Multi Objective Location Routing Inventory Problem With Time Windows

Mahdi Bashiri, Ehsan Fallahzade
Department of industrial engineering, Shahed University, Bashiri@shahed.ac.ir
Department of industrial engineering, Shahed University, e.fallahzade@shahed.ac.ir

Abstract
The location routing inventory problem (LRIP) involves selecting central depots from a set of candidates and designing a set of routes for each depot to serve customers, while minimizing total distribution and inventory costs. In order to consider other decision factors beside cost and make the problem more practical, multi objective approaches seems to be useful. This study considers the time intervals that customers must be served known as hard time windows, the time intervals that the customers like to be served known as soft time windows and also the model considers avoiding underutilization of vehicles capacity and labor. We are going to investigate the use of goal programming approach to model these problems.

Keywords: Location routing inventory problem (LRIP); Time windows; Goal programming

1. Introduction
Nowadays companies need to consider both strategic and operational decisions together in order to design and manage their supply chain more efficiently. Transportation and distribution of goods are most important segments of a supply chain. Well-managed transportation systems will guarantee not only a good service to customers but also reduction in distribution and warehousing costs.

The classic location routing problem consists of choosing appropriate location among candidates to locate the distribution center and to determine the best set of routs to serve the customers while minimizing the total cost. In the literature there are several studies that consider location routing and inventory decisions simultaneously. Among recent studies [1] and [2] presented incorporated location routing inventory problems. The objective functions of these studies are the sum of location, allocation, routing and inventory costs. Beside cost, another important factor is time. Some problems have predetermined time constraints on the period that deliveries should take place. These kinds of problems are known as the problems with Time windows. Two types of time windows exist, soft and hard; in soft time windows each customer has a preferred time interval and it like to be served in this interval. If the service reaches to the customer beyond this interval it will accept the service but with a specific penalty. However in problems with hard time window each customer has a predefined time interval and it will not accept the services which are out of this interval. There exist some comprehensive reviews among literature about these kinds of problems such as: [3] [4] [5] [6], [7] [8] [9].

In addition to minimizing total distribution cost and travel time, the real life transportation problems have other objectives such as avoiding underutilization of vehicle capacity or labor. Calvete et al. presents a multi objective vehicle routing problem (VRP) in which cost, time window and underutilization of vehicle capacity and labor are considered [10]. In this paper a multi objective location routing inventory model is going to be presented which considers location, allocation, and inventory costs; hard and soft time windows; avoiding underutilization of vehicle capacity and labor. Among the most useful multi objective methodologies, goal programming (GP) is popular. In GP an acceptable level for each goal will be defined as a target, and then the GP tries to minimize the deviation of objective functions from these targets [11], [12], [13]. There are two kinds of GP, weighted goal programming (WGP) and lexicographic (LGP). In LGP each goal gets a priority and they are going to be minimized one at a time in order of priority. In WGP deviation from each goal gets weight base on importance [13]. Then total penalties will be minimized. In this article we use WGP. The remainder of the paper is organized as follows. In section 2 the problem is described and the formulation is presented. A simulated example is solved in section 3 to illustrate the capability of the model. Some sensitivity analysis is done in section 4 . Finally we conclude the paper in Section 5.

2. Model formulation

The purpose of the model is to select distribution centers among a set of candidate locations and choose a capacity level for them, assign customers to each distribution center, find the best route for each vehicle and determine the scheduling of each route regarding to associate time windows. Therefore to use a GP the following goals are defined:

Goal 1: Minimize total cost (includes location, allocation, routing, and inventory costs)

Goal 2: Satisfy soft time windows which are the customers preference

Goal 3: Avoid underutilization of vehicle capacity

Goal 4: Avoid underutilization of labor

The model assumes that each customer demand followed a normal distribution. A heterogeneous fleet of vehicles is available; also a limitation can be applied on the number of each type of vehicles.

2.1. Index sets

K	Set of customers
J	Set of potential distribution centers
M	Merged set of customers and potential distribution centers, i.e. $K \bigcup J$
N_j	Set of capacity levels available to distribution center $j \in J$ (j ∈ J)
V	Set of vehicles

2.2. Parameters and notations

μ_k	Mean of yearly demand at customer k $\left(k \in K\right)$
σ_k^2	Variance of yearly demand at customer k $\left(k \in K\right)$
f_j^n	Yearly fixed cost for opening and operating distribution center j with capacity level n $\left(\forall j \in J, n \in N\right)$
d_{kl}	Transportation cost between node k and node l $\left(\forall k, l \in M\right)$
q	Number of visits of each vehicle in a year
h_j	Inventory holding cost per unit of product per year at distribution center j $(\forall j \in J)$
$\hat{p}_j$	Fixed cost per order placed to the supplier by distribution center j $(j \in J)$
lt_j	Lead time of distribution center j in years $(j \in J)$
g_j	Fixed cost per shipment from supplier to distribution center j $(j \in J)$
a_j	Cost per unit of shipment from the supplier to distribution center j $(j \in J)$
α	Desired percentage of customer orders that should be satisfied (fill rate), $\alpha > 0.5$
z_α	Left α -percentile of standard normal random variable Z, i.e. $P(z \le z_\alpha) = \alpha$
β	Weight factor associated with transportation cost
θ	Weight factor associated with inventory cost

Z	Lower bound on the total cost
$t_{\max}$	Total daily time that a driver can be working due to labor regulations
$\tilde{t}_{\max}$	Total daily time that a driver can be driving due to labor regulations
e_i^h	Indicates the earliest time in which the service to the customer *i* should start
l_i^h	Indicates the latest time in which the service to the customer *i* should end
e_i^s	Indicates the earliest time in which the customer *i* prefers the receive to start
l_i^s	Indicates the earliest time in which the customer *i* prefers the receive to end
t_{lk}	The time taken to travel directly from node l to node k

$$R_{klv} = \begin{cases} 1 & \text{if } k \text{ precedes } l \text{ in route of vehicle } v \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{jk} = \begin{cases} 1 & \text{customer } k \text{ is assigned to DC } j \\ 0 & \text{otherwise} \end{cases}$$

$$U_j^n = \begin{cases} 1 & \text{if DC } j \text{ is opened with capacity level } n \\ 0 & \text{otherwise} \end{cases}$$

pp_j^v = departure time of vehicle v from jth depot

p_k = departure time from customer k

$\ddot{W}, \hat{W}_k, \tilde{W}_k, \breve{W}_v$, and $\dot{W}_v$ denote the penalties per unit of deviation from each goal.
According to mentioned notation the model is presented as follow:

$$Min \quad \ddot{W}\ddot{g}^+ + \sum_{k\in K} \hat{W}_k \hat{g}_k^- + \sum_{k\in K} \tilde{W}_k \tilde{g}_k^+ + \sum_{v\in V} \breve{W}_v \breve{g}_v^- + \sum_{v\in V} \dot{W}_v \dot{g}_v^- \tag{0}$$

$$\sum_{j\in J} \sum_{n\in N_j} f_j^n U_j^n + \beta q \sum_{v\in V} \sum_{k\in M} \sum_{l\in M} d_{kl} R_{klv}$$

$$+ \sum_{j\in J} \left[\sqrt{2\theta h_j \left(\theta \hat{p}_j + \beta g_j\right) \sum_{k\in K} \mu_k Y_{jk}} + \beta a_j \sum_{k\in K} \mu_k Y_{jk} + \theta h_j z_\alpha \sqrt{lt_j \sum_{k\in K} \sigma_k^2 Y_{jk}} \right] - \ddot{g}^+ = Z \tag{1}$$

$$\sum_{v\in V} \sum_{l\in M} R_{klv} = 1 \qquad \forall k \in K \tag{2}$$

$$\sum_{l\in K} \mu_l \sum_{K\in M} R_{klv} - vc_v \times \sum_{j\in J} \sum_{k\in K} R_{kjv} + \dot{g}_v^- = 0 \qquad \forall v \in V \tag{3}$$

$$M_{kv} - M_{lv} + \left(B \times R_{klv}\right) \le B - 1 \qquad \forall k, l \in K, \forall v \in V \tag{4}$$

$$\sum_{l\in M} R_{klv} - \sum_{l\in M} R_{lkv} = 0 \qquad \forall k \in M, \forall v \in V \tag{5}$$

$$\sum_{j\in J} \sum_{k\in K} R_{jkv} \le 1 \qquad \forall v \in V \tag{6}$$

$$\sum_{l\in M} R_{klv} + \sum_{l\in M} R_{jlv} - Y_{jk} \le 1 \qquad \forall j \in J, \forall k \in K, \forall v \in V \tag{7}$$

$$\sum_{n\in N_j} U_j^n \leq 1 \qquad \forall j \in J \tag{8}$$

$$\sum_{k\in K} \mu_k Y_{jk} \leq \sum_{n\in N_j} b_j^n U_j^n \qquad \forall j \in J \tag{9}$$

$$p_k + \hat{g}_k^+ - \hat{g}_k^- = e_k^s + s_k \qquad \forall k \in K \tag{10}$$

$$p_k + \tilde{g}_k^+ - \tilde{g}_k^- = l_k^s \qquad \forall k \in K \tag{11}$$

$$\sum_{m\in M}\sum_{l\in M} (s_m + t_{lm}) R_{mlv} - t_{\max} \sum_{j\in J}\sum_{k\in K} R_{kjv} + \breve{g}_v^- = 0 \qquad \forall v \in V \tag{12}$$

$$p_k - pp_j^k + (1 - R_{jkv}) M \geq s_k + t_{jk} \qquad \forall k \in K, j \in J, v \in V \tag{13}$$

$$p_k - pp_j^k - (1 - R_{jkv}) M \leq s_k + t_{jk} \qquad \forall k \in K, j \in J, v \in V \tag{14}$$

$$p_k - p_s + (1 - R_{ksv}) M \geq s_k + t_{ks} \qquad \forall k, s \in K, v \in V \tag{15}$$

$$p_k - p_s - (1 - R_{ksv}) M \leq s_k + t_{ks} \qquad \forall k, s \in K, v \in V \tag{16}$$

$$p_k \geq e_k^h + s_k \qquad \forall k \in K \tag{17}$$

$$p_k \leq l_k^h \qquad \forall k \in K \tag{18}$$

$$\sum_{m\in M}\sum_{l\in M} t_{ml} R_{mlv} \leq \tilde{t}_{\max} \qquad \forall v \in V \tag{19}$$

$$R_{mlv} \in \{0,1\} \qquad \forall m, l \in M, v \in V \tag{20}$$

$$U_j^n \in \{0,1\} \qquad \forall j \in J, n \in N_j \tag{21}$$

$$Y_{jk} \in \{0,1\} \qquad \forall j \in J, k \in K \tag{22}$$

$$M_{kv} \geq 0 \qquad \forall k \in K, v \in V \tag{23}$$

$$p_k \geq 0 \qquad \forall k \in K \tag{24}$$

$$pp_j^k \geq 0 \qquad \forall j \in J, k \in K \tag{25}$$

$$\ddot{g}^+, \hat{g}_k^-, \hat{g}_k^+, \tilde{g}_k^-, \tilde{g}_k^+, \dot{g}_v^-, \breve{g}_v^- \geq 0 \qquad \forall k \in K, v \in V \tag{26}$$

Constraint (1) is related to the total cost which is obtained from the objective function of the model proposed by [2]. Constraints (2) make sure that each customer is placed on exactly one vehicle route. Constraints (3) allow us to formulate the goal of avoiding underutilization of vehicle capacity. Constraints (4) are the sub tour elimination. Constraints (5) ensure whenever a vehicle enters a node, it must leave again and ensuring that the routes remain circular. Constraints (6) imply that only one distribution center is included in each route. Constraints (7) link the allocation and the routing components of the model: the customer k is assigned to the distribution center j if the vehicle v, which visits the customer k, starts its trip from the distribution center j. Constraints(8) ensure that each distribution center can be assigned to only one capacity level. Constraints (9) are the capacity constraints associated with the distribution centers. Constraints (10) and (11) are soft time windows. Constraints (12) are avoiding underutilization of labor. Constraints (13) - (16) ensure feasibility of schedule for each vehicle. Constraints (17) and (18) are hard time windows. Constraints (19) are maximum driving time limitations. Constraints (20)-(22) are defining the binary variables while constraints (23)-(26) denote positive variables.

3.Experimental results

In order to verify the model we construct the following example and solve it by the proposed model. The mean and standard deviation of yearly demand at each customer are assumed to be same and equal to 20 and 1 units per year, respectively. The service time for each customer is set as 20 minutes (0.33 hours). All the vehicles must reach at customers' locations between 4am to 10am, although the customers prefer to be serviced between 6am and 8am. The vehicles capacities are 140 units. Each driver is allowed to work only 8 hours a day and the driving time should not exceed 6 hours a day. The x and y coordinates of 8 customers and 2 depots are shown inTable 1, and also the assumed depot parameters are given in 0The available capacity for each candidate node and their associate cost are shown in 0 The rest of parameters value are illustrated in Table 4.

Table 1 coordinates of nodes

#	x	y
1	34	31
2	29	32
3	24	33
4	17	29
5	8	28
6	33	27
7	24	25
8	31	23
9	25	19
10	14	24

Table 2 depots parameters

Depot #	h_j	lt_j	p_j	g_j	a_j
9	5	0.017	11	12	5
10	8	0.020	14	11	7

Table 3 capacity and the fixed cost of candidate depots

Capacity	200	280	300	350
Cost	80	100	150	180

Table 4 parameters value

Parameters	values
q	100
β	0.003
θ	0.7
z_α	1.96

Table 5 Vehicles' schedule

Vehicle 1		Vehicle 2	
Node #	Departure time	Node #	Departure time
9	5:52	9	5:40
8	6:19	5	6:19
6	6:44	4	6:48
1	7:07	7	7:16
2	7:32	9	
3	7:57		
9			

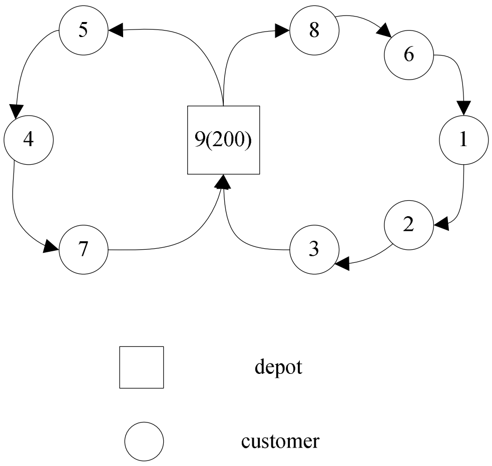

Figure 1 The sequence of each route customers

The problem has been solved by GAMS and the following result is obtained. Depot number 9 is opened with the capacity of 200 units. The schedule is shown in Table 5 0and Figure 1 depicts the sequence of customers.

4. Sensitivity analysis

In order to track the performance of the presented model we conduct some experiments base on length of time intervals. Figure 2 depicts how lengths of soft time windows affect total deviation from the goals. As it was clear, wider intervals cause less deviation from the goals. Figure 3 illustrates the effect of hard time window on the deviations from goals. When the length of hard time window is small, here less than 2 hours, it forces to use more vehicles in order to satisfy the customers demand in the short period. Using more vehicles increases the vacant space of each vehicle and also it imposes underutilization of labor. Consequently these cause more deviation from goals.

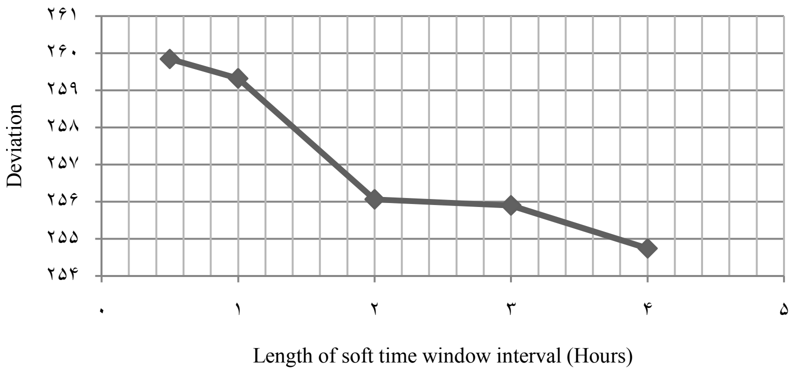

Figure 2 Effect of soft time window length on the goal value

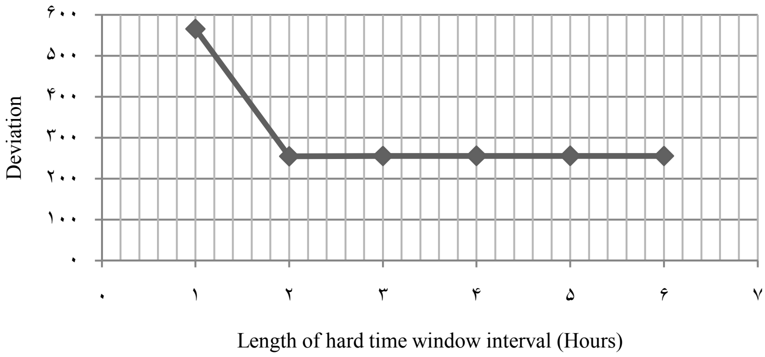

Figure 3 Effect of soft time window length on the goal value

5.Conclusion

In this paper we present a multi objective location routing inventory problem with 4 goals i.e. minimizing total cost, satisfy soft window constraint, avoid under utilization of labor and vehicle capacity. We use goal programming approach to model this problem. In order to verify the presented model an example is solved and the results are reported. Due to non-linearity of the model further studies should focus on solving methods. Multi objective meta-heuristics can be applied to solve the model.

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