A developed multi-objective supplier selection model in discount environment

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Abstract

In SCM, supplier selection has a crucial importance in supply chain. Recent years, highly attention has been concentrated on this subject. In this paper, a multi-objective model is developed based on decision on quality service level and cost of purchased goods. The difference of this research is considering discount into model. In this research, firstly, single-objective and secondly multi-objective approach is employed to solve the model. An incremental discount is added to model to find out the optimum supplier selection. According to *Lp*metric solution, results can predict optimum selection of suppliers as well as the purchased goods amount.

Key words: supplier selection, multi-objective, multiple-item, quantity discount

1. Introduction

The selection of suppliers and the evaluation of their efficiency are becoming critical challenges that face manufacturing managers [5]. Many experts believe that the supplier selection is the most significant operation of a purchasing department.

In recent years with increasing the use of quality management and just in time (JIT) concepts by a wide range of firms, and increment globally competitive business environment, firms give great attention for selecting appropriate suppliers and build long term and profitable relationships with them because it helps reduce the material purchasing costs, reducing purchase risks, improve quality of final products and services. It is a usual practice for suppliers to offer quantity discounts to encourage the buyer towards large order and to obtain operating advantages such as economies of scale or reducing the cost of transportation for the buyer. From a coordination view it has been indicated that both the buyer and the supplier can achieve better overall profits if discounting schemes are used to set transfer prices. Dolan [7] and others argue, however that the supplier is often better served by the increasing discount model, which employs a lower unit price only to those units purchased in excess of each successive break point.

2. Literature review

Although in the last several years, the process of supplier selection and evaluation as well as monitoring has been studied extensively, only a few methods address the problem from the perspective of supplier selection under multi-supplier quantity discount thus an efficient approach to solve this problem is essential.

Related studies in the literature of the subject have been divided into two main categories. First category of studies contains the papers which consider only one criterion, usually the cost as an objective function, and other criteria models as constraints of the procurement for supplier selection and evaluation problem while suppliers offer discounts on the quantity of materials being purchased.

Gaballa [8] was the first author who applied order allocation to suppliers in a real operation. He used integer programming for minimizing the total price when an all-unit discount for supplier order is defined.

Pirkul and Aras [17] developed a nonlinear programming model in all-unit discount environment when discount assigned to all orders. In theirs suggested model, the objective was minimizing the total purchasing cost and holding cost considering resource linear constraints in order to find the optimum order quantity is obtained.

Sadrian and Yoon [14,15] pertained a mixed integer programming model to optimize the sum of cost purchases in the presence of price discount constraints.

Chaudhry, et.al [4] proposed mixed integer linear programming models for the supplier selection problem. In this model price, delivery and price break are contained. The objective of the model was to minimize total price by considering both all-units and incremental discounts. The problem considered a single item and presumed that the total order quantity was known.

Rosenthal *et al.* [13] presented a mixed integer programming model for supplier selection with bundling, in which a buyer needs to buy various products from multiple suppliers with limited capacity and also with various quality and delivery proficiency which buffer bundled items at discount prices. They employed a single objective programming in their model. They were the first authors the discoursed about bundling discount when supplier selection is addressed.

Burke *et al.* [3] surveyed the impact of supplier pricing schemes and supplier capacity limitations on the optimal sourcing policy for a single buyer. In their study, authors consider the condition where the total quantity to be procured for a single period is known by the company and communicated to all suppliers. Each supplier quotes a price and capacity constraint as a maximum quantity that can supply to the buyer. In this context, the buyer makes a decision for quantity allocation between the suppliers and consequently a subset of suppliers is selected for order allocation.

Kothari *et al.* [12] presented procurement action with marginal decreasing piece wise-constant supply curves. All-unit discount is permitted by this auction. In their study authors offer fully polynomial-time approximation schemes for the cutter specification problem and the calculation of the corresponding payments of this action.

The above research used single objective programming in their model which cannot be employed for the real cases.

Considering the importance of criteria such-as quality and services supplied by suppliers and also direct and indirect impact of supplier's efficiency on organization's performance, persuade organizations to consider other criteria in supplier's evaluation and selection as well as cost. So, the next category of researches declares supplier selection under discount environment. This supplier selection and evaluation problem is a multi-objective decision making problem.

Arunkumar *et al.* [1] described a linear approach for solving a piecewise linear vendor selection problem of quantity discounts using lexicographic method. The application of lexicographic method enables the decision-makers to create the limit for defective components and late deliveries as constraints in the model. Demand can be exactly met considering the defective components available in the supply.

Xia and Wu [19] introduce a model that integrates approach improved analytical hierarchy process by rough sets theory and multi-objective mixed integer programming to support supplier selection decisions in business volume discount environments the multi-objective model is formulated in such a way as to concurrently assign the number of suppliers to employ and the order quantities allocated to the supplier so as to simultaneously minimize total purchase cost, maximize total weighted quantity of purchasing, minimize the number of defective products, and maximize on-time delivery, while satisfying supply capacity and demand constraint.

Kokangul and Susuz [10] an integration of analytical hierarchy process and non-linear integer and multi-objective programming under some constraints such as quantity discount, capacity, and budget is employed to determine the foremost suppliers and to place the optimal order quantities among them. The objective of the mathematical models created are maximizing the total value of purchase (TVP), minimizing the total cost of purchase (TCP) or maximizing TVP and minimizing TCP at the same time.

Sarfraz [16] presented an integrated model of analytical hierarchy process (AHP) and preemptive goal programming (PGP). The model attempts to presents a solution methodology. In the context, a situation where vendors offer discounts on total amount of sales values and not on the quantity or diversity of products in a conflicting multi-objective scenario wherein one needs to maximize the total purchase value and minimize the whole cost.

3. Model development

A general multi-objective for the supplier selection problem can be defined as follows [18]:

$$\begin{aligned} &\min \; Z_1 \text{و} Z_2, \dots, \mathrm{Z_k} \\ &\max \; Z_{k+1} \text{و} \mathrm{Z_{k+2}}, \dots, \mathrm{Z_p} \\ &\quad \mathit{subject\ to}: \\ &x \in X_d, \mathrm{X_d} = \{x | g(x) \le b_r, r = 1, \dots, m\} \end{aligned}$$

Where $Z_1, Z_2, \dots, Z_k$ are the negative objectives or criteria-like cost, late delivery, etc and min $Z_{k+1} \text{و} \mathrm{Z_{k+2}}, \dots, \mathrm{Z_p}$ are the positive objectives or indicators such as quality, on-time delivery, after sale service and so on. X_d is the set of feasible solutions which satisfy the constraints such as purchaser demand, supplier capacity, etc.

In this section, a mathematical model of the supplier selection decision under the condition that each supplier offers incremental quantity discount for any item is formulated. Following set of assumptions, index set, decision variable and model parameters are defined in order to describe the model.

Assumptions:
1. Incremental discount are considered.
2. No shortages of the items are permitted for any of suppliers.
3. Multi-item can be purchase required quantity from multiple suppliers.
4. The buyer can purchase required quantity from multiple suppliers.

Index:

i	Index for supplier, i=1, 2,..., n
s	Index for items, s=1, 2,..., r
j	Index for price level, j=1, 2,....,m (i,s)

Decision variable:

x_{isj}	Order quantity of item s from supplier i in price level j

Model parameter:

D_s	Demanded quantity of item s
C_{is}	Upper limit of the quantity of item s obtained by supplier i

n	Number of suppliers competing for selection
$m(i,s)$	Number of quantity ranges in supplier i's price level for item s
p_{isj}	Unit price of item s in price level j obtained by supplier i
b_{isj}	The jth price level from supplier i for item s
$y_{isj} = 1$	if the ith supplier for item s is selected at price level j and otherwise $y_{isj} = 0$
$k_{is}(\%)$	percentage of service quality level of item s from supplier i
$f_{is}(\%)$	Percentage of rejected quantity of item s from supplier i
F_s	Upper limit of rejected quantity for item s
B_s	budget constraint allocated to item s

3.1 Objective functions:

The objective function of the model is defined by three sub objective function as follows:

3.1.1 Total purchased cost minimization:

The buyer expects to minimize the total purchase cost, in which each supplier offers "incremental" quantity discount for any item, so the objective function can be stated as:

$$\text{Minimize } Z_1 = \sum_{i=1}^{n}\sum_{s=1}^{r}\sum_{j=1}^{m(i,s)}\left(p_{isj}\left(x_{isj} - b_{isj-1}\right) + \sum_{k=1}^{j-1} p_{isk}\left(b_{isk} - b_{isk-1}\right)\right) y_{isj} \tag{1}$$

3.1.2 Service quality maximization:

Supplier's service quality rating is very critical indicator for supplier selection problem. This rating value include after sale service, item delivery on time, etc. The objective function maximizes the total service quality and can be shown as follows:

$$\text{Max } Z_2 = \sum_{i=1}^{n}\sum_{s=1}^{r} k_{is} \sum_{j=1}^{m(i,s)} x_{isj} \tag{2}$$

3.2 Constraints:

There are some constraints which associated with the supplier selection problem. In the following these constraints are illustrated and modeled:

3.2.1 Demand constraint:

The first constraint which we have faced is the demand constraint that implies the total purchased quantity of any item and should be equal to the total demand item of the buyer. This is modeled as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{m(i,s)} x_{isj} = D_s \qquad s = 1, 2, \dots, r \tag{3}$$

3.2.2 Capacity constraints:

This constraint reveals that the total purchased quantity from each supplier for each item must be equal or less than the supply capacity of considered supplier for any item. So we have the following relation:

$$\sum_{j=1}^{m(i,s)} x_{isj} \leq C_{is} \quad i = 1, 2, \dots, n, s = 1, 2, \dots, r$$

(4)

3.2.3 Discount intervals constraints:

Constraints set (5) is an integrality constraint to show the binary nature of the supplier selection decision, constraint set (6) is a quantity range constraint to meet the number of quantity range for any item in a supplier's price level and constraint set (7) represents the price level per supplier for any item among which can be chosen only one or none.

(5)

$$\begin{aligned} y_{isj} &= 0 \quad \text{if } x_{isj} = 0, \\ &= 1 \quad \text{if } x_{isj} > 0, \quad i = 1, 2, \dots, n, \end{aligned}$$

(6)

$$b_{isj-1} y_{isj} < x_{isj} \leq b_{isj} y_{isj} \quad i = 1, 2, \dots, n,$$

(7)

3.2.4 Rejected quantity constraint:

Since F_s is acceptable defective rate for item s and f_{is} is percentage of rejected quantity constraint for item s can be described follows:

$$\sum_{i=1}^{n} \sum_{j=1}^{m(i,s)} f_{is} x_{isj} \leq F_s \qquad s = 1, 2, \dots, r$$

(8)

3.2.5 Budget constraint:

Since B_s is budget constraint allocated to item s and p_{isj} unit price of item s in price level j obtained by supplier i, the budget constraint for item s be illustrated as follows:

$$\sum_{i=1}^{n} \sum_{j=1}^{m(i,s)} (p_{isj} \left(x_{isj} - b_{isj-1} \right) + \sum_{k=1}^{j-1} p_{isk} (b_{isk} - b_{isk-1})) y_{isj} \leq B_s \qquad s = 1, 2, \dots, r$$

(9)

3.3The final mathematical programming model:

The final presented mathematical model for supplier selection under incremental discount policy can be developed as follows:

$$\text{Minimize } Z_1 = \sum_{i=1}^{n}\sum_{s=1}^{r}\sum_{j=1}^{m(i,s)}\left(p_{isj}\left(x_{isj}-b_{isj-1}\right)+\sum_{k=1}^{j-1} p_{isk}\left(b_{isk}-b_{isk-1}\right)\right)y_{isj}$$

$$\text{Max } Z_2 = \sum_{i=1}^{n}\sum_{s=1}^{r} k_{is}\sum_{j=1}^{m(i,s)} x_{isj}$$

s.t:

$$\sum_{i=1}^{n}\sum_{j=1}^{m(i,s)} x_{isj} = D_s, \qquad s = 1,2,\dots,r$$

$$\sum_{j=1}^{m(i,s)} x_{isj} \le C_{is}, \qquad i = 1,2,\dots,n, s = 1,2,\dots,r$$

$$\begin{aligned} y_{isj} &= 0 \quad \text{if } x_{isj} = 0, \\ &= 1 \quad \text{if } x_{isj} > 0, \qquad i = 1,2,\dots,n, \\ &\qquad\qquad j = 1,2,\dots,m(i,s), s = 1,2,\dots,r \end{aligned}$$

$$\begin{aligned} b_{isj-1}y_{isj} < x_{isj} \le b_{isj}y_{isj}, \quad i = 1,2,\dots,n, \\ j = 1,2,\dots,m(i,s), s = 1,2,\dots,r \end{aligned}$$

$$\sum_{j=1}^{m(i,s)} y_{isj} \le 1, i = 1,2,\dots,n, s = 1,2,\dots,r$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m(i,s)} f_{is}\, x_{isj} \le F_s, \qquad s = 1,2,\dots,r$$

$$\begin{aligned} &\sum_{i=1}^{n}\sum_{j=1}^{m(i,s)} \left(p_{isj}\left(x_{isj}-b_{isj-1}\right)+\sum_{k=1}^{j-1} p_{isk}\left(b_{isk}\right.\right. \\ &\left.\left.-b_{isk-1}\right)\right)y_{isj} \le B_s \quad s = 1,2,\dots,r \end{aligned}$$

$$\begin{aligned} x_{isj} \ge 0, \quad i = 1,2,\dots,j = 1,2,\dots,m(i,s), \\ s = 1,2,\dots,r \end{aligned}$$

4. Methodology solution

The method that in this research has been employed is L_p metrics.

Weighted metric method ($L_p$method): The $L_p$method belongs to the first category of MODM problems, i.e., the case where a DM gives all required information before solving the problem. It is argued in MODM references such as Hwang and Masud [9], Asgharpour [2] and Deb [6] , and it compounds multi-objective functions into a single one. This method is considered for two main reasons. The first one is that this method requires less information from a DM, and the second one is its ease of implication in practical environment.

For this research problem that is a multi-objective programming model with a maximization objective and a minimization objective, we assume that Z_1^*, Z_2^*, Z_3^* are optimum solution of objective function when appear individually into model. Then, L_p metric problem with considering all model constraint is modeled:

$$Min\left[w_1\left((Z_1-Z_1^*)/Z_1^*\right)^p+w_2\left((Z_2^*-Z_2)/Z_2^*\right)^p\right]^{1/p}$$

w_k is the weight of k objective function that it will be determined by decision maker between 0 and 1. Somehow, the summation of objective weights is equal to 1. Here, p indicates the importance of each objective function deviation from its ideal worth. When p = 1 is employed, the problem has been changed to a weighted sum of deviations. When p =2 employed, a weighted Euclidean distance of any point in the objective space from the ideal point is minimized. When p = ∞ is used, the largest deviation should be minimized, i.e.,

$$\text{Min }(\text{Max}[\text{w}_1\{(\text{Z}_1-\text{Z}_1^*)/\text{Z}_1^*\}+w_2\{(Z_2^*-Z_2)/Z_2^*\}])$$

which is equivalent to:

$$\begin{aligned}&Min\ \alpha\\&\quad s.t:\\&\alpha\geq w_1\left((Z_1-Z_1^*)/Z_1^*\right)\\&\alpha\geq w_2\left((Z_2^*-Z_2)/Z_2^*\right)\\&x\in X_d,\ \text{X}_\text{d}=\{x|g(x)\leq b_r, r=1,\dots,m\}\end{aligned}$$

5. Numerical example

In this section, authors use numerical example to test the presented model. The buyer wishes to purchase three items from the best suppliers and allocate optimum order quantities to them. Assume that three suppliers should be managed for any item. The price of any items offers in the three price level (p_{isj}in $) for any supplier are provided in table1. The supplier's capacity for any item (C_{is}), the percentage of quality level of item s from supplier i (r_{is}(%)),the percentage of quality level of item s from supplier i (f_{is}(%)), demand quantity of item s (D_s), Budget constraint allocated to item s (B_s) and Upper limit of rejected quantity for item s (F_s) presented in table 2.

Table1
Supplier quantity discount – numerical example.

$item(s)$	$supplier(i)$	b_{is0}	p_{is1}	b_{is1}	p_{is2}	b_{is2}	p_{is3}

1	1	0	18	100	17.5	200	17
	2	0	20	110	19.5	210	19
	3	0	16	150	15.5	250	15
2	1	0	10	140	9.5	240	9
	2	0	8	170	7.5	270	7
	3	0	12	130	11.5	230	11
3	1	0	22	120	21.5	220	21
	2	0	26	190	25.5	290	25
	3	0	24	180	23.5	280	23

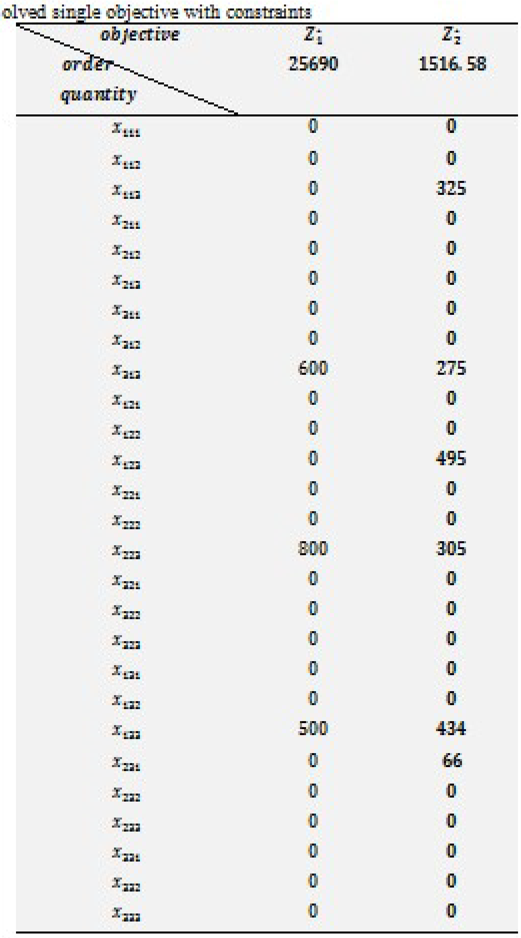
Table2
Data of supplier selection parameters.

$item(s)$	$supplier(i)$	C_{is}	$k_{is}(\%)$	$f_{is}(\%)$	D_s	B_s	F_s
1	1	900	90	5	600	10000	50
	2	750	88	3			
	3	800	85	5			
2	1	1000	96	3	800	7000	70
	2	900	83	6			
	3	1100	93	2			
3	1	1000	92	4	500	11000	30
	2	800	95	3			
	3	1100	85	5			

To solve a multi-objective supplier selection, a hypothetical numerical example is defined. At the beginning, according to L_p metric when p is unlimited and equal weights for two objectives function. In this context, we convert it to a single objective model and then the single objective model is solved with LINDO/LINGO software version 11.

To use L_p metric method, at the first, the single objective function model should be solved and the optimum results of each objective function should be substitute in L_p metric model. Then single objective model should be solved. According to each single objective model with its constraint, the result is obtained according to table 3:

Table 3
Solved single objective with constraints



objective / order quantity	Z_1^* 25690	Z_2^* 1516. 58
x_{111}	0	0
x_{112}	0	0
x_{113}	0	325
x_{211}	0	0
x_{212}	0	0
x_{213}	0	0
x_{311}	0	0
x_{312}	0	0
x_{313}	600	275
x_{121}	0	0
x_{122}	0	0
x_{123}	0	495
x_{221}	0	0
x_{222}	0	0
x_{223}	800	305
x_{321}	0	0
x_{322}	0	0
x_{323}	0	0
x_{131}	0	0
x_{132}	0	0
x_{133}	500	434
x_{231}	0	66
x_{232}	0	0
x_{233}	0	0
x_{331}	0	0
x_{332}	0	0
x_{333}	0	0

Finally, after solving the single objective model, the result is obtained according to table 4:

Table 4
Solved with L_p metric

Z_1^*	Z_2^*	X_{111}	X_{313}	X_{223}	X_{121}	X_{133}
25696	163418	1	599	779	1	500

5. Conclusions

Today's, may company are facing with decisions about supplier selection. Supplier selection in multi criteria and discount environment is the most critical operation of a purchasing or supply process. When solving multi-objective model for Z_1 and Z_2objective function, it was demonstrated that first objective function desirability was reduced. However, the utilization of Z_2 has been increased. For further potential research, authors recommend research to be conducted on fuzzy multi-objective supplier selection in incremental discount environment for different kind of discount.

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