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Structure formation and GSL in some viable $f(R)$-gravity models

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Here, we investigate the growth of matter density perturbations as well as the generalized second law (GSL) of thermodynamics in the framework of $f(R)$-gravity. We consider a spatially flat FRW universe filled with the pressureless matter and radiation which is enclosed by the dynamical apparent horizon with the Hawking temperature. For some viable $f(R)$ models containing the Starobinsky, Hu-Sawicki, Exponential, Tsujikawa, and AB models, we first explore numerically the evolution of some cosmological parameters like the Hubble parameter, the Ricci scalar, the deceleration parameter, the density parameters and the effective equation of state parameter. Then, we examine the validity of GSL and obtain the growth factor of structure formation. We find that for the aforementioned models, the GSL is satisfied from the early times to the present epoch. But in the future, the GSL for the all models but the Hu-Sawicki and AB models, is violated in some ranges of redshift. Our numerical results also show that for the all models the growth factor for larger structures like the ΛCDM model fit the data very well.

I. $F(R)$-GRAVITY FRAMEWORK

One of the representative approaches to explain the current acceleration of the universe is to consider a theory of modified gravity (MG), such as $f(R)$ gravity, in which the Einstein-Hilbert action in GR is generalized from the Ricci scalar R to an arbitrary function of the Ricci scalar [1]. A $f(R)$ model with negative and positive powers of Ricci curvature scalar R naturally admits the unification of the inflation at early times and the cosmic acceleration at late times [2]. It can also serve as dark matter (DM), [3]. The modified Einstein-Hilbert action in the Jordan frame is given by [1]

$$S_{\rm J}=\int\sqrt{-g}\,{\rm d}^4x\left[\frac{f(R)}{16\pi G}+L_{\rm matter}\right], \tag{1}$$

where G, g, R and $L_{\rm matter}$ are the gravitational constant, the determinant of the metric $g_{\mu\nu}$, the Ricci scalar and the lagrangian density of the matter inside the universe, respectively.

For a spatially flat FRW metric, taking $T_{\nu}^{\mu(m)}={\rm diag}(-\rho,p,p,p)$ in the prefect fluid form, the Friedmann equations in $f(R)$-gravity are given by [4]

$$3H^2=8\pi G(\rho+\rho_{\rm D}), \tag{2}$$

$$2\dot{H}=-8\pi G(\rho+\rho_{\rm D}+p+p_{\rm D}), \tag{3}$$

where

$$8\pi G\rho_{\rm D}=\frac{1}{2}\big(RF-f\big)-3H\dot{F}+3H^2\big(1-F\big), \tag{4}$$

$$8\pi Gp_{\rm D}=\frac{-1}{2}\big(RF-f\big)+\ddot{F}+2H\dot{F}-(1-F)\big(2\dot{H}+3H^2\big), \tag{5}$$

with

$$R=6(\dot{H}+2H^2). \tag{6}$$

Here $H=\dot{a}/a$ is the Hubble parameter. Also $\rho_{\rm D}$ and $p_{\rm D}$ are the curvature contribution to the energy density and pressure which can play the role of DE. Also $\rho=\rho_{\rm BM}+\rho_{\rm DM}+\rho_{\rm rad}$ and $p=p_{\rm rad}=\rho_{\rm rad}/3$ are the energy density and pressure of the matter inside the universe. The energy conservation laws are established for the pressureless matter, $\rho_{\rm m}=\rho_{\rm BM}+\rho_{\rm DM}$, radiation, $\rho_{\rm rad}$ and DE, $\rho_{\rm D}$. On the whole of the paper, the dot and the subscript R denote the derivatives with respect to the cosmic time t and the Ricci scalar R, respectively.

II. GROWTH RATE OF MATTER DENSITY PERTURBATIONS

The evolution of the matter density contrast $\delta_{\rm m}=\delta\rho_{\rm m}/\rho_{\rm m}$ provides an important tool to distinguish $f(R)$-gravity and generally MG models from DE inside GR and, in particular, from the ΛCDM model.

The linear evolutions of matter density contrast, in a flat FRW background is govern by [5,6]

$$\ddot{\delta}_{\rm m}+2H\dot{\delta}_{\rm m}-4\pi G_{\rm eff}\rho_{\rm m}\delta_{\rm m}=0, \tag{7}$$

where

$$G_{\rm eff}=\frac{G}{F}\left[\frac{4}{3}-\frac{1}{3}\frac{M^2a^2}{k^2+M^2a^2}\right], \tag{8}$$

and $M^2=\frac{F}{3F_{\rm R}}$. Equation (8) obviously shows that the screened mass function, i.e. $G_{\rm eff}/G$, is the time and scale dependent parameter. In the present work, we obtain the evolution of linear perturbations relevant to the matter spectrum for the scales; $k = 0.1, 0.01, 0.001\ h\ \mathrm{Mpc}^{-1}$, where h corresponds to the Hubble parameter today. For smaller scales, $k > 0.2\ h\ \mathrm{Mpc}^{-1}$, the effect of non-linearity becomes important, which is out of the scope of this paper.

III. GENERALIZED SECOND LAW OF THERMODYNAMICS

According to the GSL, entropy of the matter inside the horizon beside the entropy associated with the surface of horizon should not decrease during the time [7]. Karami et al. [8] showed that within the framework of $f(R)$-gravity, the GSL for a spatially flat FRW universe enclosed by the dynamical apparent horizon and containing the pressureless baryonic and dark matters as well as the radiation is given by

$$T_{\mathrm{A}}\dot{S}_{\mathrm{tot}} = \frac{1}{4GH^4}\left[2\dot{H}^2F - \dot{H}H\dot{F} + 2(\dot{H}+H^2)\ddot{F}\right], \quad (9)$$

where $S_{\mathrm{tot}} = S_{\mathrm{m}} + S_{\mathrm{A}}$ is the total entropy due to different contributions of the matter and the horizon. Here $S_{\mathrm{A}} = \frac{AF}{4G}$ is the geometric entropy of the horizon in $f(R)$-gravity, where $A = 4\pi\tilde{r}_{\mathrm{A}}^2$ and $\tilde{r}_{\mathrm{A}}$ is the dynamical apparent horizon which is same as the Hubble horizon for a flat FRW universe, i.e. $\tilde{r}_{\mathrm{A}} = H^{-1}$. Also $T_{\mathrm{A}} = \frac{1}{2\pi\tilde{r}_{\mathrm{A}}}\left(1-\frac{\dot{\tilde{r}}_{\mathrm{A}}}{2H\tilde{r}_{\mathrm{A}}}\right)$ is the Hawking temperature on the apparent horizon. Note that Eq. (9) shows that the validity of the GSL, i.e. $T_{\mathrm{A}}\dot{S}_{\mathrm{tot}} \geq 0$, depends on the $f(R)$-gravity model. In subsequent sections we examine the validity of the GSL for some viable $f(R)$ models.

IV. COSMOLOGICAL EVOLUTION

To obtain the evolutionary behavior for $f(R)$ models we need to solve the following equation [9]

$$(1+z)^2 y_{\mathrm{H}}'' + J_1(1+z)y_{\mathrm{H}}' + J_2 y_{\mathrm{H}} + J_3 = 0, \quad (10)$$

where

$$J_1 = -3 - \left(\frac{1-\bar{F}}{6\bar{H}^2\bar{F}_{\mathrm{R}}}\right), \quad (11)$$

$$J_2 = \frac{2-\bar{F}}{3\bar{H}^2\bar{F}_{\mathrm{R}}}, \quad (12)$$

$$J_3 = -3(1+z)^3 - \frac{1}{6\bar{H}^2\bar{F}_{\mathrm{R}}} \times \left[(1-\bar{F})\Big((1+z)^3 + 2\chi(1+z)^4\Big) + \frac{1}{3\Omega_{\mathrm{m}_0}}(\bar{R}-\bar{f})\right], \quad (13)$$

$$y_{\mathrm{H}} := \frac{\rho_{\mathrm{D}}}{\rho_{\mathrm{m}_0}} = \frac{\bar{H}^2}{\Omega_{\mathrm{m}_0}} - (1+z)^3 - \chi(1+z)^4. \quad (14)$$

We are intersected in investigating the growth of structure formation and examining the GSL in $f(R)$-gravity, hence in what follows we consider some viable $f(R)$ models including the Starobinsky [5], Hu-Sawicki [10], Exponential [9], Tsujikawa [11] and AB [12] models, Eqs. (15)-(19), respectively.

$$f(R) = R + \lambda R_{\mathrm{s}}\left[\left(1+\frac{R^2}{R_{\mathrm{s}}^2}\right)^{-n} - 1\right], \quad (15)$$

$$f(R) = R - \frac{c_1 R_{\mathrm{s}}\left(\frac{R}{R_{\mathrm{s}}}\right)^n}{c_2\left(\frac{R}{R_{\mathrm{s}}}\right)^n + 1}, \quad (16)$$

$$f(R) = R - \beta R_{\mathrm{s}}\left(1 - e^{-\frac{R}{R_{\mathrm{s}}}}\right), \quad (17)$$

$$f(R) = R - \lambda R_{\mathrm{s}} \tanh\left(\frac{R}{R_{\mathrm{s}}}\right), \quad (18)$$

$$f(R) = \frac{R}{2} + \frac{\epsilon}{2}\log\left(\frac{\cosh(\frac{R}{\epsilon} - b)}{\cosh(b)}\right). \quad (19)$$

In the next section, we only present the results and figures obtained for AB model. The overall results obtained for the rest of models are illustrated in section VI.

V. NUMERICAL RESULTS

With the help of numerical results obtained for $y_{\mathrm{H}}(z)$ in Eq. (10), we can obtain the evolutionary behaviors of H, $\omega_{\mathrm{eff}} = -1 - \frac{2\dot{H}}{3H^2}$, q, Ω_{m}, Ω_{D} and GSL for our selected $f(R)$ models. The results for the AB $f(R)$ model are displayed in Figs. 1-5. Figures show that: (i) the Hubble parameter decreases during history of the universe. (ii) The effective EoS parameter, ω_{eff}, starts from an early matter-dominated regime (i.e. $\omega_{\mathrm{eff}} = 0$) and in the late time, $z \to -1$, it behaves like the ΛCDM model. (iii) The deceleration parameter q varies from an early matter-dominant epoch ($q = 0.5$) to the de Sitter era ($q = -1$) in the future, as expected. It also shows a transition from a cosmic deceleration $q > 0$ to the acceleration $q < 0$ in the near past. The current value of the deceleration parameter is obtained as -0.6 which is in good agreement with the recent observational constraint $q_0 = -0.43^{+0.13}_{-0.17}$ (68% CL) obtained by the cosmography [13]. (iv) The density parameters Ω_{D} and Ω_{m} increases and decreases, respectively, as z decreases. (v) The GSL is always satisfied from early times to the late cosmological history of the universe.

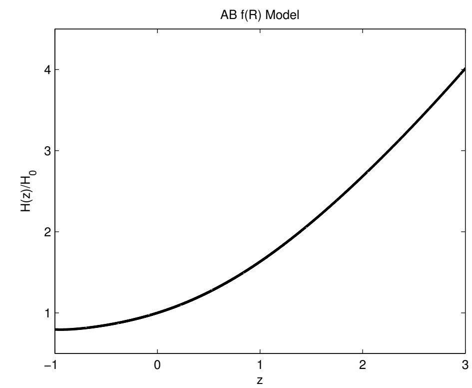

FIG. 1. The variations of the Hubble parameter H versus redshift z. Auxiliary parameters are $\Omega_{m_0} = 0.24$, $\Omega_{D_0} = 0.76$, $\Omega_{rad_0} = 4.1 \times 10^{-5}$ and $b = 1.4$, $\epsilon = R_s/(b + \log(2\cosh b))$.

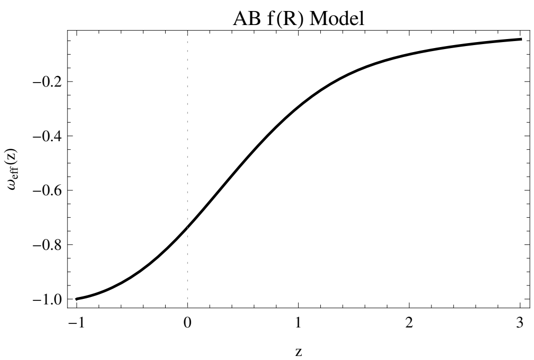

FIG. 2. The variations of the $\omega_{\rm eff}$ versus redshift z. Auxiliary parameters are as in Fig. 1.

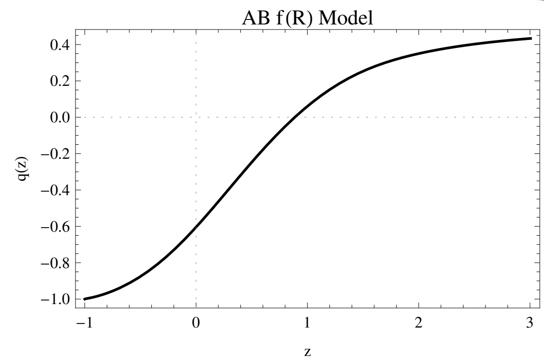

FIG. 3. The variations of the deceleration parameter q versus redshift z. Auxiliary parameters are as in Fig. 1.

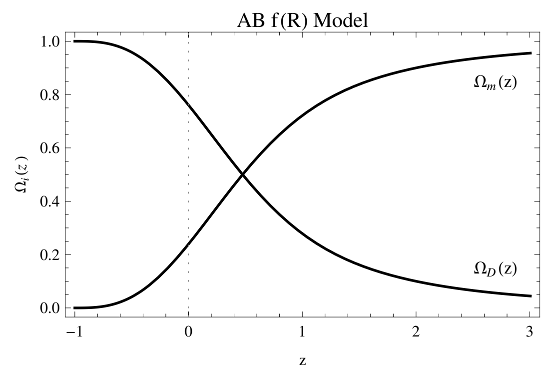

FIG. 4. The variations of the density parameter Ω_i versus redshift z. Auxiliary parameters are as in Fig. 1.

In Figs. 6-8, we plot the evolutions of growth factor f, g and $G_{\rm eff}/G$, versus z for the AB $f(R)$ model. Figures show that: (i) The evolution of the growth factor $f(z)$ for this model and ΛCDM model together with the 11 observational data of the growth factor, show that for smaller structures (larger k), the $f(R)$ model deviates from the observational data. But for larger structures (smaller k), the growth factor very similar to the ΛCDM model, fits the data very well. (ii) The linear density contrast relative to its value in a pure matter model $g = \delta/a$ starts from an early matter-dominated phase, i.e. $g \simeq 1$ and decreases during history of the universe. For a given z, g in the AB $f(R)$ model, is greater than that in the ΛCDM model. (iii) The screened mass function $G_{\rm eff}/G$ for a given wavenumber k is larger than one which makes a faster growth of the structures compared to the GR. However, for the higher redshifts, the screened mass function approaches to unity in which the GR structure formation is recovered. Note that the deviation of $G_{\rm eff}/G$ from unity for small scale structures (larger k) is greater than large scale structures (smaller k).

VI. CONCLUSIONS

Here, we investigated the evolution of both matter density fluctuations and GSL in some viable $f(R)$ models containing the Starobinsky, Hu-Sawicki, Exponential, Tsujikawa and AB models. Our results show the following.

(i) All of the selected $f(R)$ models can give rise to a late time accelerated expansion phase of the universe. The deceleration parameter for the all models shows a cosmic deceleration $q > 0$ to acceleration $q < 0$ transition. The present value of the deceleration parameter takes place in the observational range. Also at late times ($z \to -1$), it approaches a de Sitter regime (i.e. $q \to -1$), as expected.

(ii) The effective EoS parameter $\omega_{\rm eff}$ for the all models starts from the matter dominated era, $\omega_{\rm eff} \simeq 0$, and in the late time, $z \to -1$, it behaves like the ΛCDM model, $\omega_{\rm eff} \to -1$.

(iii) The GSL is respected from the early times to the present epoch. But in the future, the GSL for the all models but the Hu-Sawicki and the AB models, is violated in some ranges of redshift.

(iv) For the all models, the screened mass function $G_{\rm eff}/G$ is larger than 1 and in high z regime goes to 1. The deviation of $G_{\rm eff}/G$ from unity for larger k (smaller

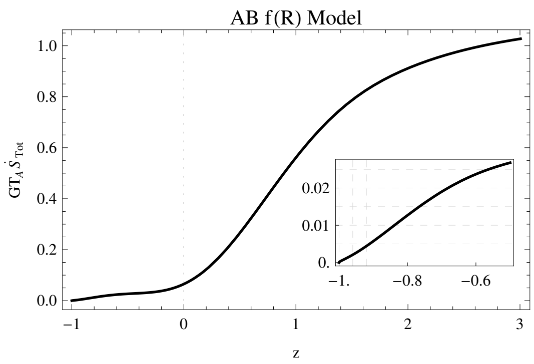

FIG. 5. The variations of the GSL versus redshift z. Auxiliary parameters are as in Fig. 1.

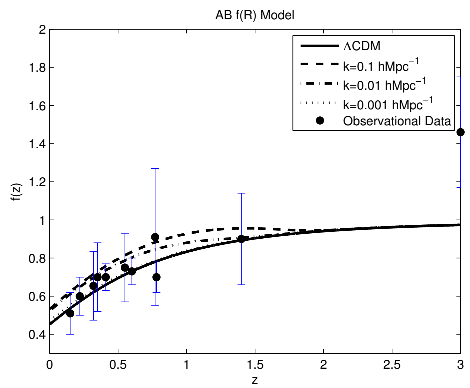

FIG. 6. The variations of the growth factor $f(z)$ versus redshift z. Auxiliary parameters are as in Fig. 1.

structures) is greater than the smaller k (larger structures). The modification of GR in the framework of $f(R)$-gravity, gives rise to an effective gravitational constant, $G_{\rm eff}$, which is time and scale dependent parameter in contrast to the Newtonian gravitational constant.

(v) The linear density contrast relative to its value in a pure matter model, $g(a) = \delta_{\rm m}/a$, for the all models starts from an early matter-dominated phase, $g(a) = 1$, and decreases during history of the universe.

(vi) The evolutionary behavior of the growth factor of linear matter density perturbations, $f(z)$, shows that for the all models the growth factor for smaller k (larger structures) like the ΛCDM model fit the data very well.

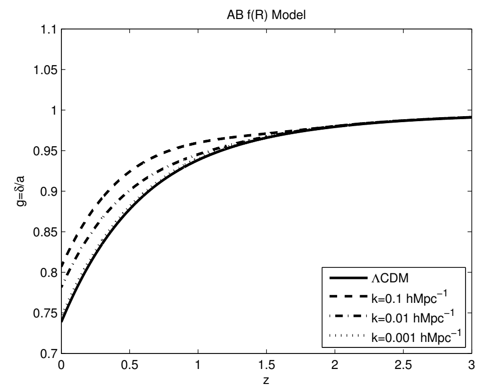

FIG. 7. The variations of the linear density contrast relative to its value in a pure matter model $g = \delta/a$ versus redshift z. Auxiliary parameters are as in Fig. 1.

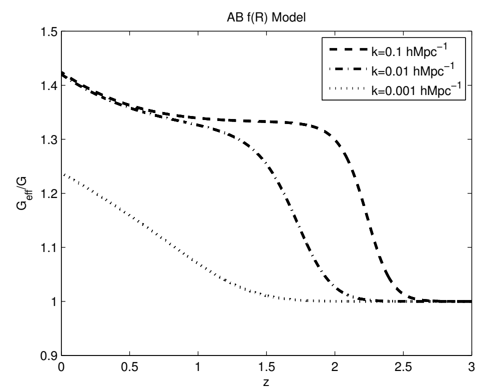

FIG. 8. The variations of the screened mass function $G_{\rm eff}/G$, versus redshift z. Auxiliary parameters are as in Fig. 1.

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