GaAS Nanostructures,Journal of Nanotechnology 5,183(1994)

[22] A.Z. GORSKI, T.SROKOWSKI " CHAOTIC AND REGULAR MOTION IN DISSIPATIVE GRAVITATIONAL BILLIARDS " H. Niewodniczzanski Institute of Nuclear Physics, Polish Academy of Sciences , Radzikowskiego 152,31-342 Krakow, Poland , 2006

[23] M.Brack and J.Roccia" Closed orbits and spatial density oscillations in the circular billiards" Institute for Theoretical Physics,University of Regensburg, D-93040 Regensburg, Germany, 6 May 2009

[24۴] S M McDonald and A N Kaufman, Phys. Rev. A **37,** 8 (1988) 3067-3086.

[25] D L Kaufman, I Kosztin and K Schulten, *Am. J. Phys*. **67**, 2 (1999).

[26] http://www.math.dartmouth.edu/~ahb/software.

[27] M J Lichtenberg and M A Liberman, Regular and Stochastic Motion, Springer-Verlag (1983) 277.

[28] E J Heller, Phys. Rev. Lett., **53** (1984) 1515.

[29] B Li, M Robnik, Phys. Rev. E **57** (1998) 4095-4105.

[30] E Vergini and M Saraceno, Phys. Rev. E **52** (1995) 2204-2207