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Multiple Allocation Hub Location Problem with Multiple Capacity Levels and Limited Flow in Linkages

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Abstract
During the two last decades, a wide rang of application of hub network design problems have received increasing attention in the fields of transportation, telecommunications, computer networks, postal delivery, less-than-truck loading and supply chain management. Hubs are special facilities used for switching, transshipment and sorting points in distribution systems. Hub facilities are used to consentrate flows in order to take advantage of economies of scale. In this research a multiple allocation hub location problem is considered and it is assumed each potential hub could have multiple capacity levels. Also, a fixed cost and a minimum capacity for all linkages are considered. In othere words, the flow between each linkages cannot exceed from a predetermined capacity. At the end, GAMS software with CPLEX solver is used to optimize the problem in small sizes and also, to check the feasibility of the new model.

Keywords: Hub location problem, multiple capacity levels, capacity of linkages.

1. Introduction
During the two last decades, a wide rang of application of hub network design problems have received increasing attention in the fields of transportation, telecommunications, computer networks, postal delivery, less-than-truck loading and supply chain management. Hubs are special facilities used for switching, transshipment and sorting points in distribution systems. Hub facilities are used to consentrate flows in order to take advantages of economies of scale. The hub location problems are involved three main issues that are: locating hub facilities, allocating origin and destination nodes to hubs and routing flows through the network. In hub networks, there are n nodes and the flow between each pair of nodes are given. The hub location problem tries to identify the set of origins, destinations and potential hub locations. There are two types of assigning the demand nodes to hubs that are: single allocation hub location problem and multiple allocation hub location problem. In single allocation, total flows of every demand node is routed from one hub, but in multiple allocation, flow of every demand node could routed from multy hubs. Also in a hub network, it is possible each pair of non-hub nodes connected to each other and sometimes it is impossible. Since location problems are affected by allocation problems and allocation problems are affected by location problems, so it is better to consider these problems together. A type of hub location problems concentrates on minimizing total transportation costs along with fixed costs for hubs in some cases that named p-hub median problem, and another type of these problems tries to minimize a maximum measure related to transportation distans or cost, so the name of this problem is p-hub center problem. Sometimes there are a limitation on the capacity of each hub for recieving and processing the flows from non-hub nodes. In this condition the problem is a capacitiated allocation hub location problem and if there is no constraint on the capacity of the hub, the problem is named uncapacitiated allocation hub location problem. Also it is possible there is a limitation on capacity of the linkage between non-hub nodes and hub nodes. Therefore, if this constraint is not met, the linkage could not be created. In the real world, it is better to consider multiple capacity levels for each hub nodes and try to choose the best capacity levels for

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creating hub nodes based on their fixed costs for creating related hubs and their capacity amounts. Therefor, only one capacity level for each hub should be selected.

Finally, in this research we have a capacitated multiple allocation hub location problem with multiple capacity levels which the capacity is related to the hubs and the linkages. In this research, it is tried to add multiple capacity levels constraint for total entering flows from non-hub nodes to hub nodes and a limitation on the flow of every linkage to the recent hub location problems in a discrete space for the first time. Thus, in this condition the objective function of the problem will be changed. The purpose of this problem is minimizing transportation cost of total flows from origin points to destination points, fixed costs of creating linkages and hubs by selecting a capacity level.

The reminder of this paper is organised as follows: in section 2 a literature review of the hub location problems is presented, section 3 describes the mathematical formulation, computational results are presented in section 4 and in the last section we make our final considerations.

2. literature review

The first works on locating hub facilities were done by ([1]-[6]). Maybe the first paper about hub location problems were published in [7], but all researchers believe that the first mathematical formulation for the hub location problem was presented by [8] about airline passenger networks. In his paper, he considered a single allocation p-hub median problem with n demand nodes. The recent survey paper was presented by [9] shows increasing greatly in hub location problems in recent years. Based on assigning demand nodes to the hubs, hub networks are classified to two types that are: single allocation hub location problems and multiple allocation hub location problems. Reference [10] considered the uncapacitated single allocation hub location problem and used three variants of a simple and efficient multi-start tabu search heuristic to solve the problem. Also, [11] proposed two genetic algorithms for solving the uncapacitated single allocation p-hub median location problem. Reference [12] presented a new formulation for the uncapacitated hub location problem. Their formulation includes a more general cost structure that satisfying the triangular inequality is not needed in it. In 2007, [13] used a dual-ascent technique for solving an uncapacitated multiple allocation hub location problem. Also they assumed the number of hubs are not determined already. Reference [14] presented a benders decomposition for the uncapacitated multiple allocation hub location problem in order to find the least expensive hub-and-spoke network, select the hubs and assign traffic to them. Sometimes there is a limitation on the amount of flow can be processed in each hub, in this condition the problem is a capacitated allocation hub location problem. Reference [15] revised a classical formulation of capacitated single allocation hub location problem and showed the incompleteness of the classical model by an example. A different approach for a capacitated single allocation hub location problem was utilized in [16] by adding an objective function to the model in order to minimize the processing time of entering flows to the hubs. In the paper presented by [17], a mixed integer linear programming model for the capacitated multiple allocation hub location problem was proposed. Also they used shortest paths and constructed an efficient heuristic algorithm. Reference [18] formulated and solved a splittable capacitated multiple allocation hub location problem with medium sized instances. The capacitated multiple allocation hub location problem was studied in [19]. In their new model, size of each hub is a part of a decision making process. Thus the presented model tries to chose a capacity level for each potential hub among a set of available capacities.

Based on the literature, since there is not a multiple allocation hub location problem with multiple capacity levels and capacity constraint on the linkages, in this research a capacitated multiple allocation hub location problem with multiple capacity levels will be presented. Also, we consider a fixed cost for creating each linkag with a capacity constraint on the allowed flow between them.

3. Mathematical formulation

It is assumed there is a complete graph $G=(V,E)$ with node set $V=(1,2,...,N)$. The node set indicates the origin and destination nodes and potential hub locations. d_{ij} is distance between node i and j and w_{ij} is the flow should be transported from node i to node j. It is assumed each origin-destination flow should be routed at last via one hub. The flow of each origin-destination includes

collection from the origin to a hub through arc *(i,k)* and distribution from a hub to the destination through arc *(m,j)*. The origin or destination node could be a hub, in this condition the collection or distribution component may be at the origin or destination node. In this model, the cost of collecting the flow from non-hub node to a hub node, transfer cost of the flow between two hub nodes and the cost of distribution from any hub node to any non-hub node are different. Thus it is assumed χ is the coefficient of the collection cost (per unit flow) from each non-hub node to each hub node; δ is the coefficient of the distribution cost from each hub node to each non-hub node and the cost for transporting flow from one hub to another hub is discounted by the parameter $\alpha, (0 \le \alpha \le 1)$ and $(\alpha \le \chi \text{ and } \alpha \le \delta)$, [16]. For each potential hub, a set of different capacity levels is considered and the presented model forces each hub node to select one capacity level that have minimum fixed cost so that the capacity constraints on the hub and linkages are meted. In this model, it is assumed each linkage has a minimum limitation so that creating each linkage is related to this limitation. Also a fixed cost is considered for creating each linkagethat it is added to the objective function.
Further assumptions of this problem are:

There are no predetermined structure about the distances and direct relation between non-hub nodes. So when two origin and destination nodes are non-hub nodes, the flow is sent from one or two hubs, when two origin and destination nodes are hub nodes, the flow is sent directly and if one of the origin or destination node is hub, the flow is sent from another hub or directly.
Now the notations of the model is presented.

Parameters are:

N	*set of nodes* $N = (0, 1, 2, ..., n)$
M	*a large number*
$Q_k = \{1,..., s_k\}$	*an available set of different capacity levels for a potential hub to be installed at node k* $(k \in N)$
$N_{Q(k)}$	*number of capacity levels for each node k*
w_{ij}	*flow to be sent from node i to node j* $(i, j \in N)$
c_{ij}	*transportation cost of one unit flow between node i and j* $(i, j \in N)$
c_{ijkm}	*cost of transporting one unit of flow from node i to node j via hub k and m* $(i, j, k, m \in N)$
F_k^q	*fixed installation cost for hub k with capacity level q* $(k \in N, q \in Q_k)$
Γ_k^q	*capacity of hub k with capacity level q* $(k \in N, q \in Q_k)$
S_{ik}	*fixed cost for creating a linkage between non-hub node i and hub k* $(k, i \in N)$
T_{ik}	*minimum capacity of each linkage between non-hub node i and hub k* $(k, i \in N)$
α	*transportation cost coefficient*
δ	*collection cost coefficient*
χ	*distribution cost coefficient*

and variables are

x_{ijkm}	*fraction of the flow originated at node i destined to node j is routed via hubs k and m* $(i, j, k, m \in N)$
h_{ik}	*1 if node i is assigned to hub k and 0 otherwise*
h_{kk}	*1 if hub k is created and 0 otherwise*
μ_{km}	*1 if node k and node m are hubs and 0 otherwise*

Since previous hub location models cannot show the linkages between hub nodes, in this research we introduce new variable μ_{km} to measure the creating cost of hub linkages.
Also transportation cost between two nodes *i* and *j* via hub *k* and *m* is as follow:

$$c_{ijkm} = \delta\, c_{ik} + \alpha\, c_{km} + \chi\, c_{mj} \qquad (1)$$

Regarding the capacity, the following relation for each $(k, i \in N_k)$ is assumed: $\Gamma_k^{q_1} < \Gamma_k^{q_2} < \ldots < \Gamma_k^{q_{s_k}}$.
Clearly, necessary conditions for the feasibility of the problem are $\sum_{k \in N} \Gamma_k^{s_k} \geq \sum_i \sum_j w_{ij}$ and $\sum_{i \in N} \sum_{k \in N} T_{ik} \leq \sum_i \sum_j w_{ij}$.

Therefore the model's formulation is as follows:

$$Min \quad \sum_i \sum_j \sum_k \sum_m w_{ij} c_{ijkm} x_{ijkm} + \sum_k \sum_{q \in Q} F_k^q z_k^q + \sum_i \sum_k S_{ik} h_{ik} + \sum_k \sum_{m>k} S_{km} \mu_{km} \tag{2}$$

s.t.

$$\sum_i \sum_j \sum_k \sum_m x_{ijkm} = 1 \quad \forall\ i, j, k, m \tag{3}$$

$$\sum_m x_{ijkm} \leq h_{ik} \quad \forall\ i, j, k \tag{4}$$

$$\sum_k x_{ijkm} \leq h_{jm} \quad \forall\ i, j, m \tag{5}$$

$$\sum_m \sum_{k \neq i} x_{ijkm} + h_{ii} \leq 1 \quad \forall i, j \tag{6}$$

$$\sum_k \sum_{m \neq j} x_{ijkm} + h_{jj} \leq 1 \quad \forall i, j \tag{7}$$

$$h_{ik} \leq h_{kk} \quad \forall\ i, k \tag{8}$$

$$\sum_{q \in Q} z_k^q \leq h_{kk} \quad \forall k \tag{9}$$

$$2\mu_{km} \leq h_{kk} + h_{mm} \quad \forall k, m \tag{10}$$

$$\mu_{km} \geq h_{kk} + h_{mm} - 1 \quad \forall k, m \tag{11}$$

$$\sum_i \sum_j w_{ij} \sum_m x_{ijkm} \leq \sum_{q \in Q} \Gamma_k^q z_k^q \quad \forall k \tag{12}$$

$$T_{ik} - \sum_j \sum_m \left(w_{ij}\ x_{ijkm} + w_{ji}\ x_{jimk}\right) \leq M\left(1 - h_{ik}\right) \quad \forall\ i, k \neq i \tag{13}$$

$$x_{ijkm} \geq 0 \quad \forall i, j, k, m \tag{14}$$

$$z_k^q \in \{o, 1\} \quad \forall k, q \in Q \tag{15}$$

$$h_{ik} \in \{o, 1\} \quad \forall i, k \tag{16}$$

$$\mu_{km} \in \{0, 1\} \quad \forall k, m \tag{17}$$

The objective function (2) evaluates the overall cost which is divided into the cost of collection, transfer, distribution, the cost for installing the hubs, the cost for creating linkages between non-hub nodes and hubs and also the cost of creating linkages between hubs. Constraint (3) ensures total flow sending from node *i* to node *j*. The next constraint shows if non-hub node *i* do not assigned to hub *k*, flow of node *i* could not send from hub *k* and also, constraint (5) ensures only if the node *j* is assigned to hub *m*, flow could sent to this node through hub *m*. Constraint (6) indicates if origin node *i* is hub, the flow between origin node *i* and destination node *j* send directly or through another middle hub. Constraint (7) shows if destination node *j* is hub, the flow between origin node *i* and destination node *j* send directly or through another middle hub. Constraint (8) insures assigning non-hub node *i* to hub node *k* is possible when hub node *k* is created. Inequality (9) is used to ensure only one capacity level is selected for every hub. Two new constraints (10) and (11) could show the linkages between all hubs and also they are used to prevent from making a non-linear constraint. Constraint (12) limits capacity of hub-node k with level q for processing the entering non-processed flows. The next constraint is used to ensure a linkage between non-hub node i and hub node k is created when minimum flow be observed. Finally (14)-(17) are domain constraints. The presented mixed-integer linear programing model has $n^4 + 2n^3 + 6n^2 + n$ constraint and $n^4 + 2n^2 + n\sum_k N_{Q_k}$ variables that $2n^2 + n\sum_k N_{Q_k}$ of them are binary variables.

4. Computational results

To test the performance of our proposed model, an exact solution is necessary, hence we use GAMS software with CPLEX solver. Since previous data in the literature is not suitable for our problem, we produce random parameters so that they create feasible solutions based on [20]. Table 1 shows the parameter values and probability distributions used for randomly generated test instances.
In this research 9 problems with 3 capacity levels are introduced. The results of proposed test instances are shown in Table 2. The first column shows the number of each problem, the second column shows the number of opened hubs, the third column shows the nodes that are hubs with their capacity levels, objective function value (OFV.) of each problem is shown in column 4 and the last column shows elapsed time of solving each problem.

Table 1. Generation of random data

No.	No. of nodes	Capacity level	(δ,α,χ)	W	C	F	S	Γ	T
1	5	1	(1, 0.75, 1)	U~(1,10)	U~(1,10)	U~(400,600)	U~(50,200)	U~(30,60)	U~(1,10)
2	5	2				U~(500,700)		U~(40,70)	
3	5	3				U~(600,800)		U~(50,80)	
4	10	1				U~(1100,1400)		U~(90,120)	U~(1,8)
5	10	2				U~(1200,1500)		U~(100,130)	
6	10	3				U~(1300,1600)		U~(110,140)	
7	15	1				U~(2000,2300)		U~(170,200)	U~(1,8)
8	15	2				U~(2100,2400)		U~(160,190)	
9	15	3				U~(2200,2500)		U~(170,200)	

Table 2. The results of CPLEX solver

No.	No. of opened hubs	(hub/capacity level)	OFV.	Elapsed time
1	2	(1/1), (5/1)	2555	0:00:00.264
2	2	(2/2), (5/1)	2466.5	0:00:00.227
3	2	(2/2), (3/2)	2296	0:00:00.166
4	5	(1/1), (3/1), (5/1), (6/1), (10/1)	11144.500	0:00:22.503
5	4	(3/2), (6/2), (8/2), (10/2)	10621.500	0:00:29.913
6	4	(3/3), (6/1), (8/3), (10/2)	10500.250	0:00:17.092
7	7	(4/1), (5/1), (6/1), (10/1), (11/1), (12/1), (14/1)	25022.750	0:15:05.797
8	6	(4/2), (6/2), (8/2), (9/2), (12/2), (15/2)	23533.750	0:10:45.935
9	6	(4/2), (5/2), (10/1), (11/3), (12/2), (15/2)	23107.500	0:7:46.787

The results show that we could decrease the objective function value and number of opened hubs by using multiple capacity levels. Therefore, when we have a capacitated hub location problem, it is better to introduce multiple capacity levels for each node in order to make the best hub network with minimum cost by choosing the best one of them for hub nodes.
Since the literature of location problems shows that these problems are NP-hard and hub location problems are a kind of location problems, therefore our proposed model is NP-hard too. In this kinds of problems necessary time for solving the problem increased exponentially by increasing the size of problem as shown in the last column of Table 2.

5. Conclusion

This research tried to extend a capacitated multiple allocation hub location problem to increase the ability of the hub network and decrease its costs. Therefore, multiple capacity levels are introdused for each potential hub node so that the best one of these levels is chosen based on the cost and capacity constraint. Also, it was assumed each linkage has a fixed cost for creating. Thus, to avoid creating linkages with less flow, a minimum capacity for each linkage is considered. Since previouse hub location models cannot show the linkages between hub nodes, a new variable was presented to show them. Our new proposed model was solved by CPLEX solver for 5, 10 and 15 nodes and 3 capacity levels for generated test instances. Our results showed we could decrease the cost and hub number of the network by choosing the best capacity level amonge multiple capacity levels. Since the proposed model is NP-hard, we suggest using a metaheuristic algorithm to solve the model for the future research.

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