Archive of SID

Fuzzy location-routing problem: modeling and a tabu search heuristic

Nader Ghaffari-Nasab[1,*] **, Solmaz Ghazanfar Ahari**[2]**, Ali Saboury**[3]
Institute for Trade Studies and Research, ngnasab@itsr.ir
Amirkabir University of Technology, solmaz.ahari@gmail.com
Iran University of Science and Technology, alisaboury@ind.iust.ac.ir

Abstract
Location-routing problem (LRP) is established as a new research area in the context of location analysis that concerns simultaneously both the problems of location of facilities and routing of vehicles among the established facilities and the existing demand points. In this paper, the location-routing problem with fuzzy demands (LRPFD) is considered which may arise in many real life situations in logistics management and a fuzzy chance constrained program is designed to model it based on fuzzy possibility theory. A tabu search (TS) heuristic is proposed to solve the problem. The efficiency of the solution procedure is demonstrated using a standard benchmark set of test problems.

Keywords: Location-routing problem, Logistics, Fuzzy sets, Possibility Theory, Tabu Search.

1. Introduction
The efficient and effective movement of goods from raw material sites to processing facilities, component fabrication plants, finished goods assembly plants, distribution centers, retailers and customers is critical in today's competitive environment. Approximately 10% of the gross domestic product is devoted to supply-related activities [1]. The above proportion can easily exceed this value within individual industries. In many real life situations, shipments are made in less-than-truckload (LTL) quantities from a facility to customers along a multiple-stop route. In the case of full truckload quantities, the cost of delivery is independent of the other deliveries made, whereas in the case of LTL quantities, the cost of delivery depends on the other customers on the route and the sequence in which customers are visited. Thus ignoring this dependence between location and routing decisions will result in sub-optimal decisions.

We define location-routing, following Nagy and Salhi [2], as "location planning with tour planning aspects taken into account". This definition stems from a hierarchical viewpoint, whereby the aim is to solve a facility location problem (the "master problem"), but in order to achieve this, we need to solve a vehicle routing problem (the "sub-problem") as well. This also implies an integrated solution approach, i.e. an approach that considers both location and routing aspects of a problem but does not address their interrelation is not classified as belonging to the LRP. Location-routing problems are closely related to both the classical location-allocation problem and the vehicle routing problems. In fact, both of the latter problems can be thought of as special cases of the LRP. If we require all customers to be directly linked to the existing depots, the LRP reduces to a standard location problem. On the other hand, if we fix the depot locations, the LRP becomes a VRP. From a practical viewpoint, location-routing is a part of distribution management, while theoretically it can usually be modeled as a combinatorial optimization problem. We note that this is an NP-hard problem, since it encompasses two NP-hard problems (facility location and vehicle routing).

[1,*] Corresponding author: PhD candidate, Industrial Engineering
[2] MSc, Industrial Engineering
[3] PhD candidate, Industrial Engineering

www.SID.ir

Archive of SID

Fuzzy logic has been used to solve many applied problems so far. The need to use fuzzy logic in problems arises whenever there are some vague or uncertain parameters. In most of the cases, there is not sufficient data for fitting a probability distribution to the customers' demands. On the other hand, based on the expert's judgment, one can easily estimate these demands. Therefore, while using probability theory is difficult and costly, fuzzy logic is used to deal with uncertainty in these problems. Credibility theory has been used in many problems with fuzzy parameters so far, in parallel with some metaheuristics. Fazel Zarandi et al. [3] addressed multi-depot capacitated LRP (MDCLRP) in which travel time between two nodes is a fuzzy variable and proposed a simulation-embedded simulated annealing (SA) procedure in order to solve the problem.

Erabo and Mingyong [4] considered the vehicle routing problem with fuzzy demands and proposed a fuzzy chance constrained program model based on fuzzy credibility theory. They used stochastic simulation and an improved differential evolution algorithm to solve the problem. Considering the literature of location-routing problem, our paper makes the following contributions to the literature. As far as the authors know, this is the first work in the literature of the LRP which considers fuzzy demands and uses credibility theory to model and solve problem. Moreover, a hybrid simulated annealing based heuristic has been proposed in which stochastic simulation is used to estimate the credibility of a solution.

In this paper, the LRP has two levels (depots and customers) and can be defined as follows: Let $G=(V,E)$ be an undirected network where V is a set of nodes comprised of a subset I of m potential depot sites and a subset $J=V \setminus I$ of n customers. E is a set of edges connecting each pair of nodes in V. Associated with each edge $(i,j)\in E$ is a traveling cost c_{ij}. Each depot site $i\in I$ has an opening cost O_i. Each customer $j\in J$ has a demand d_j of a single commodity which is assumed to be a fuzzy variable. Determination of the real values of the customers' demands prior to their realizations is often too difficult or even impossible because of their uncertain nature. In this work we assume that there is not sufficient data for fitting a probability distribution to the customers' demands. It is assumed that these demands are estimated based on the expert's judgment. Therefore, fuzzy logic is used to deal with uncertainty in this paper. A set K of identical vehicles with capacity Q is available. Each vehicle, when used by a depot i, incurs a depot dependent fixed cost F_i and performs a single route. Each route must start and terminate at the same depot. The objective is to determine which depots should be opened and which routes should be constructed to minimize the total cost. We also assume that: (*a*) a vehicle will be assigned for only one route on which there may be more than one customer, and (*b*) a customer will be visited by one and only one vehicle. The goal of our problem is: (*i*) to determine the subset of facilities (depots) to open, (*ii*) the allocation of customers to depots, and (*iii*) the routes from depots to serve customers regarding the capacities of vehicles. Fig. 1 shows a solution to a typical LRP instance with 20 customers and 6 candidate sites for depot locations. As it is depicted, in this solution three depots out of six candidate depots have been opened (depots 22, 23, and 25). The deliveries are made though five established routes (two routes are originated from depot 22, two routes from depot 23, and one route from depot 25).

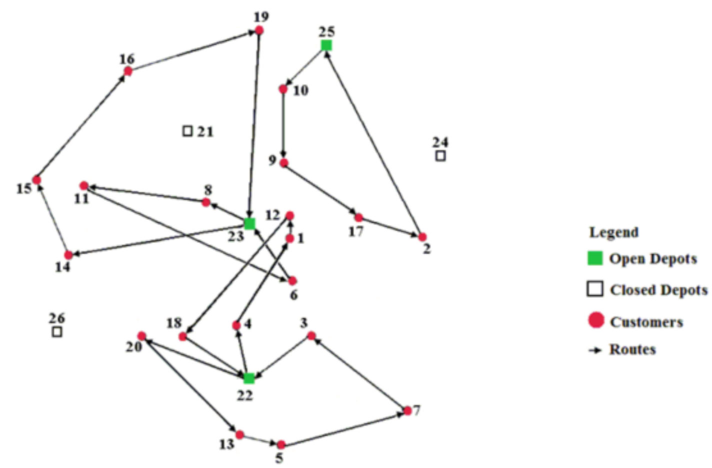

Figure 1- A feasible solution for an LRP instance with 20 customers and 6 candidate depot sites.

www.SID.ir

Archive of SID

This paper is organized as follows: In Section 2, we give some basic concepts on fuzzy theory. In Section 3, we introduce location-routing problem with fuzzy demand and present a CCP model, where we will measure fuzzy events with possibility. Then we propose a tabu search (TS) based hesuritic to solve this model in Section 4. In Section 5, we will conduct a set of experiments to reveal the effectiveness of the proposed hybrid heuristic. In the final section, we summarize the paper and provide some lines for further research.

2. Fuzzy Sets and Possibility Theory

A classic set is normally defined as a collection of elements. Each single element can either belong or not belong to this set. Such a set can be described in different ways: one can either list the elements that belong to the set; describe the set analytically by a sequence of equalities and inequalities; or define the member elements by using the characteristic function, in which 1 indicates strict membership and 0 strict nonmembership. However, in many cases, the membership (or nonmembership) is not clear. For example, "young man", "large number", "about 100 tons", "approximately 250 liters". They are not tractable by the classical set theory. In order to deal with them, Zadeh [5] firstly introduced the concept of fuzzy set and defined the membership function as the degree to which an element belongs to a fuzzy set. We call a fuzzy number (or called fuzzy quantity) a fuzzy subset $\tilde{a}$ of $\boldsymbol{R}$ with membership function $\mu_{\tilde{a}}:\boldsymbol{R}\to[0,1]$.

Possibility theory was initially proposed by Zadeh [6], and extended by many researchers such as Dubois and Prade [7]. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers with membership functions $\mu_{\tilde{a}}$ and $\mu_{\tilde{b}}$, respectively. Based on the concepts and techniques of possibility theory founded by Zadeh [6], the possibility of $\tilde{a}\le\tilde{b}$ is defined as follows:

$$Poss\left\{\tilde{a}\le\tilde{b}\right\}=\sup\left\{\min(\mu_{\tilde{a}}(x),\mu_{\tilde{b}}(x))\mid x,y\in R, x\le y\right\}. \tag{1}$$

Especially, when $\tilde{a}$ is crisp, i.e., a, we have:

$$Poss\left\{a\le\tilde{b}\right\}=\sup\left\{\mu_{\tilde{b}}(x)\mid x\in R, a\le x\right\}. \tag{2}$$

Now let us assume that $\tilde{b}$ is a triangular fuzzy number, i.e., $\tilde{b}=(b_1, b_2, b_3)$. We can now rewrite the eq. (2) as:

$$Poss\left\{a\le\tilde{b}\right\}=\begin{cases}1 & if\ a<b_2\\ \dfrac{b_3-a}{b_3-b_2} & if\ b_2<a<b_3\\ 0 & if\ b_3<a\end{cases}. \tag{3}$$

3. Fuzzy Chance Constrained Programming Model

This section presents an integer programming formulations for the location-routing problem with fuzzy demands (LRPFD). In the basic version of this model which was proposed by Prins et al. [8], the assumption of single-sourcing holds, in other words, it is assumed that the customers acquire their needed demand from a single supplier. The following notations ate used to represent the mathematical programming formulation.

Sets and parameters:

- J Set of customers indexed by j
- I Set of candidate depot sites indexed by i
- K Set of vehicles indexed by k
- V Set of all points; $V=I\cup J$
- E Set of arcs (i,j) connecting every pair of nodes $i,j\in V$
- $\tilde{d}_j$ Fuzzy demand of customer j

www.SID.ir

Archive of SID

O_i Fixed cost of opening a depot at candidate site i
F_i Fixed cost of employing a vehicle at candidate site i
c_{ij} Cost of traveling associated with arc $(i,j) \in E$
Q Capacity of vehicles; here it is assumed that all vehicles are homogeneous

Decision variables:

$$Z_i = \begin{cases} 1 & \text{if we open a depot at candidate site } i \\ 0 & \text{if not} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demands at customer } j \text{ are served by the depot at candidate site } i \\ 0 & \text{if not} \end{cases}$$

$$X_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ goes directly from node } i \text{ to node } j \\ 0 & \text{if not} \end{cases}$$

The corresponding chance constrained problem (CCP), that is, the mathematical formulation of LRPFD based on possibility theory, is as follows:

$$\min \sum_{i\in I} O_i Z_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} F_i X_{ijk} + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ij} X_{ijk} \tag{4}$$

Subject To:

$$\sum_{i\in V}\sum_{k\in K} X_{ijk} = 1 \qquad \forall j \in J \tag{5}$$

$$Poss\left\{\sum_{i\in V}\sum_{j\in J} \tilde{d}_j X_{ijk} \ge Q\right\} \le \varepsilon \qquad \forall k \in K \tag{6}$$

$$\sum_{i\in S}\sum_{j\in S} X_{ijk} \le |S|-1 \qquad \forall S \subseteq J;\ \forall k \in K \tag{7}$$

$$\sum_{j\in V} X_{ijk} - \sum_{j\in V} X_{jik} = 0 \qquad \forall j \in V;\ \forall k \in K \tag{8}$$

$$\sum_{i\in I}\sum_{j\in J} X_{ijk} \le 1 \qquad \forall k \in K \tag{9}$$

$$\sum_{m\in V} X_{imk} + \sum_{h\in V} X_{jhk} \le 1 + Y_{ij} \qquad \forall i \in I;\ \forall j \in J;\ \forall k \in K \tag{10}$$

$$Z_i \in \{0,1\},\ Y_{ij} \in \{0,1\},\ X_{ijk} \in \{0,1\} \qquad \forall i \in I \tag{11}$$

The three terms in the objective function (4) represent the sum of the fixed depot location costs and routing costs including the fixed costs of employing vehicles and the travel costs, respectively. Constraints (5) ensure that each customer belongs to one and exactly one route, and that each customer has only one predecessor in the route. Chance constraint (6) assures that all customers are visited given a vehicle's capacity within a certain confidence level. In other words, this constraint implies that the possibility of the event that customers are not visited given a vehicle's capacity be less than a predetemined small value (ε). Constraints (7) are sub-tour elimination constraints. Constraints (8) and (9) guarantee the continuity of each route, and that each route terminates at the depot where the route starts. Constraints (10) ensure that a customer must be allocated to a depot if there is a route connecting them. Finally, (11) are integrality constraints.

Using eq. (3) we can rewrite the constraint (6) as follows:

$$\frac{\sum_{i\in I}\sum_{j\in J} d_{j,3} X_{ijk} - Q}{\sum_{i\in I}\sum_{j\in J} d_{j,3} X_{ijk} - \sum_{i\in I}\sum_{j\in J} d_{j,2} X_{ijk}} \le \varepsilon$$

www.SID.ir

Archive of SID

Which is equivalent to:

$$\varepsilon\left(\sum_{i\in I}\sum_{j\in J} d_{j,2}X_{ijk}\right)+(1-\varepsilon)\left(\sum_{i\in I}\sum_{j\in J} d_{j,3}X_{ijk}\right)\le Q \tag{12}$$

The deterministic equivalent of chance constrained programming model for LRPFD can be writen as follows:

$$\min \ \sum_{i\in I} O_i Z_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} F_i X_{ijk} + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ij} X_{ijk} \tag{4}$$

Subject To:

(5),(7)-(11)

$$\varepsilon\left(\sum_{i\in I}\sum_{j\in J} d_{j,2}X_{ijk}\right)+(1-\varepsilon)\left(\sum_{i\in I}\sum_{j\in J} d_{j,3}X_{ijk}\right)\le Q \qquad \forall k\in K \tag{12}$$

4.Tabu Search based Heuristic

Tabu search, originally developed in a paper by Glover [9], is a local search has effectively tackled a variety of hard real world optimization problems. This procedure starts with an initial solution and uses a tabu list to control moves in neighborhood structure so that trapping in local optima and re-visiting the same solution not occurred. From the current solution, all the non-tabu moves are explored and the best one is selected. This move, that might be lead to better or worse solution than current solution, is recorded in tabu list. The future move is among that moves are not in tabu list, unless it fulfills aspiration level. TS is terminated when some of stopping criteria is reached.

4.1. Solution Representation

A solution is represented by a string of numbers consisting of a permutation of n customers denoted by the set {1, 2, .., n}, m potential depots denoted by the set {n+1, n+2, ..., n+m}, and N_{dummy} zeros which are used to separate routes, in addition to the vehicle capacity constraints. The ith number in {1, 2, ..., n} denotes the ith customer to be serviced. The first number in a solution is always in {n+1, n+2, ..., n+m} indicating the first depot under consideration. The parameter N_{dummy} is calculated as $\left\lceil \sum_j (d_j / C) \right\rceil$, where d_j is the mean demand of customer j, Q is the capacity of vehicle, and $\lceil . \rceil$ denotes the smallest integer which is larger than or equal to the enclosed number.

The solution representation is further explained as following. Each depot services customers between the depot and the next depot in the solution representation. The first route of this depot starts by servicing the first customer after the depot. Other customers for this depot are added to the current route one at a time. If the credibility of having enough capacity for serving the next customer falls below the dispatcher preference index, the current route is terminated. If the next number in the solution representation is a dummy zero, the current route will also be terminated. A new route will be started to service remaining customers assigned to this depot. It can be verified that this solution representation always gives a LRP solution without violating the capacity constraint of the vehicle. Fig. 2 depicts a possible coding for the solution to the LRP instance shown in Fig. 1.

21	22	4	1	12	18	0	20	13	5	7	3	0	26	24	23	8	11	6	14	15	16	19	25	10	9	17	2	0

Figure 2- A possible representation for the solution shown in Fig. 1.

4.2. Initial Solution Generation

www.SID.ir

Archive of SID

The initial solution is constructed in a quite random fashion. First, a number is randomly selected from the set of depots $\{n+1, n+2, ..., n+m\}$ and is placed in first cell of the solution string. Then, the remaining (m-1) depots as well as n customers and N_{dummy} dummy zeros are randomly allocated to the empty cells of the string. This simple initial solution generation procedure is not only a quick way to produce a feasible solution but it also produces diverse solutions which can help the algorithm does not get trapped in local optima in different runs.

4.3. Neighborhood structure

We define and use three neighborhood structures to explore the solution space in the proposed TS heuristic; "insertion", "swap", and "2-opt" moves. These three neighborhood structures can be described as follows:

The insertion move is carried out by randomly selecting the ith number of X and inserting it into the position immediately before another randomly selected jth number of X.

The swap move is performed by randomly selecting the ith and the jth numbers of X, and then exchanging the positions of these two numbers.

The 2-opt move, commonly used in solving VRP related problems, is modified and applied to improve existing routes. This is implemented by randomly selecting two customers that are assigned to the same depot, and then reverse the substring in the solution representation between them.

4.4. Parameters Used in the TS Heuristic

Parameters for the proposed tabu-based algorithm are the length of tabu tenure and the maximum number of iterations performed by TS. In the proposed tabu search heuristic, tabu tenure is equal to a randomly selected positive integer from the interval [5,10] and the maximum number of iterations performed by TS is limited to 50.

5. Numerical Study

5.1. Examining the efficiency of the TS based heuristic using LRP benchmark

To verify the performance of the proposed TS heuristic, it is applied to the LRP benchmark provided by Barreto [10]. This benchmark includes the 15 test problems of which 10 instances do not impose capacity constraint on depots. Lower bounds for these LRP instances are available at (http://prodhonc.free.fr/Instances/instances_us.htm). The capacitated LRP considered in Barreto [10] can be solved by the proposed heuristic with a little adaptation; therefore, one can apply the adapted heuristic to the benchmark to assess its performance. The proposed algorithms are coded in MATLAB R2009b on a PC with an Intel Core 2 Dou CPU (2.0 GHz) and 2 GB memory.

Table 1-Solutions obtained by the proposed heuristic for Barreto's LRP instances

No.	Instance's name in Barreto (2004)	Lower bound	TS		
			Total Cost	Gap (%)	CPU time (sec)
1	Gaskell67-22x5	585.1	585.1	0.0	3.8
2	Gaskell67-29x5	512.1	561.8	9.7	4.4
3	Gaskell67-32x5	562.2	620.7	10.4	5.5
4	Gaskell67-32x5b	504.3	504.5	0.0	5.8
5	Gaskell67-36x5	460.4	506.2	10.0	6.3
6	Christofides69-50x5	565.6	620.1	9.6	11.1
7	Christofides69-75x10	798.7	902.0	12.9	113.8
8	Christofides69-100x10	818.1	895.3	9.4	188.9
9	Perl83-12x2	204.0	204.0	0.0	1.7
10	Min92-27x5	3062.0	3062.0	0.0	4.0
Average				**6.2**	**34.5**

The results of applying TS heuristic to the 10 instances from Barreto [1] are presented in Table 1. From this table, it can be seen that the solutions found by the TS heuristic are optimal for four instances, and that, for the whole set of test problems, the average gap is less than 6.2%. This shows that the proposed TS heuristic performs well in terms of solution quality which is an indication of

www.SID.ir

efficiency of the proposed solution procedure. Also the time required to solve these problems is quite small (in a matter of seconds), which is quite short time for solving a strategic problem like LRP.

5.2. Applying the TS based heuristic to LRPFD

Now we will give some examples to show models that we have just discussed and how the proposed hybrid heuristic works. We take all the parameters of the problems same as (http://prodhonc.free.fr/Instances/instances_us.htm) and solved the problem instance "Gaskell67-32×5b" with 32 customers and 5 candidate depots. We added fuzzy demands to the same dataset and generated our test problem. We genrate the fuzzy demand using 3 levels of fuzziness; tight, normal, and loose. We set the left, mid, and right entries of the tight fuzzy demand triplets as 0.9, 1, and 1.1 of their nominal values, respectively. We set the left, mid, and right entries of the normal fuzzy demand triplets as 0.75, 1, and 1.25 of their nominal values, respectively. Finally, we set the left, mid, and right entries of the tight fuzzy demand triplets as 0.5, 1, and 1.5 of their nominal values, respectively. The results obtained from this experiment are shown in Table 2.

Table 2. Solutions obtained by the proposed heuristic for 32*5 instances using three levels of fuzziness

ε	Tight (0.9, 1, 1.1)	Normal (0.75, 1, 1.25)	Loose (0.5, 1, 1.5)
0.0	519.5	543.5	575.5
0.1	508.6	543.5	575.2
0.2	508.6	529.2	566.7
0.3	508.6	520.5	562.2
0.4	508.6	519.6	546.8
0.5	508.6	519.6	543.5
0.6	508.6	519.6	529.2
0.7	508.6	508.6	520.5
0.8	508.6	508.6	519.5
0.9	504.5	508.6	508.6
1.0	504.5	504.5	504.5

Figure 3 graphically depicts the relationship between the objective function and the value of the parameter ε for three levels of fuzziness.

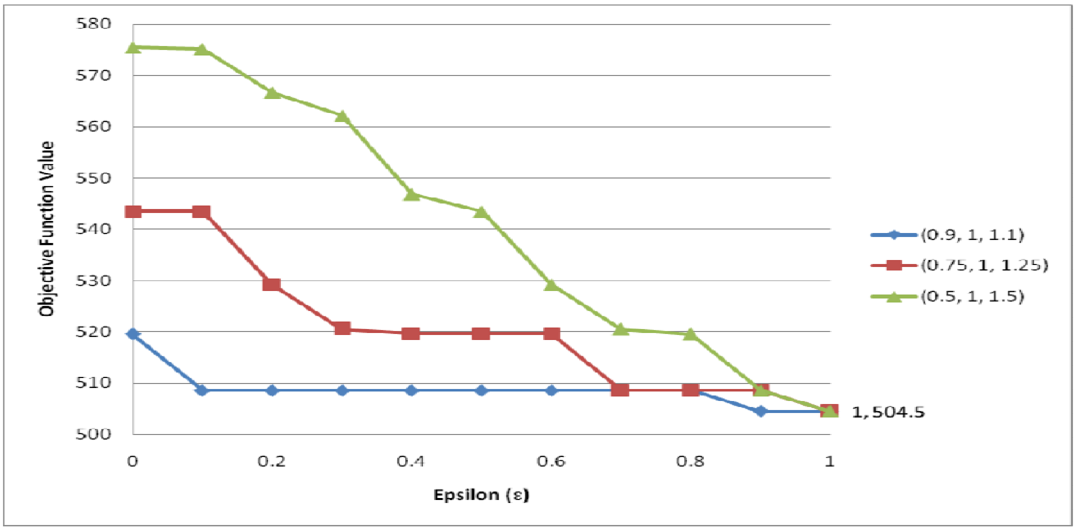

Figure 3- Graphical illustration of relation ship between bbjective function value and parameter ε for three levels of fuzziness

Observe from Table 2 and Figure 3 that, as the triangular numbers get sharper (in case of tight fuzziness level), the optimal solution gets closer to the solution of the deterministic location-routing problem. This is a quite expected result since an extremely sharp fuzzy number (fuzzy number with basis of width zero) can be interpreted as a crisp number. On the other hand, as the value of ε get larger, the value of objective function decrease. It can be mentioned that the larger values of parameter

www.SID.ir

Archive of SID

ε express the decision maker's desire to use vehicle capacity the best he/she can. As a result, these values correspond to routes with less cost. On the other hand, low values of parameter ε represent less utilization of vehicle capacity and hence for these vlaues the possibility of violation of vehicle capacity. In other words, low values of parameter ε shows that the decision maker is a risk averse person, while high values of ε indicate that the decision maker is a cost sensitive person.

6. Conclusion

This paper considered one of the most important problems in logistics and supply chain management namely the location-routing problem in which the demand of the customers is assumed to be of fuzzy nature (LRPFD). Firstly a chance constrained programming formulation based on possibility measure was proposed to model the problem and the deterministic equivalent of the proposed chance constrained programming problrm is extracted. Then a tabu search (TS) based heuristic was presented to solve the problem with the objective of minimizing the total cost provided that the possibility of violation of vehicle capacities be less than a predetemined small value (ε). Finally, the effectiveness of the proposed solution procedure was illustrated by some numerical examples of different sizes. As an interesting line for future research the interested researchers can use other fuzzy measures (such as credibility measure, etc.) in modeling the problem. Also, othe heuristic and meta-heuristic approches can be used to solve this problem.

References

[1] Simchi-Levi, D., Kaminsky, P., and Simchi-Levi, E. "*Designing and Managing the Supply Chain: Concepts, Strategies, and Case Studies"* McGraw-Hill, 2003.

[2] Nagy, G., and Salhi, S. "Location-routing: Issues, models and methods", *European Journal of Operational Research*, Vol. 177, pp. 649-672, 2007.

[3] Fazel Zarandi, M. H., Hemmati, A., and Davari, S. "The multi-depot capacitated location-routing problem with fuzzy travel times", *Expert Systems with Applications*, Vol. 38, pp. 10075-10084, 2011.

[4] Erabo, C., and Mingyong, L. "A hybrid differential evolution algorithm to vehicle routing problem with fuzzy demands", *Journal of Computational and Applied Mathematics*, Vol. 231, pp. 302-310, 2009.

[5] Zadeh, L. A. "Fuzzy sets", *Information and Control*, vol. 8, pp. 338-353, 1965.

[6] Zadeh, L. A. "Fuzzy sets as a basis for a theory of possibility", *Fuzzy Sets and Systems*, Vol. 1, pp. 3-28, 1978.

[7] Dubois, D. nad Prade, H." *Possibility Theory: An Approach to Computerized Processing of Uncertainty"*, Plenum, 1988.

[8] Prins, C., Prodhon, C., Ruiz, A., Soriano, P., and Wolfler Calvo, R. "Solving the capacitated location-routing problem by a cooperative Lagrangean relaxation granular tabu search heuristic" *Transportation Science*, Vol. 41, No. 4, pp. 470-483, 2007.

[9] Glover, F. "Tabu search", *Part i. ORSA Journal on Computing*, Vol. 1, pp. 190–206, 1989.

[10] Barreto, S., "Analysis and modeling of location-routing problems", PhD Dissertation. University of Aveiro, 2004.

www.SID.ir