An economical order quantity model for items with imperfect quality: A non-cooperative dynamic game theoretical model

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Abstract

The consideration of items with imperfect quality in an economic order quantity (EOQ) model has appealed to many researchers recently. Nonetheless, only a few papers there are that consider this assumption in a supply chain all of which are centralized ones. The centralized supply chains are not realistic in most of the real-life situations and in this paper a two-echelon decentralized supply chain consisting of a manufacturer who sells the items to a supplier with considering items with imperfect quality in a just in time (JIT) environment is proposed. Stackelberg equilibrium is applied to the model with the assuming the supplier as the leader and the manufacturer as the follower and results of the centralized and the centralized models are compared and the conclusion are drawn according to an illustration example.

Keywords: Economic order quantity (EOQ), Game theory, Stackelberg equilibrium, Just in time (JIT), Imperfect items.

1. Introduction

Vendor-buyer supply chains have taken on a special significance among the researchers in recent years and the problem of coordination of these supply chains have so far been studied by the researchers. Most of the papers published in this area deal with the supply chain as an integrated system which leads to the best coordination (see e.g.: [1], [2], [3], [4], [5]). Nonetheless, not all of the supply chains are centralized in practice. The number of the papers discussing decentralized supply chains pursues soaring, as these kinds of problems are similar to real cases. Bylka [6] applies Nash equilibrium to solve a production-distribution problem in a supply chain. The objective of the model is to minimize the overall costs by finding the number of shipments and the size of production batch per cycle. Gurnani, et al. [7] discuss a supply chain consisting of a supplier and a buyer and study three different scenarios of decision-making structures. Comparing the results of three cases from both the supplier's and the buyer's perspective, they prove that the cost structure and level of uncertainty in demand plays a vital role in players' inventive and is actually major determinant of their choice. Chen, et al. [8] discuss a dual-channel supply chain with a manufacturer as a Stackelberg leader and the manufacturer as the follower. Examining coordination schemes, they prove that manufacturer's contract with a wholesale price and the price for the direct channel would coordinate the discussed supply chain, albeit with a benefiting retailer alone.

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Implementing a complementary agreement, such as two-part tariff or a profit-sharing agreement, they illustrate how the discussed contract may result in a win-win situation.

A great amount of efforts has been devoted to elaborate and improve basic EOQ model to relax the restricting assumptions of it in order to utilize it in real inventory problems effectively as soon as it was first introduced by Harris [9]. Many papers relaxing the assumption of imperfect items within an inventory problem have been published. Nonetheless, Salameh and Jaber [10] are the first to study the imperfect items in an EOQ model very thoroughly. They consider an inventory problem in which the demand is deterministic and there is a fraction of imperfect items in the lot and are screened by the buyer and sold by them at the end of the circle at discount price. Nevertheless, there was a problem on calculating the inventory cycle time which was corrected by Cárdenas-Barrón [11].Huang [12] relaxes the assumption of perfect quality items in supply chain literature. He considers a two-echelon just-in-time (JIT) manufacturing supply chain by accounting for imperfect items. Huang [13] extends the model introduced by Ha and Kim [1] and considers the handling of the imperfect items follows the same approach introduced by Salameh and Jaber [10]. Wahab, et al. [14] consider a vendor-buyer supply chain and study the impact of defected items in three different scenarios consisting of a domestic supply chain, an international supply chain and an international supply chain with considering the environmental impact. A very comprehensive review on extensions of the Salameh and Jaber [10] EOQ model considering imperfect items has been done by Khan, et al. [15]. Introducing the basic model and its assumptions, they study the EOQ models considering imperfect items and the contributions that have been added to the basic model. To the authors' best knowledge, all of the papers published in the literature of imperfect items in supply chains study the supply chain as an integrated system, which is not very adaptive to real life situations. In this paper, we discuss the model which initially introduced by Wahab, et al. [14] with a innovative solution concept by considering the model to be a decentralized one, which is more applicable to real life situations and propose a dynamic game theoretical approach as a solution concept. Moreover, since the assumption of shifting the power to the buyer is not inconceivable (see e.g.: [16], [17]), we assume that the power is shifted to the supplier and he acts as the Stackelberg leader. A numerical example is also provided to compare the results of the decentralized model and the centralized one and subsequently the conclusions are drawn.

The rest of the paper is organized as follows: section 2 presents notation and formation of the model. In section 3 the solution concept of the problem is discussed. Section 4 presents computational results and finally section 5 concludes the paper.

2. Model formation

A two-echelon supply chain consisting of a single manufacturer who sells the products to a single supplier is considered. The manufacturer produces the items with rate of P to meet the demand of the supplier for the items *D*. Since no shortage is considered in the proposed model, the production rate must be greater than the demand (*P*>*D*). Every time the supplier places an order, the manufacturer transports the demanded items in *n* equal shipments ($Q_T = nQ$). Each shipment has *x* percentage of defected items therefore, the supplier has to scan the received batches for imperfect items. The screen rate of the supplier is *y* in the time of $t = Q/y$ and the cost of c *that* is imposed by the manufacturer. The manufacturer has to compensate for the cost of screening of the defected items. When the supplier screens the items, the defected items are stored in their warehouse and the holding cost of the defected items is paid by the manufacturer and at the end of the supplier's inventory cycle when the screening is completed the defect items will be dispatched to the manufacturer and again the manufacturer bears the transportation cost. The total expected cost function of the manufacturer and the one of the supplier will be discussed in this section. The following notation is used throughout this paper:

Q	lot size determined by the supplier
Q_T	total size of the order quantity
T	cycle of the supplier
h_s	holding cost for the supplier per item
A_s	ordering cost for the supplier per item
F_s	fixed transportation cost paid by the supplier
t	screening time
y	screening rate: $y \sim U(0, b)$

c	screening cost paid by the supplier
O_s	ordering cost of the supplier per unit time
Tr_s	transportation cost of the supplier per unit time
H_s	holding cost of the supplier per unit time
S_s	screening cost of the supplier per unit time
TC_s	total cost of the supplier per unit time
D	annual demand
x	percentage of the defected items
P	manufacturer's production rate
n	number of shipments determined by the manufacturer
h_m	holding cost for the manufacturer per item
A_m	preparation cost for the manufacturer per item
F_m	fixed transportation cost paid by the manufacturer
C_m	variable transportation cost paid by the manufacturer
H_{1_m}	holding cost of the producing items for the manufacturer per unit time
H_{2_m}	holding cost of the imperfect items for the manufacturer per unit time
S_m	setup cost of the manufacturer per unit time
Tr_m	transportation cost of the manufacturer per unit time
TC_m	total cost of the manufacturer per unit time

2.1. Manufacturer's cost function

The manufacturer's expected total cost per unit time is computed as the holding cost that is consisted of holding cost for producing items and holding cost for imperfect items in supplier's warehouse, plus preparation cost and transportation cost.

Holding cost:

As mentioned before, the holding cost of the manufacturer is consisted of two parts that will be discussed here. First the expected value of the holding cost for producing items per unit time will calculated as:

$$E(H_{1_m}) = \left\{\left(nQ\left(\frac{Q}{P} + (n-1)E(T)\right) - \frac{nQ\left(\frac{nQ}{P}\right)}{2}\right) - E(T)(Q + 2Q + \cdots + (n-1)Q)\right\} h_m/(nE(T)) \tag{1}$$

Holding cost for the imperfect items will be paid by the manufacturer as well. For *n* cycles of the supplier, the expected value of the holding cost of the imperfect items per unit time would be expressed as:

$$E(H_{2_m}) = \left(\frac{nQ^2E(x)}{2y}\right) h_s/E(T) \tag{2}$$

Setup and transportation cost:

The manufacturer bears the setup and preparation cost of A_m in every cycle of theirs and as in every batch that is sent to the supplier there is a percentage of defected items, the transportation cost of the manufacturer will be multiplied by *n*. The expected setup and transportation cost of the manufacturer per unit time is represented as:

$$E(S_m) = A_m/(nE(T)) \tag{3}$$
$$E(Tr_m) = n(F_m + C_mE(x)Q/(nE(T)) \tag{4}$$

Expected total cost of the manufacturer is sum of the above equations, substituting $E(T)$ by $(1-E(x))Q/D$, it is rewritten by:

$$E(TC_m) = DQ\left(\frac{h_m}{P(1-E(x))}\left(1-\frac{n}{2}\right)+\frac{h_m}{2D}(n-1)+\frac{h_sE(x)}{2y(1-E(x))}\right) + \frac{D}{Q(1-E(x))}\left(F_m+\frac{A_m}{n}\right)+\frac{C_mDE(x)}{(1-E(x))} \quad (5)$$

2.2. Supplier's cost function

In order to model the supplier cost function, the cycle of theirs has to be calculated. Since every shipment that the supplier receives has x percent of defected items and as only the perfect quality items are used by the supplier, their cycle $(1-x)Q/D$ will be a random variable because the x is a random variable. Therefore, the expected value of the cycle of the supplier will be $(1-E(x))Q/D$. The total cost of the supplier is considered to be consisted of the ordering and transportation cost, holding cost and screening cost.

Holding cost:

The holding cost of the imperfect items is paid by the manufacturer. Therefore, the holding cost of the supplier is only consisting of the perfect items. The expected value of the holding cost of the supplier per unit time would be presented as:

$$E(H_s) = \left(\frac{QE(1-(x))E(T))}{2}+\frac{Q^2E(x)}{2y}\right)h_s/E(T) \quad (6)$$

Ordering and transportation costs:

The supplier has the fixed ordering cost of A_s in every cycle of the manufacturer. Consequently A_s/n will be the ordering cost of the supplier in their own cycle. The transportation cost for the supplier in every cycle will be F_s. The expected value of the ordering and transportation cost of the supplier per unit time could be expressed as:

$$E(O_s) = (\frac{A_s}{n})/E(T) \quad (7)$$
$$E(Tr_s) = F_s/E(T) \quad (8)$$

Screening cost:

As all the items in a batch are screened, the screening cost of the supplier per unit time can be shown as:

$$E(S_s) = Qc/E(T) \quad (9)$$

The expected total cost of the supplier per unit time, which is the sum of the above equations, would be given by:

$$E(TC_s) = \frac{F_s+\frac{A_s}{n}}{E(T)}+\frac{\left(\frac{QE(1-(x))E(T))}{2}+\frac{Q^2E(x)}{2y}\right)h_s}{E(T)}+\frac{Qc}{E(T)} \quad (10)$$

Replacing the $E(T)$ by its value which is $(1-E(x))Q/D$ in above equation the expected value of the total cost of the supplier would be written as:

$$E(TC_s) = \frac{D}{(1-E(x))}\left(\frac{1}{Q}\left(\frac{A_s}{n}+F_s\right)+\frac{Qh_s}{2}\left(\frac{E(x)}{y}+\frac{E[(1-x)^2]}{D}\right)+c\right) \quad (11)$$

3. Solution concept of the problem

A non-cooperative dynamic game theoretical approach is proposed here as the solution concept. Therefore, the Stackelberg equilibrium with the supplier as the leader and the manufacturer as the follower is utilized to solve the problem and find the optimal number of the shipments and the quantity of each shipment in a way that minimizes both the manufacturer's and the supplier's cost functions. In order to find Stackelberg equilibrium, best response of the manufacturer and the supplier must be calculated by setting the first derivative of their expected cost function with respect to n and Q equal to zero, respectively. Applying Stackelberg equilibrium, which is proposed for dynamic non-cooperative games, the following results would be gained:

$$Q^S = \sqrt{\frac{2DF_s y}{h_s(DE(x) + y(E[1-x]^2))}} \quad (12)$$

$$n^S = \sqrt{\frac{PA_m h_s\big(DE(x) + y(E[1-x]^2)\big)}{yF_s h_m\Big(\big(1-E(x)\big)P - D\Big)}} \quad (13)$$

4. Computational results

4.1. Illustrative example

To illustrate the efficiency of the model, the numerical example that was introduced by Huang (2004) is implemented and the results are compared with those of the centralized model by Wahab et al. (2011). Let production rate P=*160 000* units/year, demand rate D=*50 000* units/year, holding cost for manufacturer $h_m = \$2$ unit/year, preparation cost for the manufacturer $A_m = \$300$/cycle, fixed transportation cost for manufacturer $F_m = \$19/\text{delivery}$, variable transportation cost for manufacturer $C_m = \$1/\text{unit}$, ordering cost for the supplier $A_s = \$100/\text{order}$, holding cost of the supplier $h_s = \$5$/unit/year, screening cost c= *$0.5/unit,* screening rate $y = 175\,200$ unit/year, transportation cost for the supplier $F_s = \$25/\text{delivery}$, the upper bound of the uniform function of the defected items b is varied between 0.001 and 0.5. On the same token, since there is no guarantee for n to be integer, the algorithm proposed by Wahab et al. (2011) is applied here as well.

Stackelberg equilibrium is utilized to draw the results of the non-cooperative dynamic game in the decentralized supply chain. Utilizing Equations (12) and (13), the optimal number of shipments and the lot size of each shipment are calculated according to Stackelberg equilibrium and results with comparison to the centralized model are given in Table 1. Comparing the results reveals the fact that the total cost of the supply chain with supplier-Stackelberg assumption is higher than the centralized model, which was expected beforehand since the centralized models that are idealistic models result in Pareto optimal solutions. Nonetheless, the total cost of the supplier is reduced as he acts as the leader and as the value of b gets higher, the difference between the total costs becomes larger.

4.2. Sensitivity analysis

For the proposed approach sensitivity analysis are done for various values of h_s and y as these two factors are crucial for both manufacturer and supplier. In doing so, the values of the other parameters are fixed and the value for the h_s and y are considered to vary. The value of b is considered to be 0.1. Figure 1 represents the variation of n and Q according to different values of h_s. While figure 2 represents the variation for the same parameters with respect to different values of y.

Form the figures it is elicited that as the value of the h_s rises, the size of the shipments reduces. Therefore, the number of shipments should ascend as well. Variation of y has no major impact on the number of the shipments or the size of the shipments when it is high. Consequently, reducing it in major amounts may result in reduction of the size of shipment and increases in number of the shipments.

Table 1. Comparison the results of the non-cooperative dynamic game with centralized model

b	Supplier Stackelberg					Centralized		
	n	Q	TC_S	TC_M	TC	n	Q	TC
0.001	7	707	29559	7538.1	37097	5	1056.1	36784
0.01	7	710	29675	7776.6	37451	5	1059.4	37141
0.02	7	713	29805	8044.2	37849	5	1063	37540
0.03	7	716	29937	8314.4	38251	5	1066.6	37944
0.04	7	719	30070	8587.3	38657	5	1070.3	38352
0.05	7	722	30204	8863	39067	5	1074	38765
0.06	7	726	30340	9141.6	39482	5	1077.7	39181
0.07	7	729	30478	9422.9	39901	5	1081.4	39602
0.08	7	732	30617	9707.2	40324	5	1085.2	40027
0.09	7	735	30757	9994.4	40752	5	1089	40457
0.1	7	738	30899	10285	41184	5	1092	40892
0.2	7	771	32412	13362	45774	5	1132	45504
0.3	7	804	34113	16796	50909	5	1173.6	50660
0.4	7	840	36039	20653	56692	6	1088.7	56457
0.5	7	874	38237	25016	63253	6	1132.4	63030

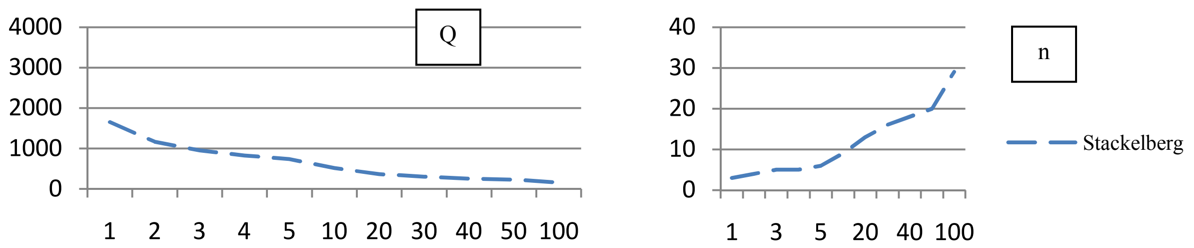

Figure 1. Effect of h_S on Q^* and n^*

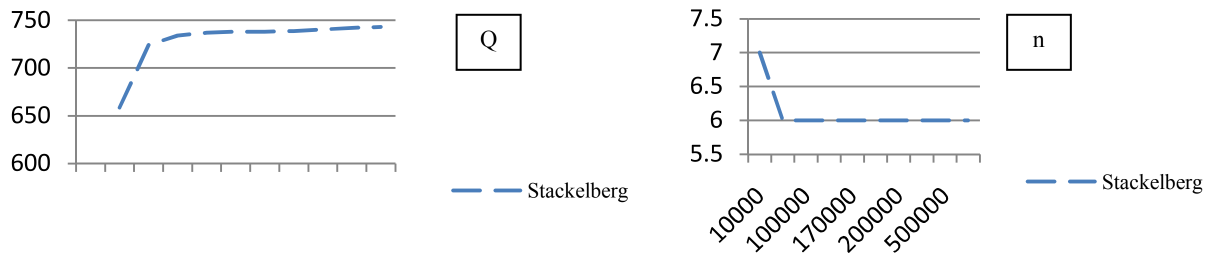

Figure 2. Effect of y on Q^* and n^*s

5. Conclusion

In this paper, a two-echelon decentralized supply chain consisting of a manufacturer and a supplier and considering the impact of the imperfect items is studied. In order to fulfill the supplier's demand the manufacturer sends the products in a number of shipments with equal shipment sizes. The objective of the model is to find the optimal size of shipments and optimal number of the shipments in a way that minimizes the both the manufacturer's and the supplier's expected total cost. A dynamic non-cooperative game theory model is considered in this paper. Stackelberg equilibrium, with the supplier acting as the leader is utilized to solve the problem and the optimal solution is obtained. In order to demonstrate the effectiveness of the model, a numerical example is introduced and the results are compared with the centralized model. Moreover, sensitivity analysis on factors that could either be controlled or reduced is done and the results are presented. It is illustrated that the results in the Stackelberg model since the supplier acts as the leader, their total cost is very less than other approaches and exactly the opposite happens to the manufacturer. Much scope there is to be extended in the future. For instance, the proposed model would be extended to a three-echelon supply chain with a downstream customer. Therefore, the items with imperfect quality would result in backorder or lost sale of the supplier. Moreover, relaxation of the assumption of the number of the shipments which is considered to be the same in this paper or the uncertainty of the demand which is not considered in this paper could extend the model and adapt it to real cases effectively. Similarly, in this paper it is assumed that the both manufacturer and the supply have complete information of each other's cost function which would be relaxed in the future works.

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