A hub median network considering the time effect by a genetic algorithm

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Abstract

Hub networks are transportation and telecommunication systems so that several origin/destination points send and receive products. Hub facilities are used to consolidate flows from several origin and route to their destinations, this leads to a better utilization of transporters. This paper considers a single allocation hub median problem, in which the number of hubs is not fixed and the fixed cost of opening hub nodes are corporated. We formulate this problem regarding the time effect in designing transportation networks. A genetic algorithm is proposed to solve such a hard problem. The associated computational results are presented on the instances of the CAB data set. Finally, the conclusion is given.

Keywords: Hub median location, Transportation, Service level, Genetic algorithm.

1. Introduction

A transport hub (or interchange) is a place where passengers and goods are exchanged between vehicles or between transport modes. Public transport hubs include train stations, rapid transit stations, bus stops, tram stop, airports and ferry slips. Hub locations are location allocation problems that are concern with selecting *p* nodes from a set of *N* nodes for hub facilities and allocating demand nodes (each of the non-hub nodes) to multiple hubs (i.e., multiple allocation) or a single hub (i.e., single allocation) based on the particular nature of application in order to route the traffic between origin-destination pairs. Instead of serving each origin–destination pair directly, hub facilities concentrate flows in order to take advantage of economies of scale. The flow from origin *i* to destination *j* includes the following cost components:

- Collection cost: The transportation cost from the origin sites to the first hub.
- Distribution cost: The transportation cost from the first hub to the destination sites that is comprised of two parts, namely first the inter-hubs link and then the link from the last hub to the final destination. Parameters λ, α , and δ are unit rates for collection (origin–hub), transfer (hub–hub) and distribution (hub–destination).

Hub location problems have some applications in transportation, telecommunication and computer networks. Solution methods have been developed for several variants of this problem, such as *p*-hub location, *p*-hub centerand hub covering. There exist several variants arising from various assumptions for each of these classes of problems; but, they often assume these things:

1. There is a direct link between each hub pair.
2. There is economy of scale generated through inter-hub links due to consolidation of flows.
3. There is no direct link between two non-hub nodes.
4. The triangle inequality holds in the cost structure and costs are proportional to the distance.

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O'Kelly (1987) presented the first recognized mathematical formulation for a hub location problem by studying airline passenger networks. His formulation is referred to the single allocation *p*-hub median problem. The *p*-hub median problem minimize the total transportation cost needed to route flow between origin–destination pairs with given *n* demand nodes, flow between origin–destination pairs and the number of hubs to locate (*p*). O'Kelly (1992) first introduced incorporation of fixed costs as hub node setup costs. Alumur [1] classified and surveyed network hub location models

Many of researchers assumed that the sub graph induced by the hub nodes is complete. Alumur [2] relaxed this fundamental assumption and provided a uniform modeling treatment to the single allocation hub location problems. They proposed models for the single allocation incomplete *p*-hub median, the incomplete hub location with fixed costs, the incomplete hub covering, and the incomplete p-hub center network design problems.

Puerto [3] provided an integrated framework for analysis of hub location problems that use ordinal information to meet the real requirements of logistics. They introduced a new type of hub location problems arising from the different role of users in the supply chain that lead to different distribution patterns. They assumed that any origin–destination delivery path is composed of two components: (1) the sub path that connect an origin site to the first hub, and (2) the sub path that links first hubs to final destinations. Also, this last component is consisting of two parts: (2.1) the inter-hubs link and (2.2) the link from the last hub to the destination. This makes it possible to distinguish the costs generated by different users according to their role in the supply chain. They assumed that each origin must support the cost to reach the first hub, while the intermediate distribution system supports the cost of delivering entities to their destinations. They determined the location of the hubs and the origin–destination delivery paths (i.e., distribution patterns) simultaneously.

Topcuoglu [4] presented a robust solution based on a genetic search framework to solve the uncapacitated single allocation hub location problem (USAHLP). In this problem, the number of hubs is not determined and fixed cost of opening hubs is included in the formulation. They used a quadratic 0-1 optimization formulation with linear constraints that was presented by O'Kelly. They exploited the features of genetic algorithms in finding the number of hubs, the location of hubs, and the assignment of spokes to the hubs.

Travel distances and times have been used to measure the level of service in transportation network design and location science since the introduction of covering models and the distance constrained *p*-median problem [5]. Campbell [6] present time definite hub location models for multiple allocation *p*-hub median problems (HMP) and multiple allocation hub arc location problems. This model determines the location of hubs with the objective of minimizing transportation costs under distance-dependent service level constraints so that ensure origin–destination travel times are competitive. Gelareh [7] proposed a 4-index formulation for the uncapacitated multiple allocation hub location problems appropriated for urban transport and liner shipping network design. They proposed a new HLP model with suitable flexibilities for transportation applications. A Benders decomposition approach is proposed. Also they solved the model with a greedy neighborhood search heuristic equipped with intensification and diversification strategies.

The aim of this research is designing hub networks with considering time effect. For this purpose, we determine penalties for exceeding the travel distance from a direct distance between each origin-destination pair. The problem is motivated from real-life observations of many hub networks. Due to take advantage of scale economies on inter-hub connections design a network so that each node will be allocated to only one hub and direct link between two nodes is not allowed. In designing transportation network cost of shipping products is an important Criterion. Also time required to send goods from origin to destination has a significant effect, on this basis time must be entered into our calculations.In the *p*-hub median problem, the fixed costs of opening facilities are ignored. In this model in addition to the total transportation costs, the objective function also includes the total cost of building hubs.

2. Problem definition

We aim to design a network so that any node can send flow to any other node in the network. Let *N* be the set of *n* demand nodes and *H* be the set of potential hub such that *H* □ *N* with *h* nodes. Our aim is to decide on the location of hub nodes and allocation of non-hub nodes to these hub nodes, so that minimize total transportation cost while minimize total time of traveling between each origin destination. We proposed an integer formulation for the single allocation *p*-hub median problem with fixed costs of opening hub nodes with two objective functions. First objective function minimizes the total transportation cost, to serve the given set of flows and second objective function minimizes total transportation time. In order to present the mathematical formulation for this problem, we first need to define some parameters:

w_{ij} flow from node *i* to node *j*.
C_{ij} cost per unit flow from node *i* to node *j* (the cost C_{ij} is proportional to the distance between *i* and *j*, and satisfying the triangle inequality).
α discount factor of hub link transportation cost.
λ discount factor for collection (non-hub to hub).
δ discount factor for distribution (hub to non-hub).
H_k fixed cost of opening a hub at node *k* □ *H*.
d_{ij} difference between actual time of shipping products between nodes *i* and *j* in the network with time of direct transportation between nodes *i* and *j*.

Then, we define the decision variables of the mathematical model:

$$X_{ik} = \begin{cases} 1 & \text{if node } i \in N \text{ is allocated to hub at node } k \in H; \\ 0 & \text{otherwise.} \end{cases}$$

The single allocation hub location with fixed costs and considering transportation time network design problem is modeled below:

$$\text{Min } \sum x_{kk} \; H_k + \sum\sum w_{ij} \; (\lambda \sum x_{ik} \; c_{ik} + \alpha \sum\sum x_{ik} \, x_{jm} c_{km} + \delta \sum x_{jm} \; c_{jm}) \quad (1)$$
$$\text{Min} \sum\sum d_{ij}/c_{ij} \quad (2)$$
$$\text{s.t. } \sum x_{ik} = 1 \; for \; all \; i \quad (3)$$
$$x_{ik} \leq x_{kk} \quad (4)$$
$$x_{ik} \in \{0;\; 1\} \text{ for all } i \in N; \; k \in H \quad (5)$$
$$\mathrm{z_{km}} \in \{0;\; 1\} \text{ for all } k, m \in H \; ; \; i \; < \; j \quad (6)$$

The objective function given in Eq. (1) sums the costs of collection, transfer, distribution, and the costs of the opening hub nodes and hub links. Eq. (2) sums diference between actual time of shiping products between nodes i and j with time of direct transportation between nodes *i* and *j*. The constraint given in Eq. (3) ensures that each node is assigned to exactly one hub. Constraint (4) ensures that no node is assigned to a location unless a hub is opened at that site. Constraints of (5), (6) represent binary requirements.

The hub median problem is NP-hard there for the exact solution potential for these problems is limited. Thus we use a genetic algorithm (GA) for finding the near-optimal solutions. By using some instances of CAB data set that contain the flow and distance table between each pair of the network, we could figure out optimom number of hubs, the location of hubs and the allocation of non-hub nodes; so that the total cost of transportation and the travel distance (or time) simultaneously will be minimized.

3. Computational results

We use 10 and 25 problem sizes of the CAB data set based on airline passenger. We put value of λ and δ equal to 1. Also it is assume that fixed cost of opening hub is equal for all nodes. We consider several experiments by varying the discount factor $\alpha \in \{0:2; 0:4; 0:6; 0:8; 1:0\}$, and fixed cost for opening a hub is varied as $f \in \{100; 150; 200; 250\}$ Tables 1 and 2 give the results of our method for various problem sizes.

Table 1. Results from GA algorithm when *n* = 10

α	*f*	Hubs	Time difference	Cost	Time
0.2	100	1,4, 6, 7, 8	112	833	6.7
	150	4, 6, 7	239	941	7.34
	200	6, 7	268	1024	9.97
	250	6,7	268	1124	9.39
0.4	100	4, 5, 6, 7, 8	112	944	6.40
	150	6, 7	268	984	6.42
	200	6, 7	268	1084	11.13
	250	6, 7	268	1184	11.34
0.6	100	4, 5, 6, 7	154	996	6.52
	150	6, 7	268	1044	8.32
	200	6, 7	268	1144	10.19
	250	6, 7	268	1244	10.46
0.8	100	4,5 ,6 ,7	154	1073	8.65
	150	6, 7	168	1104	10.34
	200	6, 7	168	1204	8.91
	250	6,7	168	1304	10.20
1	100	4, 6,7	239	1084	6.25
	150	6, 7	268	1164	11.02
	200	6, 7	268	1264	9.96
	250	5	393	1079	11.40

Table 2. Results from GA algorithm when n = 25

α	*f*	Hubs	Time difference	Cost	Time
0.2	100	1, 9, 12, 13, 18	301	1135	11.68
	150	9,12,18,21	357	1312	12.14
	200	5, 12, 18	381	1397	12.62
	250	5, 12, 18	381	1547	13.86
0.4	100	1, 6, 7, 12, 18, 21	256	1320	11.74
	150	12, 20, 21	371	1415	12.17
	200	8, 20, 21	344	1640	13.11
	250	20, 21	385	1739	15.12
0.6	100	6, 12, 14, 21, 25	303	1435	11.45
	150	8, 20, 21	344	1571	13.07
	200	20, 21	385	1681	14.16
	250	20, 21	385	1781	15.41
0.8	100	8, 18, 20, 21	314	1556	13.20
	150	8, 20, 21	344	1653	14.33
	200	20, 21	385	1729	14.87
	250	20, 21	385	1829	16.03
1	100	20, 21	385	1575	13.96
	150	20, 21	385	1675	15.34
	200	20, 21	385	1775	16.14
	250	20, 21	385	1875	16.90

5. Conclusion

This paper has dealt with the uncapacitated single allocation hub location problem (USAHLP). Opening hubs has a fixed cost that has been included in the model. Service level that imposed by travel distance (or time) is a key factor in evaluating a transportation network, the effect of this factor has been intended in our model by adding an objective function and a multi-objective integer formulation for this problem has been presented. A multi-objective genetic algorithm has been proposed to solve this model. Instances derived from the CAB data set have been carried out to test the performance of our GA-based framework. The results have shown that an increase in the service level has required modifying a given hub network by changing numbers and locations of the hubs. Relaxing this assumption, in which hubs are fully connected, may lead to more realistic problems. For further research, we can consider the cost of opening hub links.

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