



پنجمین کنفرانس ملی و سومین کنفرانس بین المللی لجستیک و زنجیره تامین  
5th national & 3rd international LOGISTICS & SUPPLY CHAIN CONFERENCE



## **Integrated machine scheduling and vehicle routing with due date constraint: case study in PEGAH dairy product**

**Seyed Hesamoddin Zegordi, Fateme Marandi**

**1-Associated professor, Tarbiat Modares University**

**2-Msc student, Tarbiat Modares University**

**F.marandi@modares.ac.ir**

### **Abstract**

This study focuses on machine scheduling and customer delivery coordination. Flow shop scheduling is developed with consideration of delivery vehicle routing problem that serves multiple customers with different locations by multiple homogeneous vehicles in order to minimize the maximum total jobs delivery time for the customers. A mixed binary integer programming model is developed to optimally solve this problem. This research is practical in different industries such as dairy products industry so this is studied in PEGAH dairy product in Tehran. Due to the complexity of the integrated problem, the optimal solution is achieved in smaller size of problem so a small-size example is proposed which comes from case study to solve the problem using the proposed model to get optimal schedule.

Key words: flow shop scheduling, vehicle routing, due date, dairy product, supply chain scheduling

## 1-Introduction

Global competition have led manufacturers to focus on delivery speed and as a conclusion to implement supply chain integration[1]. Complexity induces that planning of production and planning of delivery operations, like machine scheduling and vehicle routing, are normally decomposed and successively tackled in practice so decomposition approaches involve the danger of poorly coordinated overall plans with high total costs and an inefficient machine sequence position of just one job can already have a big negative impact on delivery tours. Traditional scheduling models focus on the determination of schedules for production such that some performance measures are optimized without considering the coordination between the machine schedule and the delivery plan. These models implicitly assume that there are infinitely many vehicles available for delivering finished jobs to their destinations so that the finished product can be transported to customers without delay. However, in reality, the number of vehicles is limited and the vehicles may need to deliver to more than one customer location to increase their utilization. Hence, in order to avoid inefficiencies and thereby remain competitive, there is a growing interest in integration approaches. What is important in such an approach implicates high challenges for compliance with promised due date. This tendency has created closer interaction between stages in a supply chain that lead to decreasing in cost and increasing profit in an integrated system between machine scheduling(production stage) and order delivery(distribution stage) to satisfy due date that causes a great competitive advantage.

A simplified version of the problem is in which each job is processed by a single machine, then delivered by a single vehicle to customers located in close proximity to each other which for example could be a single customer, a customer region, a distribution center or a warehouse. The first person, who explicitly integrated job delivery considerations into machine scheduling problems, was Potts (1980)[2]. He considers a single machine problem with job release dates and destination-dependent transportation times. A job is delivered immediately after completion. His goal is to minimize the time at which all jobs are at their destinations. Zdrzalka (1995)[3] and Woeginger (1998)[4] deal with different job families instead of job-dependent release dates. A set-up time has to be scheduled, if the next job in sequence belongs to another family. A similar problem is investigated by Li and Yuan (2009)[5]. Woeginger (1994)[6], Gharbi and Haouari (2002)[7], Liu and Cheng (2002)[8] extend the problem of Potts (1980) to parallel machines. Gharbi and Haouari (2002) and Liu and Cheng (2002) add job release dates. Liu and Cheng (2002) additionally allow for preemption. In order to deliver customer's order in batches, Mazdeh et al. (2007)[9] and Selvarajah and Steiner[10] (2009) study a single machine problem. Each job belongs to a certain destination, whereas two or more jobs can belong to the same destination. They investigate the delivery batching problem of Cheng and Kahlbacher (1993)[11], but with respect to multiple customers. The aim is to minimize the sum of flow times and delivery costs. Cheng and Kahlbacher (1993) study a single machine problem with the delivery of jobs in batches. The jobs have to be sequenced on the machine and assigned to a certain batch. The delivery time of each batch is equal to the completion time of the last job of that batch. A job is early if its completion time deviates from the delivery time of its batch. The aim is to minimize the sum of total weighted earliness and transportation costs. by incurring a machine availability constraint. Lee and Yoon (2010)[12] deal with a single machine and a single vehicle which can ship batches limited to a maximum number of jobs. The travel time to the destination can differ from the way back. The aim is to minimize the sum of the weighted production flow time, the weighted delivery flow time, and the weighted number of batches. Averbakh (2010)[13] limits the batches to a maximum number of jobs and also studies the minimization of the sum of total weighted flow times and delivery costs. Kise et al. (1991)[14] consider a variant of middle transportation with only one transporter in which the transporter can carry one job at a time. Other flow shop scheduling problems with transportation of jobs between machines have been studied by Stevens and Germill (1997)[15]., This flow

shop scheduling models focus on the transportation of jobs between machines in a flow shop environment rather than the shipping of finished goods to customers.

As vehicle routing makes sense only if there are multiple job destinations, Chang and Lee (2004)[16] examine a problem with a single machine and a single vehicle, which ships completed jobs from the machine to two destinations. The vehicle is assumed to possess a limited capacity. Each job is preassigned to one of the destinations and has a job-dependent size, which determines the amount of the vehicle capacity it needs. Due to the distances from the machine to the job destinations and from one destination to the other, there are three travel times. The objective is to minimize the maximum job arrival time. Liang et al.(2011)[17] focus on integration problem with time window constraint. Li and Vairaktarakis (2007)[18] examine a scenario with two machines. The jobs consist of two tasks. One task has to be done on the first and the other on the second machine. Both tasks of a job can be processed at the same time. Completed jobs have to be delivered to their job-dependent destination. An unlimited number of vehicles each with the same capacity is available. Because of importance of customers satisfaction many problem focus on delivering in order to satisfy customers due date, Garcia et al. (2004)[19] consider a fleet of vehicles and several plants. The vehicles can carry only one job at the same time. Each plant consists of parallel machines. The jobs, which possess job-dependent due dates and destinations, should be delivered just in time. Each job can be processed only in a subset of the plants. This problem is similar to the one of Chen and Pundoor (2006)[20] study. Li et al.(2010)[21]focus on scheduling of two parallel machines with job delivery by a single truck to one customer area. Liu et al.(2012)[22] study the same work to minimize the time required for all jobs to be completed and delivered to the customer area.

This study is an integrated problem consists of two scheduling sub-problems. The processing of the jobs on  $M$  machines in flowshop scheduling and the delivery of the jobs by a fleet of vehicles. The two sub-problems are linked and integrated by the completion times of the jobs in order to minimize the maximum total jobs delivery time for the customers. This study presents a mixed binary integer programming model for optimal schedule. The integrated problem is NP-hard. As the integrated problem is a generalization of the sub-problems, the integrated problem must be NP-hard, too. Due to the complexity of the integrated problem, the optimal solution is achieved in smaller size of problem. So a small-size example is proposed which comes from PEGAH dairy product industry to solve the problem using the proposed model to get optimal schedule. The rest of the paper is organized as follows. Section 2 defines the studied problems as well as the required notations. In section 3 a mixed BIP model is proposed. A case study is presented in section 4 and finally, conclusions drawn in section 5.

## 2-Problem description and notation definition

In this section the problem definition and the notation that will be used throughout this study is described. Then model for integrated problem will be formulated. This research studies a two-stage scheduling problem in which the first stage is job production and the second stage is job delivery to multiple customers. The focus is on the integration of production scheduling with delivery of finished products to customers that jobs are first processed in flow shop machine then delivered with consideration of routing vehicle to respective customers. The two sub-problems are linked and integrated by the completion times of the jobs. Ready time of each customers' jobs is determined by the latest completion time on the machine of the jobs. Transporters are homogeneous vehicles with finite capacity. Each product requires varying physical space while being loaded. The study aims to minimize the maximum total jobs delivery time for the customers.

The proposed problem is described as follows. A set of  $n$  jobs,  $N=\{J_1, J_2, \dots, J_n\}$ , has first to be processed in flow shop scheduling by  $m$  machines which  $M=\{m_1, m_2, \dots, m_m\}$  is machine set. The subscript symbols are:  $r$  for machines,  $1 \leq r \leq m$ , and  $i, g$  for jobs,  $1 \leq i, g \leq n$ .  $P=\{P_{ri}\}$  is the  $M \times N$  matrix of job processing time that  $P_{ri}$  is processing time of job  $i$  on machine  $r$ .

In vehicle routing problems, a set of routes is designed in order to meet customer demands. The literature in this field addresses situations with a variety of operational characteristics. In this paper CVRP is considered that  $G=(V,E)$  is a complete and undirected graph where  $V=\{0,1,\dots,c\}$  is the vertex set and  $E$  is the edge set. Vertex set  $V_c=\{1,\dots,c\}$  corresponds to  $n$  customers, whereas vertex 0 corresponds to the depot.  $T_{jl}$  is associated with each edge  $\{j,l\} \in E$  and represents the travel time spent to go from vertex  $j$  to vertex  $l$ . Each customer  $j \in V_c$  is associated with a known nonnegative physical space of customer  $j$ 's demand  $i$ ,  $e_{ij}$ , to be delivered. A set of identical vehicles, each with capacity  $Q$ , is available at the depot. It is too important to satisfy customers' due date that means the ready time which represents the latest completion time on the machine plus traveling time to customer is less than the due date. The starting time of a tour must be equal to or greater than the latest completion time of the jobs assigned to it. The objective is to minimized the maximum total jobs delivery time for the customers. The following notation was used throughout the study:

Parameters:

Parameters:

$M$  = a very large positive number;

$m$  = number of machines in production stage;

$n$  = number of orders for processing at time zero;

$c$  = number of customers;

$K$  = number of vehicles;

$t_{j,l}$  = traveling time between customer  $j$  and customer  $l$ ;

$p_{r,i}$  = job  $i$  processing time on machine  $r$ ;

$e_{ij}$  = physical space of customer  $j$ 's demand  $i$ ;

$z_{ij}$  = customer  $j$ ' demand for job  $i$ ;

$Q$  = capacity of vehicle;

$du$  = due date for delivery;

$qt$  = duration time for quality control;

Variables:

$x_{jl}$  = 1 if arc  $(j,l)$  is the solution; 0 otherwise;

$u_j$  = represent the amount of flow produced by the depot;

$r_{ij}^k$  = ready time of customer  $j$ 's job  $i$  that represents the latest completion time on the machine to deliver by vehicle  $k$ ;

$REC(k)$  = total time for vehicle  $k$  to finish delivery;

$y_{i,g}$  = 1 if job  $i$  is scheduled any time before job  $g$ ; 0 otherwise;

$c_{r,i}$  = completion time of job  $i$  on machine  $r$ ;

$y_{i,k}$ ,  $x_{j,l}$  are binary integer variable. The others are real variables that take integer values.

### 3-The optimization model

The optimization model employed mixed binary integer programming technique to find the optimal solution for the proposed problem. The optimization model is described as follows:

Minimize:

$$MAX(REC(K)) \tag{1}$$

$$co_{1,i} \geq p_{1,i} \quad 1 \leq i \leq n$$

$$co_{r,i} - co_{r-1,i} \geq p_{r,i} \quad 2 \leq r \leq m ; 1 \leq i \leq n$$

$$co_{r,i} - co_{r,g} + M(y_{i,g}) \geq p_{r,i} \quad 1 \leq r \leq m ; 1 \leq i < g \leq n$$

$$co_{r,g} - co_{r,i} + M(1 - y_{i,g}) \geq p_{r,g} \quad 1 \leq r \leq m ; 1 \leq i < g \leq n$$

$$\sum_{k=1}^K \sum_{l=1}^c x_{lj}^k = 1 \quad j=1,2,\dots,c$$

$$\sum_{k=1}^K \sum_{j=1}^c x_{lj}^k = 1 \quad l=1,2,\dots,c$$

$$\sum_{l=0}^c x_{lj}^k - \sum_{j=0}^c x_{lj}^k = 0 \quad k=1,2,\dots,K$$

$$\sum_{j=1}^c x_{0j}^k = 1 \quad k=1 \dots K$$

$$u_j + 1 \leq u_l + c(1 - x_{jl}^k) \quad k=1,2,\dots,K ; j,l=1,2,\dots,c$$

$$\sum_{k=1}^K \sum_{j=1}^c x_{0j}^k = K$$

$$\sum_{k=1}^K \sum_{l=1}^c x_{l0}^k = K$$

$$\sum_{i=1}^n \sum_{j=0}^c \sum_{l=0}^c e_{ij} x_{ij}^k \leq Q \quad k=1,2,\dots,K$$

$$r_{ij}^k \geq c_{m,i} z_{ij} + qt \quad k=1,2,\dots,K$$

$$r_{ij}^k + \sum_{l=1}^c \sum_{j=1}^c t_{lj} x_{lj}^k \leq REC(k) \quad k=1,2,\dots,K$$

$$REC(k) \leq du \quad k=1,2,\dots,K$$

(1) shows objective which is aimed to minimize the maximum total jobs delivery time for the customers by vehicle. Constraint (2) insures that the completion time of each job on machine 1 occurs no earlier than the duration of that job's processing time on machine 1. Constraint (3) insures that each job's completion time on machine r is not earlier than that job's completion time on machine r - 1 plus that job's processing time on machine r. Constraints (4) and (5) are the paired disjunctive constraints which insure that job i either precedes job ii or follows job k in the sequence, but not both.

Constraints (6) and (7) are common constraints for vehicle routing problem to start and end at depot. Constraints (8) are the flow conservation constraints for each node and ensure that circuits involving the depot are not included in the solution. Constraints (10) insures to subtour breaking. Constraints (9), (11), (12) insure to all k vehicles to service to all customer just one time. Constraints (13) guarantee that the maximum flow in any arc leaving the root is equal to Q. Constraints (14) shows ready time of each customers' jobs by the latest completion time on the machine of the jobs. Constraints (15) shows the time when vehicle finish delivery to customers in each tour. Constraints (15) satisfy that received time to customers must be less than due date.

#### 4-Case study

The PEGAH factory is one of the most immense producer of dairy products in IRAN, which produce various kinds of dairy products. In this research, milk production scheduling which are in kind of 1% and 2% fat and vehicle routing of this factory has been studied to be integrated in TEHRAN city. Yet, real-world problems are even more complex so the integrated problem is combinatorially complex and, therefore, as the size of the problem increases, it becomes harder and harder to obtain an exact solution for it in a reasonable amount of time so in order to find optimal schedule using the proposed model a smaller problem is considered which six random customers are selected in four-machine flow shop scheduling and two jobs which is delivered by two capacitated vehicles. job Processing time, travel time between customers and physical space of customers' job are given in Table 1. Quality time duration is 12 hours (720 minutes) that in this duration products must be stayed in warehouse. Customers' jobs must deliver until 8 AM to satisfy customers need, all time in model are in minutes. The proposed model for small-size example is as follow.

Minimize:

$$MAX(REC(K)) \tag{1}$$

$$co_{1,i} \geq p_{1,i} \quad 1 \leq i \leq 2$$

$$co_{r,i} - co_{r-1,i} \geq p_{r,i} \quad 2 \leq r \leq 4 \quad ; \quad 1 \leq i \leq 2$$

$$co_{r,i} - co_{r,g} + M(y_{i,g}) \geq p_{r,i} \quad 1 \leq r \leq 4 \quad ; \quad 1 \leq i < g \leq 2$$

$$co_{r,g} - co_{r,i} + M(1 - y_{i,g}) \geq p_{r,g} \quad 1 \leq r \leq 4 \quad ; \quad 1 \leq i < g \leq 2$$

$$\sum_{k=1}^K \sum_{l=1}^c x_{lj}^k = 1 \quad j=1,2,3,4,5,6$$

$$\sum_{k=1}^K \sum_{j=1}^c x_{lj}^k = 1 \quad l=1,2,3,4,5,6$$

$$\sum_{l=0}^c x_{lj}^k - \sum_{j=0}^c x_{lj}^k = 0 \quad k=1,2$$

$$\sum_{j=1}^c x_{0j}^k = 1 \quad k=1,2$$

$$u_j + 1 \leq u_l + c(1 - x_{jl}^k) \quad k=1,2 \quad ; \quad j=1,2,3,4,5,6 \quad ; \quad l=1,2,3,4,5,6$$

$$\sum_{k=1}^K \sum_{j=1}^c x_{0j}^k = K$$

$$\sum_{k=1}^K \sum_{l=1}^c x_{l0}^k = K$$

$$\sum_{i=1}^n \sum_{j=0}^c \sum_{l=0}^c e_{ij} x_{ij}^k \leq Q \quad k=1,2$$

$$r_{ij}^k \geq c_{m,i} z_{ij} + 720 \quad k=1,2$$

$$r_{ij}^k + \sum_{l=1}^c \sum_{j=1}^c t_{lj} x_{lj}^k \leq REC(k) \quad k=1,2$$

$$REC(k) \leq 1440 \quad k=1,2$$

Table 1.problem data

	$t_{ij}$							$P_{ir}$		$e_{ij}$	
plant	0	38	31	35	28	26	31			0	0
Customer1	31	0	12	9	21	22	20	42	13	45	0
Customer2	28	10	0	7	16	18	15	56	15	45	0
Customer3	33	6	9	0	21	23	21	33.6	14.4	135	45
Customer4	22	19	12	15	0	5	6	84	18	150	75
Customer5	19	19	13	16	5	0	10			60	60
Customer6	29	21	14	17	7	12	0			60	80

Figure 1 illustrates milk production process in production stage which two jobs are to be scheduled in 4-machine flow shop environment to produce milk.

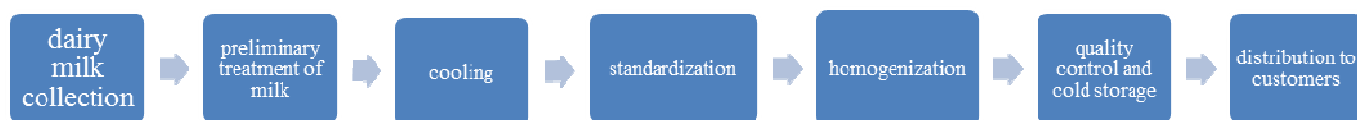


Figure 1: milk production process

The values of related variables of proposed mixed binary integer programming model that described, are solve by Lingo11 software are as follow, job1 completion time is 233.6 minutes and job2 completion time is 134 minutes. vehicle1 services customers in such an order, 0-4-6-0, vehicle2 services customers, 0-5-2-3-1-0 that 0 is plant location. The objective value is 1036.6 minutes.

### 5-Conclution and future research

This study investigate production scheduling and vehicle routing integration for jobs delivery to customers in different locations. A mixed binary integer programming model is developed to optimally solve this problem. Also, a small-size example is proposed which comes from PEGAH dairy product industry to solve the problem using the proposed model to get an optimal schedule. The PEGAH factory is one of the most immense producer of dairy products in IRAN, which produce various kinds of dairy products. In this research, production scheduling and vehicle routing of this factory has been studied to be integrated in TEHRAN city. Yet, real-world problems are even more complex so the integrated problem is combinatorially complex and, therefore, as the size of the problem increases, it becomes harder and harder to obtain an exact solution for it in a reasonable amount of time so in order to find an optimal schedule using the proposed model a smaller problem is considered which six random customers are selected in four-machine flow shop scheduling and two jobs which is delivered by two capacitated vehicles. This small-size example for proposed mixed binary integer programming model is solved by Lingo11 software to get an optimal schedule. Future research should address problems with different shop environments, including job shop. Considering multi-depot problems and also with other performance measures, including minimum mean arrival time, minimum mean flow time, mean tardiness, and multi-criteria measures, should also be studied. Metaheuristics could be used to achieve solutions for large scale problems.



## References

- 1- Stadtler, H. Supply chain management and advanced planning: basics, overview and challenges. *European Journal of Operational Research*. 2005; 163(3): 575-588.
- 2- Potts, C. N. Analysis of a heuristic for one machine sequencing with release dates and delivery times. *Operations Research*. 1980; 28 (6): 1436-1441.
- 3- Zdrzalka, S. Analysis of approximation algorithms for single-machine scheduling with delivery times and sequence independent batch setup times. *European Journal of Operational Research*. 1995; 80: 371-380.
- 4- Woeginger, G. J. Heuristics for parallel machine scheduling with delivery times. *Acta Informatica*. 1994; 31: 503-512.
- 5- Li, S., Yuan, J. Scheduling with families of jobs and delivery coordination under job availability. *Theoretical Computer Science*. 2009; 410: 4856-4863.
- 6- Woeginger, G. J. A polynomial-time approximation scheme for single-machine sequencing with delivery times and sequence-independent batch set-up times. *Journal of Scheduling*. 1998; 1: 79-87.
- 7- Gharbi, A., Haouari, M. Minimizing makespan on parallel machines subject to release dates and delivery times. *Journal of Scheduling*. 2002; 5 (4): 329-355.
- 8- Liu, Z., Cheng, T. C. E. Scheduling with job release dates, delivery times and preemption penalties. *Information Processing Letters*. 2002; 82: 107-111.
- 9- Mazdeh, M. M., Sarhadi, M., Hindi, K. S. A branch-and-bound algorithm for single-machine scheduling with batch delivery minimizing flow times and delivery costs. *European Journal of Operational Research*. 2007; 183: 74-86.
- 10- Selvarajah, E., Steiner, G. Approximation algorithms for the supplier's Supply Chain Scheduling problem to minimize delivery and inventory holding costs. *Operations Research*. 2009; 57 (2): 426-438.
- 11- Cheng, T. C. E., Kahlbacher, H. G. Scheduling with delivery and earliness penalties. *Asia-Pacific Journal of Operational Research*. 1993; 10: 145-152.
- 12- Lee, I. S., Yoon, S. H. Coordinated scheduling of production and delivery stages with stage-dependent inventory costs. *Omega*. 2010; 38: 509-521.
- 13- Averbakh, I. On-line integrated production-distribution scheduling problems with capacitated deliveries. *European Journal of Operational Research*. 2010; 200: 377-384.
- 14- Kise, H., Shioyama, T., Ibaraki, T., Automated two-machine flowshop scheduling: A solvable case, *IIE Transactions*. 1991 ; 23: 10-16.
- 15- Stevens, J.W., Germill, D.D. Scheduling a two-machine flowshop with travel times to minimize maximum lateness, *International Journal of Production Research*. 1997; 35: 1-15.
- 16- Chang, Y.C., Lee, C.Y. Machine scheduling with job delivery coordination. *European Journal of Operational Research*. 2004; 158: 470-487.

17-Liang, C.H., Zhou, H., Zhao, J. Integrated optimization approach for production-distribution planning in supply chain.2011;26(1):27-36.

18-Li, C.L., Vairaktarakis, G. L. Coordinating production and distribution of jobs with bundling operations. IIE Transactions.2007;39: 203–215.

19-Garcia, J. M., Lozano, S., Canca, D. Coordinated scheduling of production and delivery from multiple plants. Robotics and Computer-Integrated Manufacturing.2004;20:191–198.

20-Chen, Z.-L., Pundoor, G. Order assignment and scheduling in a supply chain. Operations Research.2006; 54 (3):555–572.

21-Liu, C.H., Wu, T.L., Lin, P.S. A hybrid genetic algorithm-based approach to solve parallel machine scheduling with job delivery coordination.Proceedings of the International MultiConference of Engineers and Computer Scientists. 2010;1674-1678

22-Liu, C.H., Leu, B.Y., Hsu, S.Y. Scheduling of parallel machines with job delivery coordination.International Journal of Innovative Computing, Information and Control.2012;8(1B):553-566.