A fuzzy model for a distribution network problem in a multi-product supply chain system

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Abstract
This study presents a fuzzy mathematical model for a distribution network design problem in a multi-product supply chain management that contains locating production plants and distribution centers as well as determining the best strategy for distributing the product from plants to warehouses and from the warehouses to customers with triangular and trapezoidal membership functions for customer demands. Furthermore the model determines the capacities level of the facilities (plants or distribution centers (DCs)) and the design of the network (selecting the optimum numbers and locations of plants and distribution centers to open, so that all uncertain customer demands of all product types are satisfied) with a minimum total cost. In this study we develop a fuzzy mixed-integer non-linear mathematical programming model for designing a supply chain distribution network and present a case-study of Iran to show the superiority and efficiency of the model for a large dairy company and solve it by a professional software package (LINDO).

Keywords: distribution centers, MINLP, supply chain system, fuzzification, capacities level.

1. Introduction
Nowadays, the supply chain design problem has been obtaining an important place because of competitive nature of the market globalization. This competitive nature of marketplace is more important in a large Iranian dairy company and the complexity of decision-making process with contradictory objectives of the different business aspects; clarify the need for designing the most optimum supply chain network in such an industry. Also the choice of facilities (plants and distribution centers (DCs)) to be opened and the distribution centers design to satisfy the uncertain customer demands with minimum total cost were desired to achieve, too.

Supply chain management (SCM) is a strategy through which integration between different functions such as marketing, distribution, planning, manufacturing and purchasing can be achieved. It includes the choice of facilities (plants and distribution centers (DCs)) to be opened and the distribution centers design to satisfy the customer demands with minimum total cost. If the facilities have a certain capacity, the problem is known as a capacitated location-allocation problem. In this problem, customers demand multiple units of different commodities and receive these products from several plants. Both kinds of problems (capacitated location problems and uncapacitated location problems) can be formulated as mixed integer programming problems.

Researchers have used heuristic approaches to solve the multi-stage design problem which is difficult to solve optimally, especially if capacity constraints are imposed on both plants and DCs (as in this study we do so). A heuristic approach based on Lagrangean relaxation for the single-source, multi-commodity, multi-plant, capacitated facility location problem have proposed [1]. Also for a multi-source, multi-commodity, multi-location framework another heuristic approach based on

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Lagrangean relaxation and simulated annealing has proposed [2]. Also a heuristic approach based on simulated annealing for the designing of distribution network and management in supply chain environment has developed [3]. Some researchers have been applied genetic algorithm to solve SCN design problems in location-allocation problems ([4], [5]). A multi- site, multi-commodity production network have been modeled with a mixed integer linear programming that combines aspects related to production, distribution and marketing and involves production sites (plants) and sales points [6].

Reference [7] shows a mixed integer programming for modeling a multi-period, two-echelon, multi-commodity capacitated plant location problem to minimize the total cost for meeting demands for all kinds of commodities specified over the planning horizon at different customer locations while satisfying the capacity requirements of the production plants and warehouses. They assumed that the capacities of plants and warehouses and transportation costs change over the time periods. Then they used a Lagrangean relaxation to solve the problem and with the use of feasible solutions of the original problem constructed from the solutions at the lower bounds obtained by the relaxed problems, a heuristic procedure finds the optimal solution for the problem. They implemented the model by using C++ and CPLEX 6.0. At last good performance of this approach for a wide range of problems (with randomly generated data) is shown by the computational tests.

Reference [8] modeled a capacitated, multi-commodity, multi- period, multi-stage facility location problem with an algorithm that comprises of three phases: in phase I which is called dynamic cycle, they try to identify facilities that should be opened or closed in the optimal solution. In phase II, the optimal static solutions for each time period are obtained, and a list of candidate static facility configurations is generated using a branch and bound procedure for each period. In phase III optimal solution is obtained from these lists by dynamic programming (the straight forward application). The periods form the stages while the static configurations form the states within each stage in dynamic programming. They acclaimed that when the facility reopening and closing costs are significant in the multi-period problem, their algorithm is effective. They assumed that in addition to the deliveries from the plants to warehouses and from the warehouses to customers, there are direct deliveries from the plants to the customers. With these assumptions, finally they implemented the model for an instance with three periods, four potential locations, five customers, two commodities and two factories. In this instance, closing penalties are the same as the set-up costs of the corresponding facilities, while reopening penalties are twice of the closing penalties. The most advantage of using their algorithm in comparison to others is reduction in total computation time required for solving the entire problem.

Another logistic chain network problem modeled with a 0-1 mixed integer linear programming that includes the choice of the facilities (plants and distribution centers) to be opened and the distribution network design, with the goal of satisfying the demand requirements imposed by customers with minimum total cost. They used the spanning tree-based genetic algorithm. They coded their model by the use of a professional software package LINDO and Visual C language. At last they demonstrated the efficiency and superiority of their model by comparing its numerical experiment results for four different sizes of test problems with the results of traditional matrix-based genetic algorithm [9].

Reference [10] proposed a mathematical modeling framework for dynamic multi-commodity capacitated facility location and relocation of facilities in strategic supply chain planning with respect to many aspects such as: dynamic planning horizon, generic supply chain network structure, external supply of materials, inventory opportunities for goods, distribution of commodities, facility configuration, availability of capital for investments, and storage limitations. In this study, gradual relocation of facilities over the time horizon is considered. Capacity expansion and capacity reduction scenarios for coping with fluctuating demands are also considered. They assumed that commodities can be transported between any type of facilities (in addition to the common deliveries, direct deliveries from plants to customers, deliveries between plants or warehouses is allowed). The objective function minimizes the costs which are divided into two categories: the business costs, which result from operating the supply chain network, and investment costs for facility relocation, which are constrained by the available budget. At last they solved the MIP with the branch and bound algorithm of ILOG CPLEX 8.0.

Reference [11] proposed a steady-state genetic algorithm for the design of a single-source, multi-commodity, multi-stage supply chain network design (includes the choice of facilities to be opened) to satisfy the customer demand with the minimum total cost. They assumed that the number of customers (suppliers) and their demand (capacities) are known; the number of potential plants, DCs and their maximum capacities are known; and customers are supplied products from a single DC. At last, they investigated the effectiveness of the ssGA by comparing its results (for 50 randomly generated data instances in each class) with those obtained from other approaches such as CPLEX, Lagrangean heuristic, hybrid GA and simulated annealing.

Reference [12] proposed a fuzzy mixed integer multiple goal programming for modeling a channel allocation problem that consists of determination in channel mix and capacity allocation for each distribution channel. In addition to manufacturing capacity, customer's demand, channel capacity, channel quota flexibility, budget limitations that they considered in constraints; they also considered the business competitive advantages such as maximizing net profits, minimizing the rate of end user claims, and minimizing the rate of late lading in their objective functions. They assumed that quantity discount is not allowed. In their special case (in this study, realistic data from Taiwan's largest steel company is implemented for the effectiveness of the model), they set higher priority to the profit maximization goal than the other two goals, and thus they ranked the three goals into two priority levels to meet requirements from decision makers.

Reference [13] proposed an integrated cost-based and mixed-integer production-distribution programming model for the dynamic location and allocation problem with safety stock optimization and customer service level. Their target is to make a decision about the number of facilities (warehouses and distribution centers), their locations and the assignment of customer demand to them, and simultaneously consider inventory control, production rates, and service-level determination in a stochastic environment for a two-stage distribution system. They used the model in an international Italian electronics company. In first step, they assessed the current network to understand if their model is suitable for this company, after that with applying this model to case study they could reduce the global cost of network by at least 10 percent (with reduction of safety stock levels, number of distribution centers, optimization of flows and transportations).

Reference [14] proposed a fuzzy linear programming approach for the optimization of a multi-echelon supply chain network with uncertain facility capacities. They used triangular and trapezoidal membership functions to obtain the fuzzy capacities of the facilities (plants or distribution centers (DCs)) and the design of the network configuration (includes choosing the location of facilities and determining the optimum flow of materials from supply sources to consumption points) with a minimum total cost which is segmented into five parts: the purchasing cost and the shipping cost from suppliers; the transportation costs between plants and DCs; the distribution costs between DCs and customer zones; the fixed operating costs; and the opportunity costs from not having the material at the right time (not to select the shortest shipping, transportation or distribution time). Finally with the use of a professional software package (LINDO), they solved the model and claimed that the trapezoidal membership functions are better than the triangular membership functions in real cases and increasing the interval width of the trapezoidal membership functions decreases the total cost in the supply chain network.

A multi-period, multi-stage stochastic supply chain network design problem with respect to the financial decisions and risk management ($SCND_{MSFR}$) is modeled. They have optimized the location of the facilities, the flow of commodities and the investments to make in alternative activities to those directly related with the supply chain design and simultaneously they considered uncertainty for demand and interest rates. With the use of a multi-stage stochastic mixed-integer linear programming, they maximized the total net financial benefit. The objective function includes the total revenue, the cost of operation of the facilities and the cost of shipment of commodities. In addition to the capacity constraints, they have a constraint for investment to not exceeding the total budget available. Also an alternative formulation is proposed which is based upon the paths in the scenario tree. Then they implemented the model for an instance with three periods, five potential locations, 20 customers, eight different investments (that vary in their duration, returns and variability), three different loans (each of them starting at the beginning of a different period) by using the C++ optimization modeling library.

At last with the use of randomly generated data, they presented that the stochastic approach is worth considering in these types of problems [15].

Reference [16] proposed a mixed integer programming model to optimize the distribution network design problem in a supply chain system (includes locating production plants and distribution warehouses, as well as determining the best strategy for distributing the products from the plants to warehouses and from warehouses to customers) with multiple levels of capacities available to the warehouses and plants. The objective function minimizes the total costs that are segmented into three parts: the costs to serve the demands of customers from the warehouses, the costs of transportations from the plants to the warehouses, and the costs associated with opening and operating the warehouses and the plants. The model selects the optimum numbers, locations and capacities of plants and warehouses to be opened so that all customer demand is satisfied. He provided a heuristic solution procedure with Lagrangean relaxation and designed the distribution network in the supply chain system. He coded the model by Borland Delphi and solved it for twenty-eight problem sets (each set contains ten problems) which are generated randomly.

Reference [17] extended Amiri's work in developing the mathematical model for a distribution network problem in a multi-product supply chain system that involves the optimum numbers, location and capacities of plants and warehouses to open for satisfaction of all customer demands of all product types at minimum total costs of the distribution network. They proposed a mixed-integer mathematical programming for designing such a problem that provides the best strategy for distributing the products from the plants to the warehouses and from the warehouses to the customers as well. The objective is minimizing the cost of the system and satisfaction of all customer demands of all product types without exceeding the capacities of the warehouses and plants. They assumed that just the customer zones are fixed and predetermined. They considered a real-case data from a pharmaceutical company in Iran including 15 product types, 30 customer zones, 10 potential warehouses, 5 potential plants and just 1 warehouse capacity level and 1 plant capacity level. Finally they coded the model by Lingo 6.0.

Our study is an extension of Rabbani et al.'s work and we considered the fuzzy model for a large Iranian dairy company. Our contributions are adding capacity levels of warehouses and plants and simultaneously considering uncertain customer demands for opening and operating plants or DCs in a multi-product supply chain management. The objective in designing this distribution network is to determine the minimum total cost system design such that the demands of all customers are satisfied without exceeding the capacities of the warehouses and plants. Usually, this involves making trade-offs among the cost components of the system that include the costs of opening and operating the plants and warehouses and the transportation costs. Also it contains locating production plants and distribution centers as well as determining the best strategy for distributing the product from plants to warehouses and from the warehouses to customers with respect to triangular and trapezoidal membership functions for customer demands.

The structure of this study is as follows: in section 2 we introduce the assumptions, description and formulation of the model. Application of the framework to a simulated case-study with variety of warehouse and plant capacity levels and considering financial decisions in a certain time horizon is then presented in Section 3. Finally, the work is summarized and results are provided in Section 4.

2. Model Formulation

This model assumes that the final network of the presented mathematical model is derived from a general potential network (Figure1). It assumes that the customer zones are fixed and predetermined and this model is trying to select the optimum supply chain network, incurred of the optimum numbers, locations and capacity levels of plants and warehouses to open with the object of serving all customer demands of all product types at minimum total costs.

The proposed model uses the following notations:

N number of customer(s) zones
M number of potential warehouse sites
L number of potential plant sites
R number of capacity levels available to the potential warehouses

H number of capacity levels available to the potential plants
P number of different product types
C_{lij} cost of supplying one unit of product l demand to customer zone i from warehouse at site j
D_{ljk} cost of supplying one unit of product l demand to warehouse at site j from plant at site k
F_{rj} fixed cost for opening and operating warehouse with capacity level r at site j
G_{hk} fixed cost for opening and operating plant with capacity level h at site k

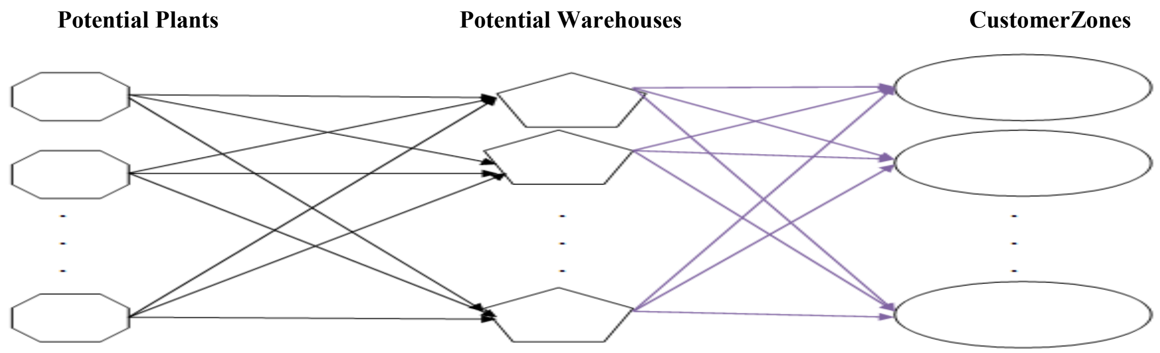

Figure 1 A supply chain network

a_{li} product l demand of customer zone i
b_{rj} capacity with level r for the potential warehouse at site j
e_{hk} capacity with level h for the potential plant at site k.

The decision variables are defined as follows:

X_{lij} = fraction (regardinga_{li}) of product l demand of customer zone i delivered from warehouse at site j

Y_{rljk}= fraction (regardingb_{rj}) of product l shipment from plant at site k to warehouse at site j with capacity level r

S_{lj}= fraction of capacity of warehouse at zone j that is assigned to hold product type l

T_{lk}= fraction of capacity of plant at zone k that is assigned to produce product type l

$$U_{rj}=\begin{cases}1 & \text{if a warehouse with capacity level r is located at site j}\\ 0 & \text{otherwise}\end{cases}$$

$$V_{hk}=\begin{cases}1 & \text{if a plant with capacity level h is located at site k}\\ 0 & \text{otherwise}\end{cases}$$

The problem is formulated as follows:

$$\min \sum_{l\in P}\sum_{i\in N}\sum_{j\in N} C_{lij}\,\widetilde{a_{li}}X_{lij} + \sum_{l\in P}\sum_{r\in R}\sum_{j\in M}\sum_{k\in L} D_{ljk}b_{rj}Y_{rljk} + \sum_{j\in M}\sum_{r\in R} F_{rj}U_{rj} + \sum_{k\in L}\sum_{h\in H} G_{hk}V_{hk} \quad (1)$$

s.t.

$$\sum_{j\in M} X_{lij} = 1 \qquad \forall\, i \in N, l \in P \quad (2)$$

$$\sum_{l\in P} S_{lj} = 1 \qquad \forall\, j \in M \quad (3)$$

$$\sum_{l\in P} T_{lk} = 1 \qquad \forall\, k \in L \quad (4)$$

$$\sum_{i\in N} \widetilde{a_{li}}X_{lij} \le \sum_{r\in R} b_{rj}U_{rj}S_{lj} \qquad \forall\, l \in P, j \in M \quad (5)$$

$$\sum_{i\in N} \widetilde{a_{li}}X_{lij} \le \sum_{k\in L}\sum_{r\in R} b_{rj}Y_{rljk} \qquad \forall\, l \in P, j \in M \quad (6)$$

$$\sum_{j\in M}\sum_{r\in R} b_{rj}Y_{rljk} \le \sum_{h\in H} e_{hk}V_{hk}T_{lk} \qquad \forall\, l \in P, k \in L \quad (7)$$

$$\sum_{l\in P} S_{lj} = \sum_{r\in R} U_{rj} \qquad \forall\, j \in M \quad (8)$$

$$\sum_{l\in P} T_{lk} = \sum_{h\in H} V_{hk} \qquad \forall\, k \in L \quad (9)$$

$$\sum_{r\in R} U_{rj} \le 1 \qquad \forall\, j \in M \quad (10)$$

$\sum_{h \in H} V_{hk} \leq 1 \qquad \forall\ k \in L \qquad (11)$

$Y_{rljk} \leq U_{rj} \qquad \forall\, r \in R,\ l \in P,\ j \in M, k \in L \qquad (12)$

$X_{lij}, S_{lj}, T_{lk}, Y_{rljk} \geq 0 \qquad \forall\, r \in R,\ l \in P,\ j \in M, k \in L \qquad (13)$

$U_{rj} \in (0,1) \qquad \forall\, r \in R,\ j \in M \qquad (14)$

$V_{hk} \in (0,1) \qquad \forall\, k \in L,\ h \in H \qquad (15)$

The model minimizes total costs consist of these factors:

1. the costs to satisfy the demands of customers from the open warehouses
2. the costs of transportation from the open plants to the open warehouses
3. the costs of opening and operating the warehouses and the plants in different levels

Constraint (2) guarantees that the all customer demands of all product types are served by open warehouses. Constraints (3) and (4) guarantee that the assigned fractions of open warehouses and plants capacity levels to different product types do not exceed warehouses and plants capacity levels. Constraints (5) and (6) ensure that the total fuzzy annual amount of customer demands of all product types served by an open warehouse and this amount of commodities is in the warehouse capacity limitation and in the total transportation limitation to the warehouse from all open plants. Constraint (7) ensures the capacity limitations of the plants with considering their total transportations of all product types to the warehouses. Constraints (8) and (9) represent that the assigned fraction of warehouses and plants capacities to different product types is just distributed to one capacity level. Constraints (10) and (11) guarantee that a warehouse and a plant are assigned to at most one capacity level. Constraint (12) ensures not exceeding the capacity level of the located warehouse. Finally, Constraint set (13) indicates the non-negativity restrictions and Constraints (14) and (15) ensure the binary variables.

As it mentioned before, the new model will be solved by using the LINDO software. With considering the literature review that is presented before in this study, it is obvious that LINDO is one of the most popular approaches in solving such a model among its competitors like CPLEX, GAMS, LINGO, … .

3. Case Study

In this study, designing a distribution network for a large dairy company is desired. This company wants to optimum its costs by using this network. This company produces 10 million units of 5 main categories of different kind of products. The parameters of that company are as follows:

- 4 main customer zones, 3 potential warehouses, 3 potential plants, 5 product types, 3 warehouse capacity levels and 3 plant capacity levels 3.
- Total number of variables is 244 including 18 integers and total number of constraints is 219 (with 1150 Jacobian elements) including 120 non-linear ones. The Hessian of the Lagrangian has 0 elements on the diagonal, 90 elements below the diagonal. These data are generated by LINDO.

The proposed model is solved with respect to different scenarios that are as follows (see Table1):

Scenario 0: fuzzy customer demands with triangular membership functions are considered (Figure2).

Scenario 1, 2, 3: fuzzy customer demands with trapezoidal membership functions are considered as 10%, 30% and 50% of $\tilde{\tilde{a}}$ (Figure3).

Table 1: fuzzy customer demands of the example with respect to scenarios.

Customer Zones	Product Types	Indices	Scenario 0	Scenario 1	Scenario 2	Scenario 3
1st customer zone	1st product type	a_{11}	20	[18, 22]	[14, 26]	[10, 30]
	2nd product type	a_{21}	17	[15.3, 18.7]	[11.9, 22.1]	[8.5, 25.5]
	3rd product type	a_{31}	47	[42.3, 51.7]	[32.9, 61.1]	[23.5, 70.5]
	4th product type	a_{41}	26	[23.4, 28.6]	[18.2, 33.8]	[13, 39]
	5th product type	a_{51}	31	[27.9, 34.1]	[21.7, 40.3]	[15.5, 46.5]

2nd customer zone	1st product type	a_{12}	31	[27.9, 34.1]	[21.7, 40.3]	[15.5, 46.5]
	2nd product type	a_{22}	41	[36.9, 45.1]	[28.7, 53.3]	[20.5, 61.5]
	3rd product type	a_{32}	29	[26.1, 31.9]	[20.3, 37.7]	[14.5, 43.5]
	4th product type	a_{42}	10	[9, 11]	[7, 13]	[5, 15]
	5th product type	a_{52}	36	[32.4, 39.6]	[25.2, 46.8]	[18, 54]
3rd customer zone	1st product type	a_{13}	12	[10.8, 13.2]	[8.4, 15.6]	[6, 18]
	2nd product type	a_{23}	38	[34.2, 41.8]	[26.6, 49.4]	[19, 57]
	3rd product type	a_{33}	41	[36.9, 45.1]	[28.7, 53.3]	[20.5, 61.5]
	4th product type	a_{43}	26	[23.4, 28.6]	[18.2, 33.8]	[13, 39]
	5th product type	a_{53}	50	[45, 55]	[35, 65]	[25, 75]
4th customer zone	1st product type	a_{14}	50	[45, 55]	[35, 65]	[25, 75]
	2nd product type	a_{24}	29	[26.1, 31.9]	[20.3, 37.7]	[14.5, 43.5]
	3rd product type	a_{34}	16	[14.4, 17.6]	[11.2, 20.8]	[8, 24]
	4th product type	a_{44}	31	[27.9, 34.1]	[21.7, 40.3]	[15.5, 46.5]
	5th product type	a_{54}	41	[36.9, 45.1]	[28.7, 53.3]	[20.5, 61.5]

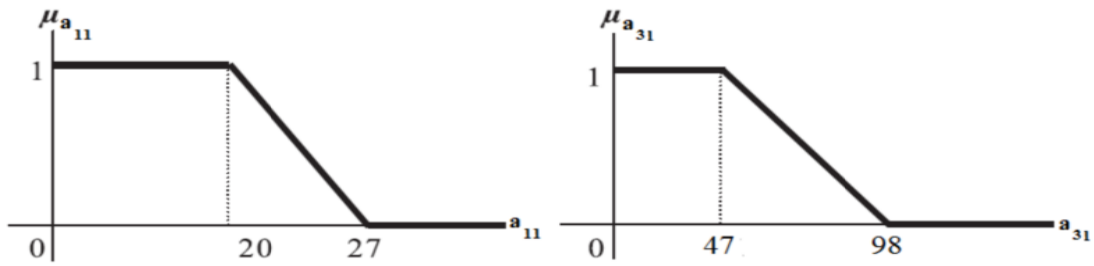

Figure2: Triangular membership functions for customer demands in scenario 0.

$$\mu_{a_{11}} = \left\{\begin{array}{lll} 1 & \text{if} & a_{11} < 20 \\ 1-\frac{a_{11}-20}{7} & \text{if} & 20 \leq a_{11} \leq 27 \\ 0 & \text{if} & a_{11} > 27 \end{array}\right\}$$

$$\mu_{a_{31}} = \left\{\begin{array}{lll} 1 & \text{if} & a_{31} < 47 \\ 1-\frac{a_{31}-47}{51} & \text{if} & 47 \leq a_{31} \leq 98 \\ 0 & \text{if} & a_{31} > 98 \end{array}\right\}$$

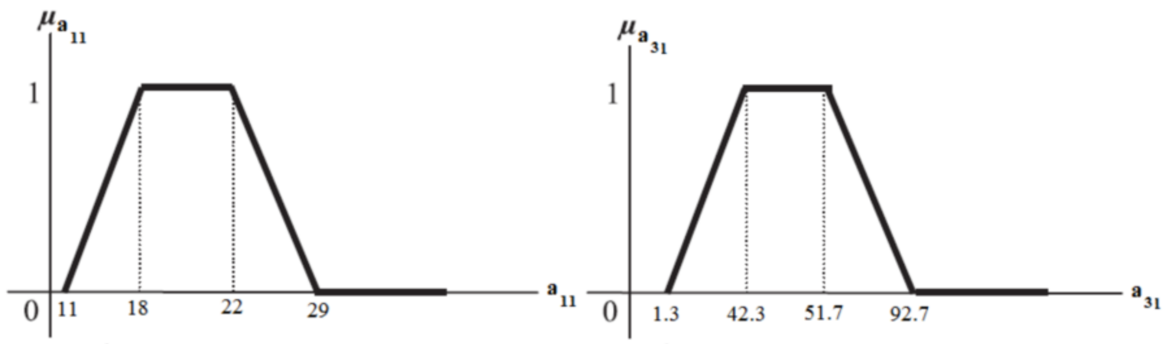

Figure3: Trapezoidal membership functions for customer demands in scenario 1

$$\mu_{a_{11}} = \begin{cases} 1 - \frac{18-a_{11}}{7} & \text{if} \quad 11 \le a_{11} < 18 \\ 1 & \text{if} \quad 18 \le a_{11} < 22 \\ 1 - \frac{a_{11}-22}{7} & \text{if} \quad 22 \le a_{11} < 29 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{a_{31}} = \begin{cases} 1 - \frac{42.3-a_{31}}{41} & \text{if} \quad 1.3 \le a_{31} < 42.3 \\ 1 & \text{if} \quad 42.3 \le a_{31} < 51.7 \\ 1 - \frac{a_{31}-51.7}{41} & \text{if} \quad 51.7 \le a_{31} < 92.7 \\ 0 & \text{otherwise} \end{cases}$$

Then, the new model is solved by using the LINDO software on a Core-i5 with 2.40 GHz. The run terminated giving a local optimum solution after 5 sec CPU time and 123 iterations. Some computational results of the output of the model are shown in Figures 2, 3 and Tables 1, 2 and 3.

As it mentioned before, T_{lk} is the fraction of capacity of plant that is located at site k and is assigned to produce product type l. In figure 4, that percentage of capacity of plant at zone 1 which is assigned to produce product type l, is 21.3% (1st column in Figure 4), type 2, is 31.7% (2nd column in Figure 4) and type 3, is 40.4% (3rd column in Figure 4) and so on, respectively. This means if our company decides to produce 100 units of all product types at plant 1, 21 products would be type 1, 32 products would be type 2, 40 products must be type 3 and 7 products must be type 4. Although the plant 1 must use all its capacity with level 1, lack of demand may cause the other plants to not to use all of their capacities. Also by assessing V_{hk} that is the capacity level h of plant at zone k, the company must open a plant at zone 1 with capacity level 1.

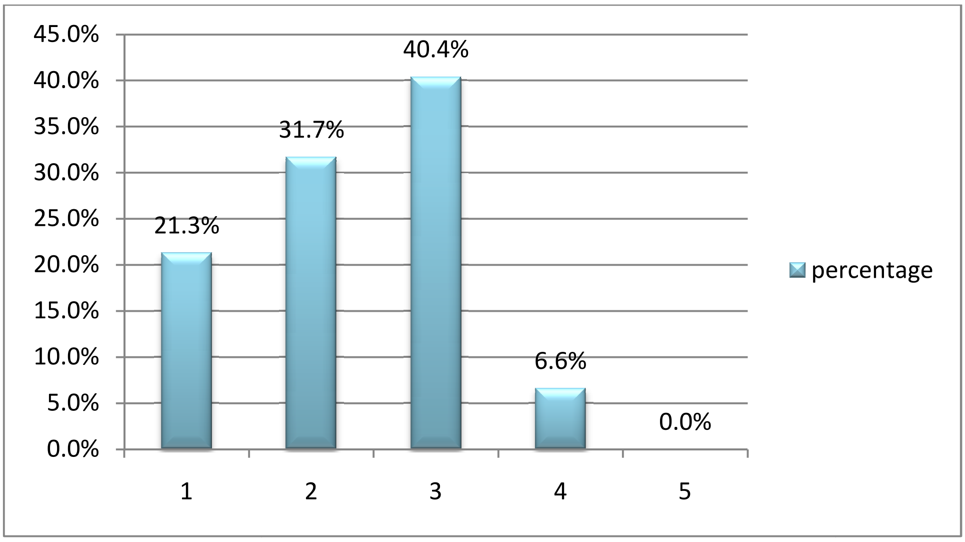

Figure 4 Capacity distribution of Plant 1

Similarly, S_{lj}^{t} is the fraction of capacity of warehouse that is located at site j and is assigned to hold product type l. In figure 5, that percentage of capacity of warehouse at site 2 which is assigned to hold product type 1, is 18.2% (1st column in Figure 5), type 3, is 12% (3rd column in Figure 5) and type 4, is 32.5% (4th column in Figure 5) and so on, respectively. This means if our company decides to hold 100 units of all product types at warehouse 2, 18 products would be type 1, 12 products would be type 3, 33 products would be type 4 and 37 products would be type 5. Although the warehouse 2 must use all its capacity with level 2 in the third period, lack of received production from plants to the other warehouses may cause the other warehouses to not to use all of their capacities in order to be stored and distributed. Also by assessing U_{rj} that is the capacity level r of warehouse at zone j, the company must open a warehouse at zone 2 with capacity level 2.

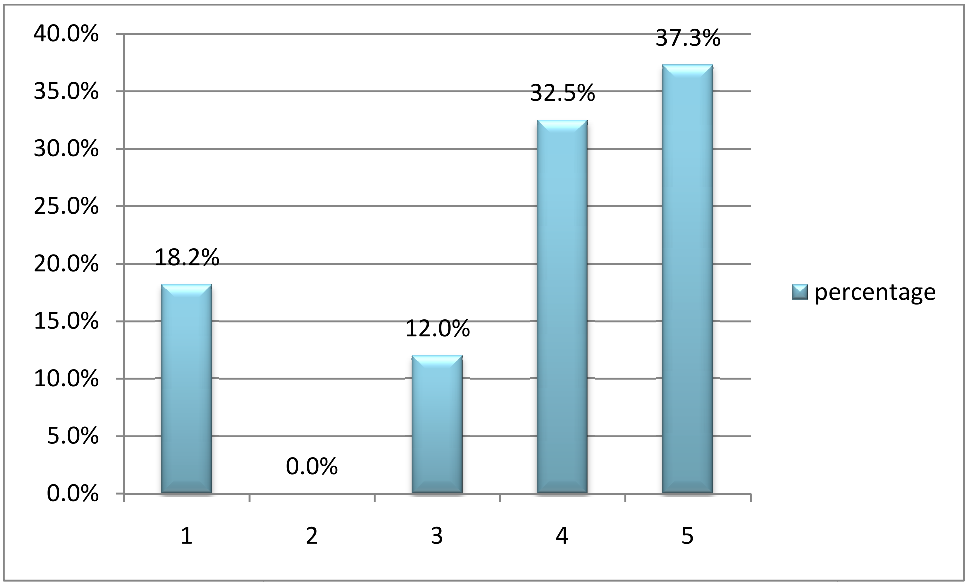

Figure 5 Capacity distribution of Warehouse 2

As can be seen from Figure 6, while the minimum total cost is obtain during scenario 3 with trapezoidal fuzzy functions, we can conclude that trapezoidal membership functions provide better results than triangular membership functions. Also, it seems that by relaxing the width of a trapezoidal membership function, the objective function decreases.

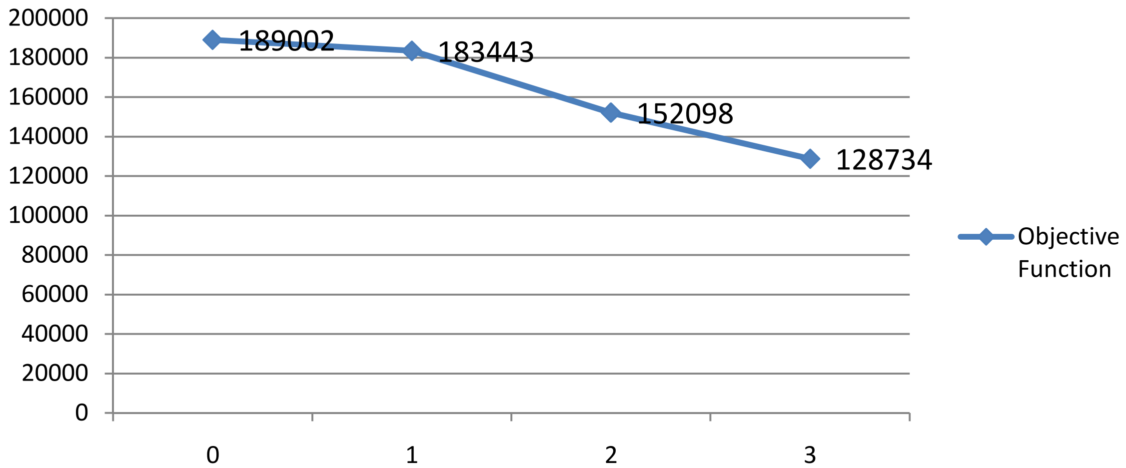

Figure 6 Objective function values with respect to triangular/ trapezoidal fuzzy customer demands in all of the scenarios

So, by applying this model to case study, the total costs of 128,734 via scenario 3 achieved and simultaneously we considered uncertain customer demands in different customer zones. Also the best strategy for distributing the product from plants to warehouses and from the warehouses to customers is achieved.

4. Conclusion

This study presented an extension to a mathematical model for the problem of designing a distribution network in the multi-product supply chain system with respect to fuzzy customer demands with triangular and trapezoidal membership functions that involves simultaneously determining the best location of both plants and warehouses and selecting the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customer zones. We added capacity levels of warehouses and plants for opening and operating plants or DCs in a multi-product supply chain management. We developed a fuzzy mixed-integer non-linear programming model for the above problem and the application of the developed framework was carried out to the case study of a large dairy company in Iran. At last, 9 percent reduction in total costs confirmed the superiority and efficiency of the model.

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