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Fuzzy models for Transfer Point Location Problem

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Abstract
Recently, the transfer point location problem in which demand points have uniformly distributed coordinates and are weighted for the case of minimax objective and planar topology has been introduced. In the real situations, different points of an area can be considered with different possibility of occurrence. By such consideration, the model will be more applicable. In this paper, we propose a new TPLP where demand points are weighted and have possibilistic coordinates. The proposed model is formulated in fuzzy unconstraint linear programming which leads to construct the general model based on experts' viewpoints and the fuzzy decision is made using the optimization techniques. To reach the optimum or near optimum values for all decision variables a Fuzzy Logic Controller is developed. Finally, the application of the proposed model is demonstrated by using two numerical examples.

Keywords: Transfer Point Location Problem, Decision Support System, Fuzzy decision making, Possibilistic unconstraint linear programming, Fuzzy decision variables

1. Introduction

The Transfer Point Location Problem (TPLP) is definable as Berman et al. [1] said: Suppose we want to select locations for a new facility which will serve n demand points. In addition, we would be able to use transfer points, which play role as hub centers, to combine the service to some demand points. The unit cost of the traveling from such transfer point to the facility is reduced by a factor of α, which is limited to (0,1). Unit costs of the traveling from demand points to the transfer point are the same as that of traveling directly to the facility.

In the real world, many applications of TPLP would be considerable. The classic application of the TPLP, which is introduced by Berman et al [1], includes a hospital that accepts the injured via a helicopter. Then, injured are transferred to the transfer point, the heliport, by ambulance at the common speed and from there flown by helicopter to the hospital. The travel time from the transfer points to the hospital is shorter than that from the demand points to the transfer point. In this application, the location of the hospital is known, and the objective is to detect the location of the heliport. Other applications of TPLP are as follow: location of transfer point in disaster relief Logistic systems, location of transfer point for collection of agricultural crops, location of transfer point in military Logistic systems, location of transfer point in industrial product distribution systems and everywhere we deal with collection or distribution of some goods.

There are many related problems in the TPLP literature. The hub and spoke models are the most pertinent models to TPLP. Although Goldman [2] was the first paper addressing the network hub location problem, the research on hub location got started with pioneering studies of O'Kelly [3-5]. According to O'Kelly [5], Hub was central facility which acts as a switching point in network which connecting a set of interacting nodes. Continuous hub location problem was about locating hub facilities on a plane rather than on the nodes of a network. For more study on Continuous hub location

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problem one may refer to O'Kelly [5,6], Aykin [7,8], Campbell [9], O'Kelly and Miller [10], and Aykin and Brown [11].

In the collection depots problem, the objective is to locate the single facility in order to serve a set of demand nodes. Each service consists of a trip to the demand node, collecting materials, dropping the materials at one of the available collection depots and returning to the single facility. Drezner and Wesolowsky [12] first introduced the collection depots problem. Berman et al.[13] investigated Properties of solutions to minimax and minisum versions of the collection depots location problem on networks. As a natural extension of this problem Berman and Huang [14] and Tamir and Halman [15] proposed the multiple facilities location problem on a network.

Another field of the location problem that is related to TPLP is the round-trip location problem. In this model [16, 17], a service person starts at a facility, visits two, in TSLP more than two [18-21], demand points sequentially, then the service person returns to the facility. Recently, Chien-Chang Chou [22] proposed a new Fuzzy Multiple Criteria Decision Making Method (FMCDM) for solving the transshipment container port selection problem under fuzzy environment which play a role similar to transfer points in a transportation network.

Recently, transfer point location problem found an independent essence in the literature of the location problems. Hence, there is some literature related to models of the transfer point location problem. Berman et al. [1] introduced three models in the transfer point location problem and established properties and solution approaches to these three models. In their model, both planar and network variants, as well as the minisum and minimax objectives, were considered. In the first model, the Transfer Point Location Problem (TPLP), the set of demand points supposed to be given. Also, no weights were incorporated in the model. The problem was to find a location for the transfer point with respect to minimize the sum of (or maximum) distance to all the demand points through the transfer point. In the second model, the Multiple Location of Transfer Points (MLTP), it was needed to collect the points into subsets, each served by a single transfer point. In both two above models, the location of facility was supposed to be given. But, the third model, the Facility and Transfer Points Location Problem (FTPLP), was also extended to find the optimal location for the facility. Berman et al. [23] proposed the Multiple Transfer Points Location Problem (MTPLP) as a natural extension of TPLP, where the establishment of multiple transfer points was allowed and also the location of the single facility is known. They also investigated both minisum and minimax versions of the model in the plane. By investigating the minisum MTPLP and minimax MTPLP on a network, they demonstrated that above versions of the model can be regarded as a p+1-median problem and a p+1-center problem, respectively, with single given facility. They also mentioned above versions of the model can be regarded as a p+q-median problem and a p+q-center problem with q fixed facilities, respectively. Furthermore, Berman et al. [24] considered generalized models called Multiple Location of Transfer Points (MLTP) and Facility and Transfer Points Location Problem (FTPLP), which made the location of multiple facilities possible. They formulated the problem both in the plane and on a network with two minisum and minimax objectives. The location of facilities was given in MLTP, although an optimal location of both facilities and transfer points was needed to be found in FTPLP. Sasaki et al. [25] formulated the minisum MLTP and the minimax MLTP as a p-median problem and a p-center problem. They also proposed a new flow-based formulation for the minisum FTPLP and the minimax FTPLP. Respectively, using the proposed formulation they solved the minisum MLTP and also solved the minisum FTPLP with q=1 by an enumeration-based approach.

The research on one-center problem got started with pioneering works of Sylvester [26]. A nondeterministic extension of the problem using random weights of demands was first studied by Frank [27] on a network topology. Probabilistic weights in the one-dimensional location problem were proposed by Wesolowsky [28]. Berman et al. [29] have investigated a minimax stochastic (1-center) location problem in a plane. Averbakh and Bereg [30] studied interval data minimax regret 1-center location problems in the plane. In their models and for the case of rectilinear distances, uncertainty was incorporated in both weights and location coordinates of customers. While, for the case of Euclidean distances, uncertainty was only considered in weights. Foul [31] considered a one-center problem on the plane in which demand points were assumed to have a bivariate uniform distribution in a given rectangle. Berman et al. [1] only studied TPLP for cases in which demand points were not weighted and had known coordinates for the case of minimax and minisum objectives and planar topology. According to S. A. Hosseinijou and M. Bashiri [32] for the case of minimax objective and planar topology demand points were weighted and their coordinates had bivariate uniform distribution. Meanwhile, they only studied TPLP for the case in which demands were uniformly distributed, had a similar probability all over the coordinate. Also, they used expected value approach to formulate the problem.

In this paper, we propose a new TPLP where demand points are weighted and have possibilistic coordinates. We formulate the problem as an unconstrained nonlinear programming. By solving the formulated possibilistic model, optimum values for decision variables will be obtained in the form of fuzzy numbers. This means that based on the problem situation (parameters realization) several points could be optimum location for the transfer point with different degrees of occurrence possibility. Because of the complexity of the proposed model especially for practical persons such as managers, a special type of Decision Support Systems (DSS) called Fuzzy Logic Controller (FLC) is designed to reach the optimum or near optimum values for all decision variables without solving the original nonlinear programming problem directly. Other application of this approach was investigated by Sadjadi et al. [33] in the pricing problem.

This paper is organized as follows. In “section 2,” after introducing the deterministic minisum TPLP, the fuzzy minisum TPLP and the necessary notations and assumptions for fuzzy minisum TPLP are introduce. In “section 3,” the Fuzzy Logic Controller is explained. In “section 4,” two numerical examples are illustrated to demonstrate the implementation of our proposed model. Finally, in “section 5,” to epitomize the contribution of the paper the conclusion remarks are drawn.

2. The Minisum transfer point location problem model

2.1. Deterministic Minisum transfer point location problem (TPLP)

According to Berman et al. [1] the notation of TPLP is as follow:

n	number of demand points
α	factor multiplying by the travel to the transfer point
(x_0, y_0)	facility location
(x, y)	transfer point location
$d(x, y)$	distance between the transfer point and the facility
wi	weight associated with demand point i, i =1,..., n
D_i	distance between demand point i and the facility
$d_i(x, y)$	distance between demand point i, i =1,..., n, and the transfer point
d_{ij}	distance between demand points i and j

Except the (x_0, y_0) which is representing the location of facility, all terms of the above notation clearly is the same as that of Berman et al. [1]. The reason Berman et al. [1] established a condition for α is that the location of the transfer point must be unique, i.e., it is beneficial to have a transfer point and the locations of the transfer point and the facility do not coincide with each other. In this case, for the extreme point $\alpha = 1$ there is no need to have a transfer point and the locations of the transfer point and the facility coincide with each other. In this model, the set of demand points and the location of facility are given. Also, as depicted above, it is assumed that demand pints must use the transfer point which the location of it must be unique. The problem is to find a location of transfer point, (x, y), such that the sum of distance from the facility to all demand points through the transfer point, F(x, y), is minimized. Berman et al. [1] formulated the Minisum transfer point location problem as follows:

$$SUM = min_{x,y}\{\sum_{i=1}^{n} w_i d_i(x,y) + \alpha\, d(x,y)\} \quad (1)$$

Suppose distances are Euclidean. Since the Euclidean distances are convex and the sum of a set of convexs is convex, the objective function of the Minisum TPLP (Eq. 1) is convex and thus a local optimum is the global one. Fig. 1 shows geometric representation of TPLP problem.

It must be mentioned that there are some considerations about the model. First, each demand point is assigned its weight, i.e., each demand point has a unique weight representing its’ degree of

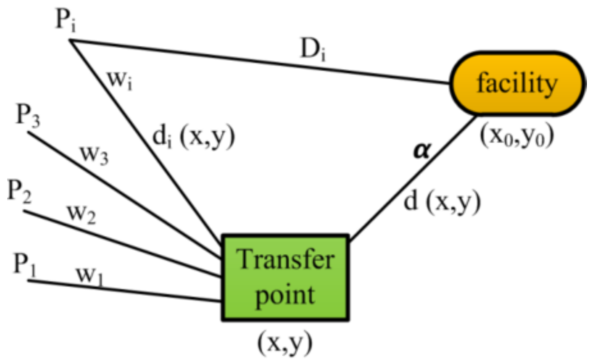

Figure 1. Geometric representation of basic minisum TPLP model

importance of the demand. Second, location of all demands are known, i.e., demand for some services is created at a set of points. Third, enough explanations have not been provided in Berman et al. [1] for justifying and supporting application of the model. Some of the above considerations make models less applicable in the real world conditions. For example, considering demand areas instead of the demand points is more realistic in the real world situation. The reason we consider the demand areas instead of demand points is because the locations of demands are not clearly known and also demand points have uncertainty in coordinates. Although S. A. Hosseinijou and M. Bashiri [32] considered the situation in which demands are generated in areas instead of the points in the Minimax TPLP model, they only considered demands with uniform distribution, i.e., have a similar (uniform) probability all over the coordinates. In fact, in the real world situation demands are not distributed identically in an area, i.e., different points of an area might have different possibilities of occurrence. By considering different possibilities of occurrence in the area, the model can be more applicable in the real world situation. In the next section, by considering this issue we developed the problem.

2.2. Fuzzy minisum transfer point location problem

A fuzzy TPLP model for the case of minisum objective is developed under following notations and assumptions:

Notations:

n	number of demand points
	factor multiplying by the travel to the transfer point
	Location of the facility
	Weight associated with demand point i, i =1,..., n
$\tilde{P}_i: (\tilde{x}_i, \tilde{y}_i)$	Transfer point location
	Possibilistic Coordinates of demand point i
	Distance between the transfer point and the facility
	Distance between demand point i and the facility
d_{ij}	Distance between demand point i, i =1,..., n, and the transfer point

Assumptions:

I: Coordinates of the demands point are uncertain and represented in the form of triangular fuzzy number $(\tilde{x}_i, \tilde{y}_i) = \left((x^L, x^M, x^R), (y^L, y^M, y^R)\right)$.

II: D^2 distance measure is used between the demand point and transfer point and between transfer point and the facility.

III: It is beneficial to have a transfer point and the locations of the transfer point and the facility do not coincide with each other, even at the extreme point $\alpha = 1$ it might be needed to have a transfer point in some special cases.

Geometric representation of the proposed general model is shown in Fig. 2. Each demand has a regional possibility distribution, and also each coordinate has a known possibility of occurrence of emergency services. Possibility distributions for $\tilde{x}_i$, $\tilde{y}_i$ may be different for various demand areas. Also, specific weight is assigned to each demand.

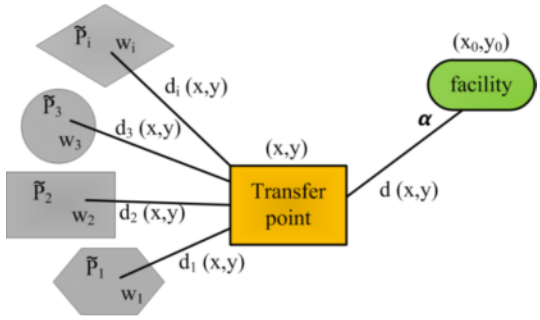

Figure 2. Geometric representation of Fuzzy minisum TPLP model

In this section we develop the fuzzy TPLP model. As said before, the coordinates of demands, i.e., $\tilde{P}_i = (\tilde{x}_i, \tilde{y}_i)$, are fuzzy in nature. Thus, the decision variable, i.e., (x, y), should be derived in the form of fuzzy sets, i.e., we are interested in deriving the possibility distributions of the minimum of total distance for the objective function, i.e., F, and the decision variable of the coordinates of the transfer point, i.e., (x, y), which are formulated as follows:

$$F = \min_{x,y}\left\{\sum_{i=1}^{n} w_i d_i^2(x,y) + \alpha\, d^2(x,y)\right\}$$

$$= \min_{x,y}\{\textstyle\sum_{i=1}^{n} w_i[(\tilde{x}_i - x)^2 + (\tilde{y}_i - y)^2] + \alpha[(x - x_0)^2 + (y - y_0)^2]\} \qquad (2)$$

$$\frac{\partial F}{\partial x} = 0 \ \text{ then } \ x^* = \frac{\alpha x_0 + \sum_{i=1}^{n} w_i \tilde{x}_i}{\alpha + \sum_{=1}^{n} w_i} \qquad (3)$$

$$\frac{\partial F}{\partial y} = 0 \ \text{ then } \ y^* = \frac{\alpha y_0 + \sum_{i=1}^{n} w_i \tilde{y}_i}{\alpha + \sum_{i=1}^{n} w_i} \qquad (4)$$

As said before, distances are Euclidian. Since the distances are convex functions and the sum of a set of convex functions is convex, the objective function of the Minisum TPLP (Eq. 1) is convex and thus a local optimum is the global one. So, the proposed problem is an unconstrained nonlinear convex optimization problem which can numerically be solved by using standard mathematical programming [34], but, solving this problem for real world problems could be a bit hard for practical persons such as managers. Hence, a new Fuzzy Logic Controller is designed to gain the optimum or near optimum values for the decision variables and the objective function. In the next section, the proposed FLC is illustrated.

3. Fuzzy Logic Controller

Special types of decision support systems (DSS) are fuzzy logic controllers which use of rules to model process in a simple way. In the FLCs, designer constructs rules that link the input variables with the outputs by terms of linguistic variables, Zimmerman (1996). In this study, Mamdani's controller is developed to construct a rule based to inference the decision variable (x, y), i.e., the transfer point location. The designed steps of the FLC include:

1. Assign linguistic terms to fuzzy input variables where input variables are coordinates parameters x, y. Parameters $\tilde{x}_i$ and $\tilde{y}_i$ are described with three and three linguistic terms, respectively. As an example, we only describe $\tilde{x}_1$ and $\tilde{y}_1$ as shown in Figs. 3 and 4.

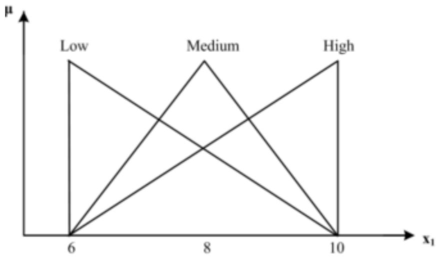

Figure 3. Linguistic variables x1

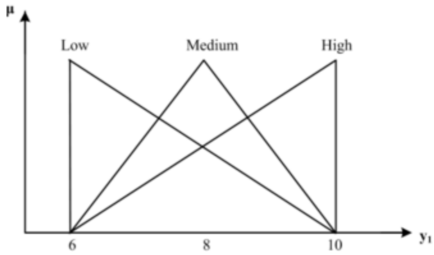

Figure 4. Linguistic variables y1

2. Calculate the fuzzy numbers for the outputs. In the traditional fuzzy controllers, for example Mamdani's controller, outputs are defined by a decision maker (DM) as the linguistic terms identical with the inputs and we do not have any contribution in estimating the outputs. However, in this work, by solving a possibilistic unconstrained nonlinear programming discussed in Section 2.2 for each set of fuzzy parameters $(\tilde{x}_i, \tilde{y}_i)\ \forall\, i = 1 \dots n$, the decision variable (x, y), i.e., transfer point location, is estimated in the form of fuzzy numbers.
3. Construct the rule base. As said in stage 2, in order to construct the rules, for each set of parameters $(\tilde{x}_i, \tilde{y}_i)\ \forall\, i = 1 \dots n$ a possibilistic unconstrained nonlinear programming must be solved and that the possibility distributions of the decision variable derived from the possibilistic unconstrained nonlinear programming is applied as the consequences. For example, for the case in which we have one fuzzy demand area, $(\tilde{x}_1, \tilde{y}_1)$ is considered as (low, medium) = ((6,6,10), (6,8,10)) and other parameters take their values with respect to example 1, then the decision variable (x, y) is assigned to this rule as ((8.17, 8.95, 8.95), (0.088, 0.090, 0.097). In this way, 9 rules of TPLP model are constructed as Table 1.

Table 1. Fuzzy control rule based of the first example

$\tilde{x}_1$	$\tilde{y}_1$	x	y	F
L	L	(14.55,14.55,15.38)	(11.4,11.4,12.23)	(103.08,124.76,124.99)
L	M	(14.55,14.55,15.38)	(11.95,12.49,13.05)	(95.84,113.91,119.3)
L	H	(14.55,14.55,15.38)	(12.78,13.59,13.6)	(91.64,105.05,11.68)
M	H	(15.10,15.65,16.20)	(12.78,13.59,13.6)	(79.2,87.28,102.53)
H	H	(15.93,16.74,16.75)	(12.78,13.59,13.6)	(71.53,71.53,89.72)
H	M	(15.93,16.74,16.75)	(11.95,12.49,13.05)	(75.74,80.39,97.33)
M	M	(15.10,15.65,16.20)	(11.95,12.49,13.05)	(89.41,96.14,110.14)
M	L	(15.10,15.65,16.20)	(11.4,11.4,12.23)	(90.65,106.99,115.83)
H	L	(15.93,16.94,16.75)	(11.4,11.4,12.23)	(82.97,106.99,115.83)

4. Defuzzification. Based on the fuzzy control rules of TPLP model shown in Table 1, defuzzification is applied to estimate the decision variable.

In the next section we provide three different numerical examples to show the application of the proposed model of this paper.

4. Numerical example

In this example, the Fuzzy Minisum TPLP problem is designed with one demand area. In this example, the coordinates of single demand have a possibility distribution described by linguistic terms, as shown in Figs. 3 and 4. The possibilistic unconstrained nonlinear programming presented in Eq. 5 is proposed to describe this problem.

$$\min_{x,y} F = w_1\left[\left((6,8,10) - x\right)^2 + \left((6,8,10) - y\right)^2\right] + \alpha[(x - x_0)^2 + (y - y_0)^2] \qquad (5)$$

Table 2, which consists of 16 distinct sub-examples, shows the optimal solution, (x^*, y^*), in column 5 and its objective function, $F(x^*, y^*)$, in column 6 and also actual solution, $F^*(x, y)$, in column 7 for different weights of demand, i.e., column 2, and the facility, i.e., column 3, when $(x_0, y_0) = (25, 18)$. Also, column 8 in table 2 shows the difference between values in column 6 and 7.

Fig. 5 plots optimal trajectory of transfer point, for different weights of demand and the facility when $(x_0, y_0) = (25, 18)$, with respect to the values of table 2, for this example. According to Fig. 5, for each set of the demand and the facility weights there is one optimal point shown in this figure. It must be mentioned that it is beneficial to have a transfer point, the locations of the transfer point and the facility should not coincide with each other even for the case in which the facility weight α is greater than the demand weight w1, except $\alpha = 1$ and w_1 is too small. This issue is because of the process of optimal solution of the problem depicted above.

Table 2. Optimal solution for example 1 when $(x_0, y_0) = (25, 18)$, $\tilde{P}_1 = ((6, 8, 10), (6, 8, 10))$

Sub e.g.	w	α	α/w	(x*,y*)	F(x*,y*)	F*(x, y)	Error
1	1	0.01	0.01	(8.17,8.36)	3.96	3.86	0.1
2	1	0.1	0.1	(9.55,9.15)	36.39	35.40	0.1
3	0.9	0.2	0.22	(11.1,10.04)	65.50	63.72	1.78
4	0.8	0.3	0.37	(12.64,10.92)	87.33	84.96	2.37
5	0.7	0.4	0.57	(14.18,11.81)	101.89	99.12	2.77
6	0.55	0.45	0.82	(15.65,12.65)	99.07	96.37	2.70
7	0.55	0.55	1	(16.5,13.3)	110.08	107.08	3.00
8	0.5	0.6	1.2	(17.23,13.58)	109.17	106.20	2.97
9	0.49	0.63	1.29	(17.56,13.74)	110.33	107.32	3.01
10	0.47	0.64	1.36	(17.8,13.88)	108.47	105.52	2.95
11	0.45	0.65	1.44	(18.05,14.02)	106.44	103.54	2.90
12	0.44	0.68	1.55	(18.32,14.18)	106.93	104.02	2.91
13	0.4	0.7	1.75	(18.82,14.46)	101.89	99.12	2.77
14	0.3	0.8	2.66	(20.36,15.35)	87.33	84.96	2.37
15	0.2	0.9	4.5	(21.91,16.23)	65.50	63.72	1.78
16	0.1	1	10	(23.46,17.12)	36.39	35.40	0.99

Scenario analysis. Fig. 6 shows the trend of the objective function for different sets of the demand and the facility weights, i.e., different values of $^{\alpha}/_{w_1}$, in table 2. According to the Fig. 6 before the ratio $^{\alpha}/_{w_1} = 1$, the objective function is ascending and after there the objective function is descending. This issue depicts that the worst case of the objective function is occurred when $\alpha = w_1$, means the location of transfer point is exactly between the demand area and facility location. In the real world situations, by changing the factors which affects the weights, i.e., w and α, in a way that make the ratio $^{\alpha}/_{w_1}$ more or less than that value, the objective value will decrease. In this way, we have two strategies. First, making the ratio large enough such that the location of transfer point and the facility are the same, i.e., there is no need for a transfer point. Second, if there is necessity to have a transfer point, decrease the ratio in order to move the transfer point to the center of demand area (see Fig. 5).

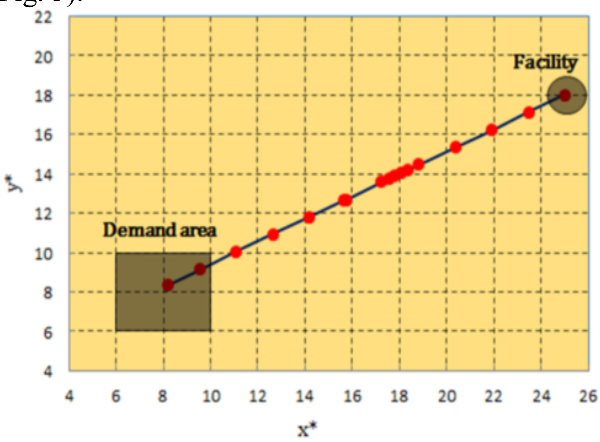

Figure 5. Location of transfer point for values of $^{\alpha}/_{w}$ when $(x_0, y_0) = (25, 18)$

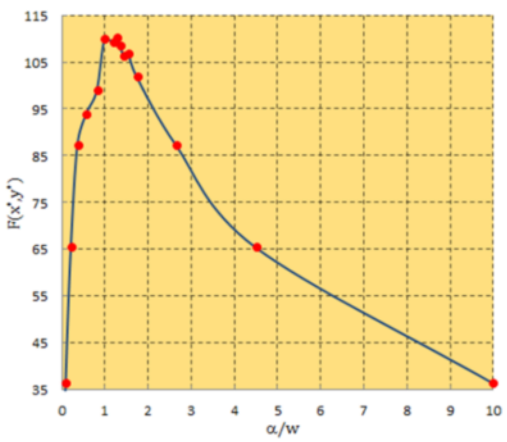

Figure 6. Trend of the objective function for different values of $^{\alpha}/_{w}$

Model validation. the reason model validation is essential to confirm that the derived transfer point location, i.e., (x^*, y^*), from our model really minimizes the value of the objective function for a fuzzy demand within the defined area is because demand pattern is fuzzy in our problem and we applied a Fuzzy Logic Controller (FLC) to model this uncertain behavior. This can be performed by simulating several demands exist in the demand area and derive the optimal location of transfer point, consequently, and then comparing the developed FLC results with actual results computed by Eq. 6.

$$F_{actual}(x,y) = w_1[(x_1 - x)^2 + (y_1 - y)^2] + \alpha[(x - x_0)^2 + (y - y_0)^2] \quad (6)$$

Result of validation of the example 1 for sixth sub-example in table 2 when $\alpha = 0.45$, $w = 0.55$ and the optimal recommended location of the transfer point derived from our model solution, $(x^*, y^*) = (15.65, 12.56)$, are shown in table 3.

Table 3. Result of validation of example 1 when $\alpha = 0.45$, $w = 0.55$ and $(x^*, y^*) = (15.65, 12.56)$

Scenario No.	$(\tilde{x}, \tilde{y})$	F(x*,y*)	F*(x, y)	Error
1	(7.48,8.26)	91.19	99.45	8.26
2	(6.50,9.05)	116.06	104.53	11.52
3	(9.24,6.87)	78.89	92.13	13.24
4	(8.01,7.28)	97.06	99.89	2.82
5	(10,00,8.24)	81.10	79.26	1.83
6	(6.01,7.50)	114.78	116.54	1.76
7	(8.88,6.85)	87.54	95.08	7.55
8	(9.67.7.86)	80.14	83.61	3.47
9	(7.78,9.06)	94.18	93.17	1.01
10	(8.40,6.99)	101.96	98.20	3.76
11	(6.67,8.25)	116.50	106.69	9.81
12	(9.25,9.67)	78.92	78.57	0.35
13	(8.64,9.45)	99.25	84.34	14.91
14	(7.01,7.62)	116.41	106.77	9.64
15	(6.98,8.00)	117.30	105.12	12.19

15 different scenarios are generated in defined demand coordinates, which have possibility distribution in nature. Column 2 in table 3 depicts the location of demand in each scenario. Column 3 depicts the value of the objective function ($F(x^*, y^*)$) for the optimal recommended location of the transfer point. Column 4 shows the value of the objective function ($F^*(x, y)$) for the actual, or optimal, location of transfer point derived from actual solution plotted in Fig. 7. Also, column 5 shows the difference between values in column 3 and 4. Actually, column 5 is our model error. Figure 7 plots contour lines for sub-example 6 of table 2. In this figure, the optimal recommended location of the transfer point derived from our model solution is pictured by a black star shape and the actual solution is pictured by a white star shape. Thereupon, the reason one can conclude that the model solution is near optimal is because the contour of model solution is eminently close to the contour of actual solution.

Fig.8 shows the surface of objective function for our model solution for sixth sub-example in table 2 when the ration $\frac{\alpha}{w} = \frac{0.55}{0.45} = 0.82$. It is clear from the surface that the behavior of objective function when moving to the optimal recommended location of the transfer point ($(x^*, y^*) = (15.65, 12.56)$) is descending and the optimal location of the transfer point has the minimum objective function.

Table 4 represents the result of our analysis of table 3. The first row of this table indicates the average minimum cost for model solution which is an average value of column 3 of table 3. The second row indicates the average minimum cost for optimal solution which is an average value of column 4 of table 3. The third row is absolute average error of two prior rows. The fourth row indicates the relative average error. This row is suitable for validation of the model. But, to make the validation of the model more robust fifth row is presented to show the average of relative error for 100 different scenarios.

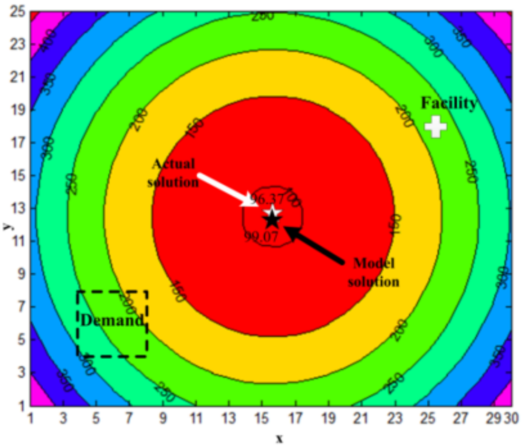

Figure 7. Contour lines for validation of example 1 when $\alpha = 0.2$

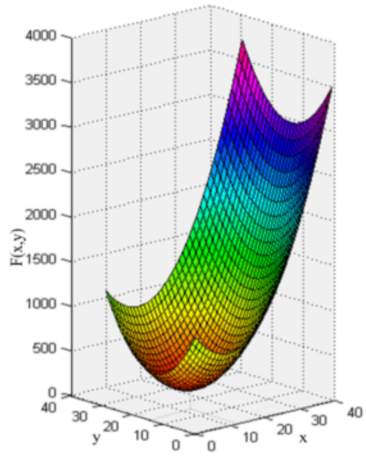

Figure 8. Surface of the objective function for the example 1 for $\frac{\alpha}{w} = \frac{0.55}{0.45}$

Table 4. Analysis of validation results of example 1 when $\alpha = 0.45$, $w = 0.55$ and $(x^*, y^*) = (15.65, 12.56)$

Average minimum cost for model solution $\overline{F(x^*, y^*)}$	98.09
Average minimum cost for optimal solution $\overline{F^*(x, y)}$	96.29
Absolute average error $\overline{F(x^*, y^*)} - \overline{F^*(x, y)}$	1.86
Relative average error $(\overline{F(x^*, y^*)} - \overline{F^*(x, y)}) / \overline{F^*(x, y)}$	1.94%
Average of relative error (for 100 scenarios) $\sum_{i=1}^{100}((F_i(x^*, y^*) - F_i^*(x, y)) / F_i^*(x, y)) / 100$	3.83%

There is a small difference between $\overline{F(x^*, y^*)} = 98.09$ and $\overline{F^*(x, y)} = 96.29$ that is 1.86. One can come to the conclusion that the small relative average error 1.94% is a good enough reason for the validation of the model. But, to prove the robust validation of the model the value of fifth row, 3.83%, did it. On the other hand, to demonstrate how much the value of the average of relative error for 100 different scenarios, i.e., 3.83% is robust and to show the pattern of the dispersion of the errors, we provide the cdf curve of average of relative error for 100 scenarios shown in Fig. 9. This figure depicts that about 80% of the errors are less than 10% and over 50% are less than 6% and also the dispersion of the errors before error 5% is much greater than after that. It is so clear from the above issues that our model solution does well and the result of it is close to the actual solution. Thus, we can conclude that our model solution is near optimal with respect to fuzzy nature of the problem. It must be mentioned that so many scenarios were studied to come to these results.

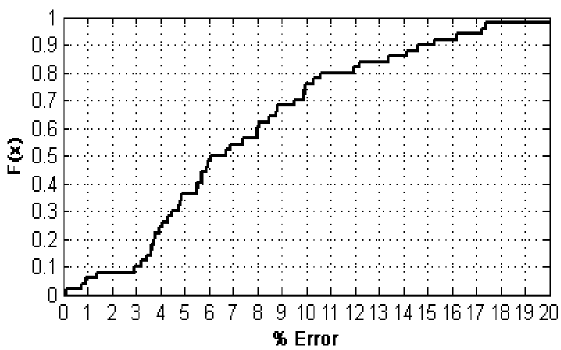

Figure 9. Cdf curve of average of relative error when $\alpha = 0.45$, $w = 0.55$

5. Conclusion

In this paper, a new TPLP where demand points are weighted and have possibilistic coordinates has been proposed. The model has been formulated as a possibilistic unconstraint linear programming which leads to construct the general model based on experts' viewpoints and the fuzzy decision has been made using the optimization techniques. Also, a new developed Fuzzy Logic Controller has been developed. By solving the new model with developed Fuzzy Logic Controller, various optimal solutions with different possibility degree have been obtained. We can summarize novel contributions of this paper as below:

	In TPLP
Berman et al. (2007)	This work
• Deterministic coordinates	• Fuzzy coordinates
• Demands are not weighted	• Demands are weighted
• No conceptual explanation	• Conceptual justifications
	In FLC model
Mamdani's Inference System	This work
• Consequences are asked by expert	• Consequences are derived based on Optimum knowledge

To demonstrate the applicability of the proposed model two numerical examples have been illustrated with respect to fuzzy nature of the problem.

More developments in this field are possible. one can study the problem with the probability distributions of the demand weights and coordinated. Network version of fuzzy minisum TPLP can be considered as other extension of presented model. Also, it is possible to study a fuzzy version of the multiple transfer points location problem.

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