Intermediate inflation in a non-canonical setting

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We study the intermediate inflation in a non-canonical scalar field framework with a power-like Lagrangian. we obtain an inverse power law inflationary potential and approximate the total e-fold number of inflation in our model. Then, we estimate the inflationary observables and show that in contrast with the standard canonical intermediate inflation, our non-canonical model is compatible with the observational results of Planck 2015. Subsequently, we obtain an approximation for the energy scale at the initial time of inflation and show that it can be of order of the Planck energy scale. Therefore, we can resolve one of the mysteries of the inflation theory.

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I. INTRODUCTION

Inflationary scenario is one important part of modern cosmology. In this scenario, it is believed that a rapid expansion has occurred in the very early stages of our universe. Consideration of this fast accelerated expansion can resolve some of basic problems of the Hot Big Bang cosmology, such as horizon problem, flatness problem and relic particle abundances problem.

In the standard model of inflation, a canonical kinetic term is included in Lagrangian and usually this term is dominated by the potential term. But also there are some models of inflation in which the kinetic term can be different from the standard canonical one [1–5]. These models are known as the non-canonical models of inflation.

In this paper, we focus on the intermediate inflation in a non-canonical setting. First, we turn to obtain the inflationary potential for our model. Then, we approximate the total e-fold number of inflation in our model. In addition, we estimate the inflationary observables and compare them with the observational results of Planck 2015 [6]. Subsequently, we find an approximation for the energy scale at the start of inflation.

II. INTERMEDIATE INFLATION IN A NON-CANONICAL FRAMEWORK

Let us consider the following action

$$S = \int \mathrm{d}^4x \, \sqrt{-g}\, \mathcal{L}(X,\phi), \tag{1}$$

where $\mathcal{L}$, ϕ and $X \equiv \partial_\mu\phi\partial^\mu\phi/2$ are the Lagrangian, the inflaton scalar field and the kinetic term, respectively. The energy density ρ_ϕ and pressure p_ϕ of the scalar field for the above action are given by [1–5]

$$\rho_\phi = 2X\left(\frac{\partial\mathcal{L}}{\partial X}\right) - \mathcal{L}, \tag{2}$$

$$p_\phi = \mathcal{L}. \tag{3}$$

In this work, we consider the flat FRW metric. Therefore, the kinetic term turns into $X = \dot{\phi}^2/2$. Also, dynamics of the universe is determined by the Friedmann equation

$$H^2 = \frac{1}{3M_P^2}\rho_\phi, \tag{4}$$

together with the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P^2}\left(\rho_\phi + 3p_\phi\right), \tag{5}$$

where $M_P = 1/\sqrt{8\pi G}$ is the reduced Planck mass, a is the scale factor and $H \equiv \dot{a}/a$ is the Hubble parameter.

The first and second slow-roll parameters are defined as

$$\varepsilon = -\frac{\dot{H}}{H^2}, \tag{6}$$

$$\eta = \varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon}, \tag{7}$$

respectively. In the slow-roll approximation, we have $\varepsilon \ll 1$ and $|\eta| \ll 1$.

In this paper, we assume that in the action (1), the Lagrangian has the power-like form

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi), \tag{8}$$

where α is a dimensionless parameter and M is a parameter with dimensions of mass [3–5]. For $\alpha = 1$, the above Lagrangian turns into the standard canonical Lagrangian $\mathcal{L}(X,\phi) = X - V(\phi)$.

Inserting the Lagrangian (8) into Eqs. (2) and (3), we find the energy density and pressure of the scalar field ϕ as

$$\rho_\phi = (2\alpha - 1)\, X \left(\frac{X}{M^4}\right)^{\alpha-1} + V(\phi), \tag{9}$$

$$p_\phi = X \left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi). \tag{10}$$

We can show that by use of the slow-roll conditions for the Lagrangian (8), the first slow-roll parameter (6) is related to the potential $V(\phi)$ as

$$\varepsilon_V = \left[\frac{1}{\alpha}\left(\frac{3M^4}{V(\phi)}\right)^{\alpha-1}\left(\frac{M_P V'(\phi)}{\sqrt{2}\, V(\phi)}\right)^{2\alpha}\right]^{\frac{1}{2\alpha-1}}, \tag{11}$$

Also, in the slow-roll regime, the potential energy dominates the kinetic energy and thus the Friedmann equation (4) reduces to

$$H^2(\phi) = \frac{1}{3M_P^2} V(\phi). \tag{12}$$

Moreover, in the slow-roll regime, the evolution equation of the scalar field is [4]

$$\dot{\phi} = -\theta \left\{\left(\frac{M_P}{\sqrt{3}\alpha}\right)\left(\frac{\theta V'(\phi)}{\sqrt{V(\phi)}}\right)\left(2M^4\right)^{\alpha-1}\right\}^{\frac{1}{2\alpha-1}}, \tag{13}$$

where $\theta = 1$ when $V'(\phi) > 0$ and $\theta = -1$ when $V'(\phi) < 0$.

In this paper, we are interested in studying the intermediate inflation with the scale factor

$$a(t) = \exp\left[A (M_P t)^f\right], \tag{14}$$

where $A > 0$ and $0 < f < 1$ [7–9].

With the help of Eqs. (4), (5), (9) and (10) for the intermediate scale factor (14), we find the inflationary potential as [5]

$$V(\phi) = V_0 \left(\frac{\phi}{M_P}\right)^{-s}, \tag{15}$$

where

$$s = \frac{4\alpha(1-f)}{2\alpha + f - 2}, \tag{16}$$

and

$$V_0 = \frac{3 \times 2^{\frac{6\alpha(1-f)}{2\alpha+f-2}}\, \alpha^{\frac{2(2\alpha-1)(1-f)}{2\alpha+f-2}}\, \bar{M}^{-\frac{8(\alpha-1)(1-f)}{2\alpha+f-2}}}{(2\alpha+f-2)^{\frac{4\alpha(1-f)}{2\alpha+f-2}}} \times (Af)^{\frac{4\alpha-2}{2\alpha+f-2}} (1-f)^{\frac{2(1-f)}{2\alpha+f-2}} M_P^4, \tag{17}$$

where $\bar{M} \equiv M/M_P$. We see that the potential driving the intermediate inflation in our non-canonical framework, like the potential of the standard canonical case [9], has inverse power law form.

III. ENERGY SCALE AT THE INITIAL TIME OF INFLATION

It is convenient to express the amount of inflation with respect to the e-fold number defined as

$$N \equiv \ln\left(\frac{a_e}{a}\right). \tag{18}$$

The above definition leads to

$$dN = -H dt = -\frac{H}{\dot{\phi}} d\phi. \tag{19}$$

Here, we are interested in obtaining the evolution of the scalar field ϕ in terms of the e-fold number N. To this end, in Eq. (19) we replace H and $\dot{\phi}$ from Eqs. (12) and (13), respectively. We notice that the potential (15) has inverse power law form, thus $V'(\phi) < 0$ and consequently we take $\theta = -1$ in Eq. (13). Now, we can solve the differential equation (19). To determine the initial condition, we use the first potential slow-roll parameter (11) which for our inflationary potential (15), reads

$$\varepsilon_V = \frac{2^{\frac{3\alpha f}{2\alpha+f-2}}\, \alpha^{\frac{f(2\alpha-1)}{2\alpha+f-2}}\, \bar{M}^{-\frac{4f(\alpha-1)}{2\alpha+f-2}} (1-f)^{\frac{2(\alpha+f-1)}{2\alpha+f-2}}}{(Af)^{\frac{2(\alpha-1)}{2\alpha+f-2}} (2\alpha+f-2)^{\frac{2\alpha f}{2\alpha+f-2}}} \times \left(\frac{\phi}{M_P}\right)^{-\frac{2\alpha f}{2\alpha+f-2}}. \tag{20}$$

This is a decreasing function during inflation and hence the relation $\varepsilon_V = 1$ is related to the initial time of inflation [10]. Consequently, the value of the scalar field at the start of inflation is obtained as

$$\phi_i = \frac{2\sqrt{2}\alpha^{\frac{2\alpha-1}{2\alpha}}\, \bar{M}^{-\frac{2(\alpha-1)}{\alpha}} (Af)^{\frac{1-\alpha}{\alpha f}} (1-f)^{\frac{\alpha+f-1}{\alpha f}}}{2\alpha+f-2} M_P. \tag{21}$$

With this initial condition, the differential equation (19) gives

$$\phi = \frac{2\sqrt{2}\alpha\mu^{\frac{2(\alpha-1)}{\alpha}} (1-f)^{\frac{1}{2\alpha}}}{\alpha^{\frac{1}{2\alpha}} (2\alpha+f-2)(Af)^{\frac{\alpha-1}{\alpha f}}} \times \left[f(N_i - N - 1) + 1\right]^{\frac{2\alpha+f-2}{2\alpha f}} M_P, \tag{22}$$

where N_i is the e-fold number corresponding to the initial time of inflation.

In the slow-roll approximation, the power spectrum of scalar perturbations for our non-canonical model (8) acquires the form [3–5]

$$\mathcal{P}_s = \frac{1}{72\pi^2 c_s}\left(\frac{6^\alpha \alpha V(\phi)^{5\alpha-2}}{M_P^{14\alpha-8}\bar{M}^{4(\alpha-1)} V'(\phi)^{2\alpha}}\right)^{\frac{1}{2\alpha-1}}\Bigg|_{aH=c_s k}. \tag{23}$$

This quantity should be evaluated at the sound horizon exit specified by $aH = c_s k$ where k is the comoving wavenumber and c_s is the sound speed defined as [1–5]

$$c_s^2 = \frac{\partial p_\phi/\partial X}{\partial \rho_\phi/\partial X}. \qquad (24)$$

For our non-canonical model (8), it reduces to

$$c_s = \frac{1}{\sqrt{2\alpha - 1}}, \qquad (25)$$

which is a constant quantity. Using Eqs. (15) and (22) in Eq. (23) and after some simplifications, we get

$$\mathcal{P}_s = \frac{\sqrt{2\alpha - 1}(Af)^{2/f}}{8\pi^2 (1-f)} \left[f\,(N_i - N - 1) + 1\right]^{3-\frac{2}{f}}. \qquad (26)$$

In the above equation, we see that for the value of $f = 2/3$, the scalar power spectrum is independent of the e-fold number N and we expect a Harrison-Zel'dovich scale invariant spectrum. The scalar spectral index is defied as

$$n_s - 1 \equiv \frac{d \ln \mathcal{P}_s}{d \ln k}. \qquad (27)$$

Here, we use the equation $aH = c_s k$ and we note that H is approximately constant during slow-roll inflation, and also c_s is constant for our non-canonical model. Therefore, we get

$$d \ln k = -dN. \qquad (28)$$

Using this relation together with Eqs. (27) and (26), we obtain the scalar spectral index as

$$n_s = 1 - \frac{2 - 3f}{f\,(N_i - N - 1) + 1}. \qquad (29)$$

In addition, using Eqs. (28) and (29), we find the running of the scalar spectral index as

$$\frac{dn_s}{d\ln k} = \frac{(2-3f)\,f}{\left[f\,(N_i - N - 1) + 1\right]^2}. \qquad (30)$$

The power spectrum of the tensor perturbations for our non-canonical model (8) is given by [2]

$$\mathcal{P}_t = \frac{2}{3\pi^2}\left(\frac{V(\phi)}{M_P^4}\right)_{aH=k}, \qquad (31)$$

where it should be calculated at the horizon exit specified by $aH = k$. Inserting Eqs. (15) and (22) into Eq. (31), we obtain

$$\mathcal{P}_t = \frac{2(Af)^{2/f}}{\pi^2}\left[f\,(N_i - N - 1) + 1\right]^{-\frac{2(1-f)}{f}}. \qquad (32)$$

The tensor spectral index is defined as

$$n_t \equiv \frac{d\ln \mathcal{P}_t}{d \ln k}. \qquad (33)$$

With the help of Eqs. (28), (32) and (33), we obtain the following relation for the tensor spectral index

$$n_t = -\frac{2\,(1-f)}{f(N_i - N - 1) + 1}. \qquad (34)$$

Another important inflationary observable is the tensor-to-scalar ratio defined as

$$r \equiv \frac{\mathcal{P}_t}{\mathcal{P}_s}. \qquad (35)$$

Substituting Eqs. (26) and (32) into (35), we obtain the tensor-to-scalar ratio as

$$r = \frac{16\,(1-f)}{\sqrt{2\alpha - 1}\left[f\,(N_i - N - 1) + 1\right]}. \qquad (36)$$

Now, using Eqs. (25), (34) and (36), we see that the consistency relation for our non-canonical model [3–5]

$$r = -8 c_s n_t \qquad (37)$$

is satisfied.

If we evaluate the inflationary potential (15) at ϕ_i given by Eq. (21), we find the potential energy at the initial time of inflation as

$$V_i \equiv V(\phi_i) = 3(Af)^{2/f}(1-f)^{-\frac{2(1-f)}{f}} M_P^4. \qquad (38)$$

Here, we take $\alpha = 20$ and $f = 1/4$ and set $A = 4.119$ as determined in [5]. In addition, we take the e-fold number of the horizon exit as $N_* = 60$. Now, if we fix $\mathcal{P}_s|_{N_*} = 2.207 \times 10^{-9}$ from Planck 2015 TT,TE,EE+lowP data combination [6] in Eq. (26), then we find the e-fold number corresponding to the initial time of inflation as $N_i = 201$. We define the total e-fold number as $N_{\rm tot} \equiv N_i - N_e$ where N_e is the e-fold number corresponding to the end time of inflation and vanishes according to definition (18). Therefore, our non-canonical inflationary model predicts the total e-fold number of inflation as $N_{\rm tot} = 201$. It should be reminded that this value is an approximation according to Linde's idea about the eternal inflation [11,12].

In order to show the consistency of our discussion, we estimate the inflationary observables and compare them with the Planck 2015 observational results. For this purpose, we evaluate the inflationary observables at $N_* = 60$. Therefore, using Eq. (29) we obtain $n_s = 0.9653$ which lies in the range with 68% CL allowed by Planck 2015 TT,TE,EE+lowP data ($n_s = 0.9644 \pm 0.0049$) [6]. Also, from Eqs. (30) and (36), we get $dn_s/d\ln k = 0.0002$ and $r = 0.0534$, respectively, which are in agreement with Planck 2015 TT,TE,EE+lowP data at 68% CL [6]. Furthermore, from Eq. (34) we see that our model predicts the tensor spectral index as $n_t = -0.0417$ that can be checked by precise measurements in the future. Therefore, we conclude that in contrast with the standard canonical inflation, our non-canonical model is consistent with the Planck 2015 observational results.

At this point, we obtain an approximation for the energy scale at the start of inflation. To this end, we use Eq.

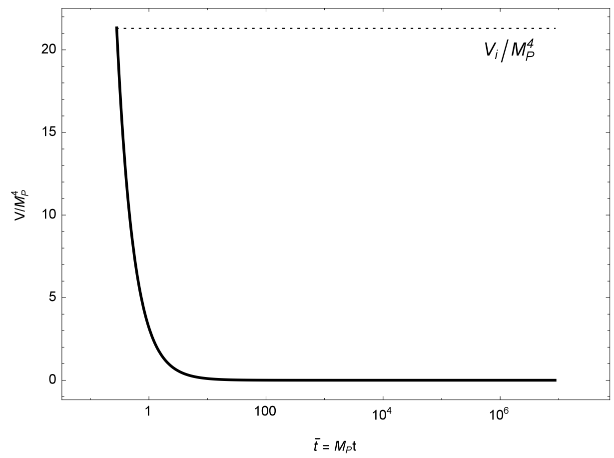

FIG. 1. Evolution of the inflationary potential (15) versus the dimensionless time $\bar{t} = M_P t$. The dotted line specifies the potential energy at the initial time of inflation.

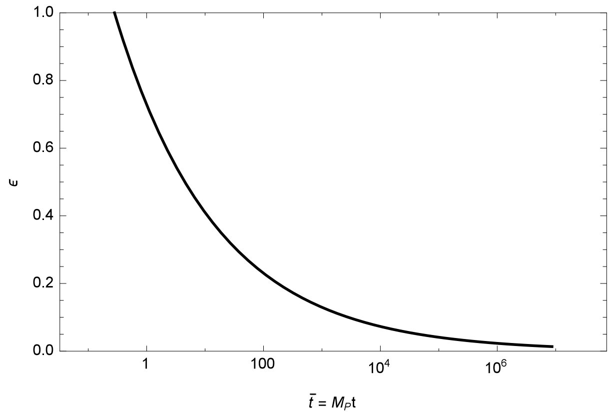

FIG. 2. Evolution of the first slow-roll parameter (6) versus the dimensionless time $\bar{t} = M_P t$.

(38) and find the potential energy at the initial time of inflation as $V_i = 21.30 M_P^4$. Therefore, we find the energy scale at the start of inflation as $V_i^{1/4} \sim M_P \sim 10^{18}$GeV which is of order of the Planck energy scale. Therefore, we can provide a reasonable explanation for one of the mysteries of the inflation theory that the energy scale defined by the energy density of the universe at horizon exit is a few orders of magnitude less than the Planck energy scale and is approximately of order $M_P/100 \sim 10^{16}$ GeV according to observational results, while we expect that inflation occurs at the energy scale of order $M_P \sim 10^{18}$GeV [13]. In fact, we resolve this problem by implying that inflation begins from the energy scale of order M_P but it converges rapidly to the energy scale of order $M_P/100$ at which the slow-roll behaviour occurs so that the horizon exit takes place at this energy scale. In order to show this remark more concretely, we use Eq. (15) to plot the evolution of inflationary potential versus dimensionless time. This plot is demonstrated in Fig. 1. It should be noted that we could solve this problem in our inflationary model because the slow-roll conditions are not perfectly satisfied during a short period of time at the beginning of inflation. To show this fact, we use Eq. (6) for the intermediate scale factor (14) and plot the evolution of the first slow-roll parameter relative to dimensionless time in Fig. 2. This figure shows that after a short period of time, inflation rapidly enters the slow-roll regime ($\varepsilon \ll 1$) in which the horizon crossing takes place.

IV. CONCLUSIONS

Here, we investigated the intermediate inflation characterized by the scale factor $a(t) = \exp\left[A(M_P t)^f\right]$ where $A > 0$ and $0 < f < 1$ in a non-canonical framework with a power-like Lagrangian. We showed that in our non-canonical framework, the intermediate inflation is driven by the inverse power law potential. Having the inflationary potential in hand, we turned to find an approximation for the total e-fold number of inflation in our model. Subsequently, we estimated the inflationary observables and showed that in contrast with the standard canonical intermediate inflation, our non-canonical model of intermediate inflation can be compatible with Planck 2015 results. Finally, we obtained an approximation for the energy scale at the initial time of inflation and showed that it can be of order of the Planck energy scale. Therefore, we could provide a convincing explanation for one the mysteries of the inflation theory.

[1] C. Armendariz-Picon, T. Damour, V. Mukhanov, Phys. Lett. B **458**, 209 (1999).
[2] J. Garriga, V.F. Mukhanov, Phys. Lett. B **458**, 219 (1999).
[3] S. Unnikrishnan, V. Sahni, A. Toporensky, JCAP **08**, 018 (2012).
[4] S. Unnikrishnan, V. Sahni, JCAP **10**, 063 (2013).
[5] K. Rezazadeh, K. Karami, P. Karimi, arXiv:1411.7302.
[6] P.A.R. Ade, et al., arXiv:1502.02114.
[7] J.D. Barrow, Phys. Lett. B **235**, 40 (1990).
[8] J.D. Barrow, A.R. Liddle, Phys. Rev. D **47**, 5219 (1993).
[9] J.D. Barrow, N.J. Nunes, Phys. Rev. D **76**, 043501 (2007).
[10] X.M. Zhang, J.Y. Zhu, JCAP **02**, 005 (2014).
[11] A.D. Linde, Phys. Lett. B **175**, 395 (1986).
[12] A.D. Linde, Mod. Phys. Lett. A **1**, 81 (1986).
[13] S. Weinberg, *Cosmology*, Oxford University Press (2008).