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The effect of inclination angle of the coronal loop plane on the resonant absorption of kink waves

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Here, we investigate the effect of inclination angle on the resonant absorption of standing fast kink body waves in the solar coronal loops. To this aim, we consider a typical coronal loop as a straight, zero-β, nonaxisymmetric and longitudinally stratified cylindrical magnetic flux tube. With the help of connection formulae, we derived and solved numerically the dispersion relation governing the quasi normal kink modes. Consequently, we obtained both the frequencies and damping rates of the fundamental and first-overtone kink modes. We concluded that as the inclination angle of the loop plane increases: (i) the frequencies and their relevant damping rates decrease. (ii) The frequency ratio ω_2/ω_1 of the first overtone and its fundamental mode increases. (iii) The ratio of the oscillation frequency to the damping rate remains unchanged.

I. INTRODUCTION

Since the first observation of transverse oscillations of coronal loops by Nakariakov *et al.* [1] many studied have been made to explain and interpret such decaying oscillations. Verwichte *et al.* [2] observed two values 1.81 ± 0.25 and 1.64 ± 0.23 for the period ratio P_1/P_2 in different loops.

Karami, Nasiri & Amiri [3] showed that for both kink ($m = 1$) and fluting ($m = 2$) modes, in the presence of longitudinal density stratification one sees that the frequencies ratio of resonantly damping oscillations is less than 2 which justifies the observations. On the other hand, as the stratification parameter increases both the frequencies and their relevant damping rates increase. But the ratio of any frequency to its relevant damping rate does not experience any change with changing the stratification parameter.

Karami *et al.* [4] studied the effect of an elliptic shape and its stage of emergence on the resonantly damped oscillations of stratified coronal loops. Their results indicated that both the elliptical shape and stage of emergence of the loop alter the kink frequencies and damping rates of the tube as well as the ratio of frequencies of the fundamental and its first-overtone modes. Their obtained results were in agreement with the findings of Morton & Erdélyi [5] for the period ratio P_1/P_2.

One of the geometrical aspects of coronal loops that can plays role in interpreting coronal and the loops seismology, is the inclination of the loop plane from a plane normal to the photosphere. Aschwanden *et al.* [6] investigated seven different kink oscillations event of AR 8270 and estimated some geometrical characteristics of the loops including the inclination angle.

In the present paper we consider an inclination angle θ for the loop plane and would like to demonstrate how it affects the resonant absorption in a longitudinally stratified loop.

II. EQUATIONS OF MOTION AND MODELING OF THE FLUX TUBE

The linearized MHD equations for a zero-β plasma are as follows

$$\frac{\partial \delta\mathbf{v}}{\partial t} = \frac{1}{4\pi\rho}\{(\nabla\times\delta\mathbf{B})\times\mathbf{B} + (\nabla\times\mathbf{B})\times\delta\mathbf{B}\} + \frac{\eta}{\rho}\nabla^2\delta\mathbf{v}, \qquad (1)$$

$$\frac{\partial \delta\mathbf{B}}{\partial t} = \nabla\times(\delta\mathbf{v}\times\mathbf{B}) + \frac{\mathbf{c^2}}{\mathbf{4\pi\sigma}}\nabla^2\delta\mathbf{B}, \qquad (2)$$

with $\delta\mathbf{v}$ and $\delta\mathbf{B}$ being the Eulerian perturbations of velocity and magnetic fields. Also $\mathbf{B}$, ρ, σ, η and c are the constant background magnetic filed, the mass density, the electrical conductivity, the viscosity and the speed of light, respectively.

We make some simplifying assumptions the same as the paper of Karami & Asvar [7]. The perturbed quantities $\delta\mathbf{v}$ and $\delta\mathbf{B}$ can be expanded as [3,8]:

$$\delta\mathbf{B}(\mathbf{r},\mathbf{z}) = \sum_{\mathbf{k=1}}^{\infty}\delta\mathbf{B}^{(\mathbf{k})}(\mathbf{r})\psi^{(\mathbf{k})}(\mathbf{z}),$$
$$\delta\mathbf{v}(\mathbf{r},\mathbf{z}) = \sum_{\mathbf{k=1}}^{\infty}\delta\mathbf{v}^{(\mathbf{k})}(\mathbf{r})\psi^{(\mathbf{k})}(\mathbf{z}), \qquad (3)$$

where $\psi^{(k)}(z)$s form a complete set of orthonormal eigenfunctions of Alfvén operator L_A, and satisfy the relation $L_A\psi^{(k)} = \eta_k\psi^{(k)}$ [3,8]. One can write the density function as $\rho(r,z) = \rho_0(r)\rho(z)$. We assume $\rho_0(r)$ to be as that of [4], i.e.,

$$\rho_0(r) = \begin{cases} \rho_{\rm in}, & (r < R_1), \\ \left[\frac{\rho_{\rm in}-\rho_{\rm ex}}{R-R_1}\right](R-r) + \rho_{\rm ex}, & (R_1 < r < R), \\ \rho_{\rm ex}, & (r < R), \end{cases} \qquad (4)$$

where R is the loop radius and $R - R_1$ indicates the width of the inhomogeneous layer l. For the longitudinal direction z, we choose [9]

$$\rho(z) = \exp\Big[-\mu\cos(\theta)\sin\big(\frac{\pi z}{L}\big)\Big]. \quad (5)$$

Here the stratification parameter μ is defined as $\mu := \frac{L}{\pi H}$, with L and H being the length of the loop and the density scale height respectively. Solving Eqs. (1) and (2)for body waves in the interior region and in the absence of dissipation(i.e., out of the inhomogeneous layer) will lead to the solutions of [4], with all definitions and symbols being held here:

$$\delta B_z^{(\text{in})}(r,z) = \sum_{k=1}^{+\infty} A^{(\text{in,k})} J_{\text{m}}(|k_{\text{in,k}}|r)\psi^{(\text{in,k})}(z), \quad (6)$$

$$\delta v_r^{(\text{in})}(r,z) = -\frac{i\omega B}{4\pi} \times \sum_{k=1}^{+\infty} \frac{k_{\text{in,k}}}{\eta_{\text{in,k}}} A^{(\text{in,k})} J'_{\text{m}}(|k_{\text{in,k}}|r)\psi^{(\text{in,k})}(z), \quad (7)$$

where $k^2_{\text{in,k}} = \frac{\eta_{\text{in,k}}}{B^2/4\pi}$. For the exterior region the Bessel function J_{m}, index "in", and $|k_{\text{in,k}}|$ are replaced by the modified Bessel function K_{m}, "ex" and $k_{\text{ex,k}} = -\frac{\eta_{\text{ex,k}}}{B^2/4\pi}$, respectively.

III. CONNECTION FORMULAE AND DISPERSION RELATION

In the inhomogeneous layer of the tube -where the density decreases from its interior to its exterior constant value- the tube oscillation frequency equals the Alfén frequency. Consequently the solutions amplitude become unlimited and one can not obtain any analytical solution there. In the thin boundary approximation the jumps of the solutions across the layer - or the so called “connection formulae”- help to relate the interior and exterior analytical solutions. According to [4], the jumps are as follows

$$[\delta B_z] = 0,$$

$$[\delta v_r] = -\sum_{k}^{+\infty} \frac{B\tilde{\omega}m^2 \left\langle \phi^{(\text{in,k})} \mid \delta B_z^{(\text{in,k})} \right\rangle}{4r_A^2 \left\langle \phi^{(\text{in,k})} \mid L_{A1} \middle| \phi^{(\text{in,k})} \right\rangle} \phi^{(\text{in,k})}. \quad (8)$$

Here L_{A1} denotes the derivative of L_A in Alfvén radius. Also $\phi^{(\text{in,k})}$'s are the eigenfunctions of L_A with vanishing eigenvalues. They are defined as follows [4]

$$L_{A1} = \frac{\partial L_A}{\partial r}\Big|_{r=r_A} = \tilde{\omega}^2[1+S_{kk}]\frac{\partial \rho_{(0)}(r)}{\partial r}\Big|_{r=r_A\approx R}, \quad (9)$$

and

$$\phi^{(\text{in,k})} = \sqrt{\frac{2}{L}} \sum_{j=1}^{+\infty} \phi_j^{(\text{in,k})} \sin\Big(\frac{j\pi}{L}z\Big), \quad (10)$$

where

$$\phi_j^{(\text{in,k})} = \begin{cases} \frac{k^2 S_{kj}}{\rho_{\text{in}}(1+S_{kk})(j^2-k^2)} & j \neq k \\ 1 & j = k \end{cases}, \quad (11)$$

$$S_{kj} = -\sqrt{\frac{2}{L}}\,\mu\cos(\theta) \times \int_0^L \sin\big(\frac{k\pi}{L}z\big)\sin\big(\frac{\pi z}{L}\big)\sin\big(\frac{j\pi}{L}z\big)dz\ . \quad (12)$$

To obtain a dispersion relation and consequently the complex frequencies $\tilde{\omega} = \omega + i\gamma$ of various modes, one should substitute exterior and interior solutions into the jump conditions (8) and set $r = R$, where the resonant occurs. The form of the dispersion relation and all its relevant functions and symbols in this paper are the same as Eqs. (28)-(32) in [4].

IV. NUMERICAL RESULTS

To obtain the frequencies and damping rates we take the stratification parameter $\mu = 0.9$. Also we assume the necessary parameters of the coronal loop and its surrounding medium to be the same as those in [4], i.e., $L = 10^5$ km, $R = 10^3$ km, $B = 100$ G, $\rho_{\text{in}} = 2\times10^{-14}$ gr cm^{-3}, $\rho_{\text{ex}}/\rho_{\text{in}} = 0.1$, $v_{A_{\text{in}}} = 2\times10^3$ km s^{-1}, $v_{A_{\text{ex}}} = 6.4\times10^3$ km s^{-1}. We normalize frequencies and damping rates with respect to $\omega_{A_{\text{in}}} := \frac{v_{A_{\text{in}}}}{L} = 0.02$ rad s^{-1}.

In Figs. 1 and 2 we plot the frequency ω, the damping rate $|\gamma|$ and the ratio $\omega/|\gamma|$ for both fundamental and first-overtone modes of kink ($m = 1$) waves, versus the loop plane inclination angle θ. Figures show that: (i) as the loop becomes more inclined, both frequencies ω_1 and ω_2 decrease. For instance, by increasing the angle of inclination from 0 to 75^o the decrease of ω_1 and ω_2 are 45% and 32% respectively. (ii) The damping rates $|\gamma|$also decrease when the inclination angle increases. The percentage of decrease in this case is 46% for $|\gamma_1|$ and 23% for $|\gamma_2|$, when θ increases from 0 to 75^o. (iii) The ratio $\omega/|\gamma|$ - which is proportional to the number of oscillations take place before damping completely - does not affected by the loop plane inclination angle θ. Therefore, we expect two loops with different inclination angles to have the same number of oscillations.

In Fig. 3, the frequency ratio ω_2/ω_1 of the first overtone and its fundamental mode is plotted versus the inclination angle. Figures shows that: (i) for a stratified loop, the frequency ratio ω_2/ω_1 increases with increasing

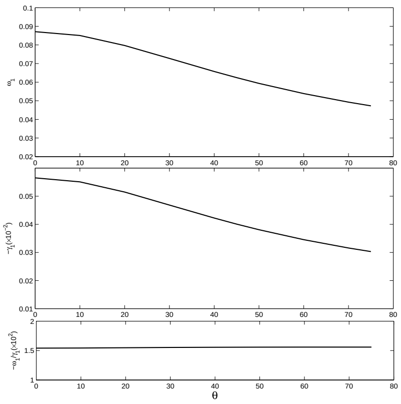

FIG. 1. The oscillation frequency ω_1, the damping rate $|\gamma_1|$ and the ratio $\omega_1/|\gamma_1|$, versus the loop plane inclination angle θ for the fundamental kink modes ($m = 1$) with the stratification parameter $\mu = 0.9$. The loop parameters are $L = 10^5$ km, $R/L = 0.01$, $l/R = 0.02$, $\rho_{\rm ex}/\rho_{\rm in} = 0.1$, $\rho_{\rm in} = 2 \times 10^{-14}$ gr cm^{-3} and $B = 100$ G.

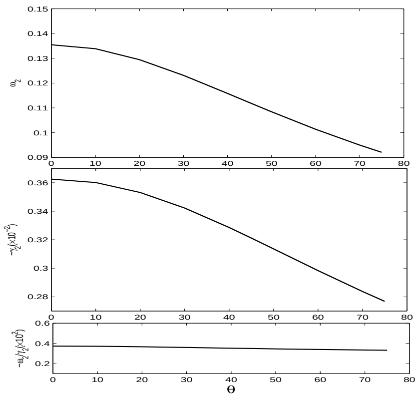

FIG. 2. Same as Fig. 1, but for the first-overtone kink modes ($m = 1$).

the loop plane inclination angle. Therefore, the inclination angle is one of the important aspects that should be taken into account for a comprehensive interpreting of coronal seismology using frequency ratios. When the inclination angle varies from 0 to 75^o then the frequency ratio ω_2/ω_1 increases from 1.55 to 1.94, i.e. it changed approximately 25%.

V. CONCLUSIONS

The main goal of this work was to illustrate that how an inclination with respect to the vertical plane - which is observed in coronal loops - does affect the resonant absorption of kink waves in such oscillating loops. To do so, we modeled a coronal loop as a longitudinally stratified flux tube. Using a connection formulae, we derived the relevant dispersion relation and solved it numerically in thin tube thin boundary approximation. Consequently, we obtained the frequencies and damping rates of the

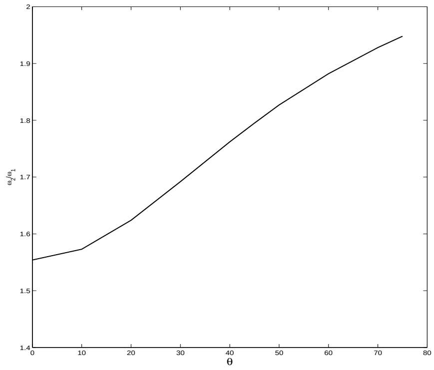

FIG. 3. The frequency ratio ω_2/ω_1 of the first overtone and its fundamental mode versus the inclination angle θ for kink modes ($m = 1$). Auxiliary parameters as in Fig. 1.

fundamental and first-overtone kink modes. Our numerical results show that the frequencies and their relevant damping rates decrease with increasing the loop plane inclination angle. But the ratio of the oscillation frequency to the damping rate which shows the number of oscillations, remains unchanged. Also the frequency ratio ω_2/ω_1 of the first overtone and its fundamental kink mode increases when the inclination angle of the loop plane increases.

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