



The Effect of magnetic diffusivity parameter on Two Dimensional ADAFs with Outflow

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ABSTRACT

The aim of this paper is to investigate the role of magnetic diffusivity parameter on the structure of ADAFs. We use the self-similar assumption in radial direction to solve the MHD equations for hot accretion disks. The toroidal component of magnetic field and all three components of the velocity field $\mathbf{v} \equiv (v_r, v_\theta, v_\phi)$ are considered in our work. Our results indicate that the outflow region, where the radial velocity becomes positive in a certain inclination angle θ_0 , always exists. We see that the stronger magnetic diffusivity parameter does not have any sensible effect on the inclination angle, θ_0 . Numerical calculations of our model have revealed that the magnetic diffusivity parameter has a significant effect on the vertical structure of accretion disks.

Key words: accretion, accretion disks, MHD, magnetic diffusivity, outflow.

INTRODUCTION

Accretion disks are challenging and controversial issues, in recent decades, which have attracted researchers to study the various models. Many theoretical models have been proposed for better recognitions of accretion disks. One of them is the standard accretion disk model which is presented by Shakura & Sunyaev (1973). Another model is advection-dominated accretion flows model (ADAF). Structure of accretion disks is undergoing the thermal conduction, magnetic field, viscosity, etc. Magnetic fields have a great significance in accretion disks. They can have two sources. Disks could either itself has a continual magnetic field or its magnetic field be derived from external sources (see Moffat 1978). The effects of a magnetic field on the structure of ADAFs were also studied extensively (Balbus & Hawley 1998; Kaburaki 2000; Shadmehri 2004; Meier 2005; Shadmehri & Khajenabi 2005, 2006; Ghanbari et al. 2007; Abbassi et al. 2008, 2010; Bu et al. 2009). Indeed the crucial role of magnetic field in a hot flow is expected because of the high temperature of accreting gas in ADAFs ($10^9 - 10^{12}$ K). For the first time Lynden-Bell (1969) considered the role of magnetic field in the context of active galactic nuclei and found how it might be responsible for angular momentum transport and the origin of anomalous disk viscosity. Narayan & Yi (1995) by using the self-similar method in radial direction, solved the disk structure along θ direction. The self-similar approach adopted by Narayan & Yi (1995) is only partially supported by numerical simulations, i.e., there exists a new class of accretion flow, which is hot and optically thin and it is advection dominated. Also recent researches indicate that outflow is commonly observed to be associated with the hot accretion flows (Stone & Pringle 2001, De Villiers, Hawley, Krolik & Hirose 2005; Ohsuga & Mineshige 2011, Yuan et al. 2012a, 2012b). Moreover, numerical simulations indicate that v_θ is non-zero (Stone, Pringle & Begelman 1999; Ohsuga & Mineshige 2011; Yuan, Bu & Wu 2012). The ADAFs solutions with wind were reported by Abbassi et al. (2008, 2010), Mosallanezhad et al. (2013; hereafter MAB) where the effects of wind and outflow are achieved by adding relevant terms in MHD equations.





THE BASIC EQUATIONS AND SELF-SIMILAR SOLUTIONS

Here, we consider a steady state ($\partial/\partial t = 0$) and axisymmetric ($\partial/\partial\varphi = 0$) situation. Spherical coordinates are used (r, θ, φ). We neglect self-gravity of the accreting flow and relativistic effects, and use only the Newtonian gravity of the central object in our model. The magnetic field is considered with toroidal configurations $\mathbf{B} = (0, 0, B_\varphi)$ and we adopt α -prescription for effective turbulent viscosity. Thus the MHD equations, are as follows:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0 \quad (1) \quad , \quad \rho \frac{D\mathbf{v}}{Dt} = -\rho \nabla \psi - \nabla P + \frac{1}{c} \mathbf{J} \times \mathbf{B} + \nabla \cdot \mathbf{T} \quad (2)$$

$$\rho \left[\frac{De}{Dt} + P \frac{D}{Dt} \left(\frac{1}{\rho} \right) \right] = Q_+ - Q_- + Q_{cond} \equiv Q_{adv} + Q_{cond} \quad (3) \quad , \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B} - \frac{4\pi}{c} \eta \mathbf{J} \right) \quad (4)$$

where $\rho, \mathbf{v} \equiv (v_r, v_\theta, v_\varphi), \psi, p, \mathbf{B}, \mathbf{J} \equiv (c/4\pi) \nabla \times \mathbf{B}$ and \mathbf{T} are the mass density, velocity vector, gravitational potential, gas pressure, magnetic field, current density, and the tensor of viscous stress, respectively. Now we reformulate the basic equations (1)-(4) in spherical coordinates and by using Self-similar method, investigate the set of our equations in two dimensions (r, θ) as follows

$$v_r(r, \theta) = v_r(\theta) \sqrt{\frac{GM_*}{r}} \quad , \quad v_\theta(r, \theta) = v_\theta(\theta) \sqrt{\frac{GM_*}{r}} \quad , \quad v_\varphi(r, \theta) = v_\varphi(\theta) \sqrt{\frac{GM_*}{r}}$$

$$\rho(r, \theta) = \rho(\theta) r^{-n} \quad , \quad P(r, \theta) = P(\theta) GM_* r^{-n-1} \quad , \quad B_\varphi(r, \theta) = b(\theta) \sqrt{GM_*} r^{-\frac{n}{2}-\frac{1}{2}}$$

We arrived to these equations

$$\rho(\theta) \left[\left(n - \frac{3}{2} \right) v_r(\theta) - v_\theta(\theta) \cot \theta - \frac{dv_\theta(\theta)}{d\theta} \right] - v_\theta \frac{d\rho(\theta)}{d\theta} = 0 \quad (5)$$

$$\rho(\theta) \left[\frac{1}{2} v_r^2(\theta) + v_\theta^2(\theta) + v_\varphi^2(\theta) - v_\theta(\theta) \frac{dv_r(\theta)}{d\theta} - 1 \right] + (n+1)P(\theta) + \frac{1}{8\pi} (n-1)b^2(\theta) = 0 \quad (6)$$

$$\rho(\theta) \left[v_\varphi^2(\theta) \cot \theta - \frac{1}{2} v_r v_\theta - v_\theta(\theta) \frac{dv_\theta(\theta)}{d\theta} \right] - \frac{dP}{d\theta} - \frac{1}{4\pi} b(\theta) \left\{ b(\theta) \cot \theta + \frac{db(\theta)}{d\theta} \right\} = 0 \quad (7)$$

$$v_\varphi(\theta) \left[\frac{3}{2} \alpha (n-2) P(\theta) \left(1 + \frac{b^2(\theta)}{8\pi P(\theta)} \right) - \rho(\theta) \left\{ v_\theta \cot \theta + \frac{1}{2} v_r(\theta) \right\} \right] - \rho(\theta) v_\theta(\theta) \frac{dv_\varphi(\theta)}{d\theta} = 0 \quad (8)$$

$$P(\theta) \left\{ \gamma v_\theta(\theta) \frac{d\rho(\theta)}{d\theta} - (n\gamma - n - 1) v_r(\theta) \rho(\theta) \right. \\
 + f(\gamma - 1) \left(1 + \frac{b^2(\theta)}{8\pi P(\theta)} \right) \left[\frac{9}{4} \alpha \rho(\theta) v_\varphi^2(\theta) \right. \\
 \left. \left. + \frac{\eta_0}{4\pi} \left\{ \left(b(\theta) \cot \theta + \frac{db(\theta)}{d\theta} \right)^2 + \left(\frac{1}{2} (n-1) b(\theta) \right)^2 \right\} \right] \right\} - \rho(\theta) v_\theta \frac{dP(\theta)}{d\theta} \\
 + \frac{\lambda_0}{\rho} \left\{ P + \cot \theta \left(\dot{P} - \frac{P\dot{\rho}}{\rho} \right) + \left(\ddot{P} - \frac{P\ddot{\rho}}{\rho} \right) - \frac{2\dot{P}\dot{\rho}}{\rho} + \frac{2P\dot{\rho}^2}{\rho^2} \right\} = 0 \quad (9)$$

$$\eta_0 \frac{P(\theta)}{\rho(\theta)} \left\{ \left(1 + \frac{b^2(\theta)}{8\pi P(\theta)} \right) \left[\frac{d^2 b(\theta)}{d\theta^2} + \frac{db(\theta)}{d\theta} \cot \theta + \frac{1}{4} n(n-1) b(\theta) - \frac{b(\theta)}{\sin^2 \theta} \right] \right. \\
 \left. + \left(b(\theta) \cot \theta + \frac{db(\theta)}{d\theta} \right) \left[\left(1 + \frac{b^2(\theta)}{8\pi P(\theta)} \right) \left(\frac{\dot{P}}{P} - \frac{\dot{\rho}}{\rho} \right) + \frac{b^2(\theta)}{8\pi P(\theta)} \left(\frac{2\dot{b}}{b} - \frac{\dot{P}}{P} \right) \right] \right\} + \frac{n}{2} v_r(\theta) b(\theta) \\
 - v_\theta \frac{db(\theta)}{d\theta} - b(\theta) \frac{dv_\theta(\theta)}{d\theta} = 0 \quad (10)$$

where λ_0, Φ_s are two thermal conduction parameters that are simply related to each other by $\lambda_0 \cong 5\rho(\theta)C_s(\theta)\Phi_s$. Parameter Φ_s is the saturated thermal flux and it is smaller than unity. Tanaka& Menou (2006) applied some approximations and they concluded that $\lambda_0 \approx 8.4\Phi_s$.

Here we have $\rho(\theta), p(\theta), b(\theta), v_r(\theta), v_\theta(\theta), v_\varphi(\theta)$, the variable θ and eight input parameters ($\alpha, f, \gamma, n, \eta_0, \beta_0, \Phi_s, \lambda_0$), in which β_0 is the ratio of the gas pressure to the magnetic pressure at the equatorial plane which is considered to be constant. $\beta_0 = \frac{P_{gas}}{P_{mag}} \Big|_{90^\circ} = 8\pi \frac{P}{b^2} \Big|_{90^\circ}$



The set of OEDs can be solved numerically with proper boundary conditions. We assume the structure of the disk is symmetric to the equatorial plane, and then we have

$$\theta = 90^\circ : \quad v_\theta = \frac{dp}{d\theta} = \frac{dP}{d\theta} = \frac{dv_r}{d\theta} = \frac{dv_\phi}{d\theta} = \frac{db}{d\theta} = 0 \quad , \quad \rho(90^\circ) = 1$$

Now by putting the above boundary conditions into the equations (5)-(10), we obtain

$$\begin{aligned} v_r|_{90^\circ} &= E_1 P|_{90^\circ} \quad , \quad \frac{dv_\theta}{d\theta}|_{90^\circ} = \left(n - \frac{3}{2}\right) v_r|_{90^\circ} \quad , \quad v_\phi^2|_{90^\circ} = \frac{E_1 E_3 - E_4}{E_2} P|_{90^\circ} \quad , \quad P|_{90^\circ} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \\ \frac{1}{2} v_r^2|_{90^\circ} + v_\phi^2|_{90^\circ} + \left[(n+1) + \frac{(n-1)}{\beta_0} \right] P|_{90^\circ} - 1 &= 0 \quad , \quad \frac{d^2 b}{d\theta^2}|_{90^\circ} = (E_1 E_5 + E_6) \frac{b}{P}|_{90^\circ} \\ E_1 &= 3\alpha(n-2)(1 + \beta_0^{-1}) \quad , \quad E_2 = \frac{9}{4} f\alpha(\gamma-1)(1 + \beta_0^{-1}) \quad , \quad E_3 = n\gamma - n - 1 \\ E_4 &= \frac{1}{2} f\eta_0(\gamma-1)\beta_0^{-1}(n-1)^2(1 + \beta_0^{-1}) \quad E_5 = \frac{-n}{2\eta_0}(1 + \beta_0^{-1}) \quad , \quad E_6 = 1 - \frac{1}{4}n(n-1) \\ A &= \frac{E_1^2}{2} \quad , \quad B = \frac{E_1 E_3 - E_4}{E_2} + (n+1) + (n-1)\beta_0^{-1} \quad , \quad C = -1 \end{aligned}$$

Magnetic field is an important quantity for determining the behavior and structure of the disk. We investigate the structure of our thick disk in the presence of magnetic field. Fig. 1 shows the effect of various values of magnetic field parameter β_0 on the profiles of the physical variables along θ direction. In which we adopt $\alpha = 0.3$, $\gamma = 5/3$, $f = 1$, $\eta = 0.3$, $n = 1$ and $\Phi_s = 0.03$. In this Fig, dotted, dashed and solid lines correspond to $\beta_0 = 10^4$, $\beta_0 = 10^3$ and $\beta_0 = 10^2$ respectively. According to Fig. (1-a), we see that β_0 affects a little on the inflow region ($v_r(\theta) < 0$). But it makes tangible changes in the amount of outflow. In other words, the toroidal component of magnetic field prevents the material outflow from the surface of the disk. Moreover, as seen in Fig. (1-b) the stronger magnetic field does not have any sensible effect on the v_ϕ . But in Fig. (1-c) we see that the stronger magnetic field enlarges the value of v_θ in the midplane (the negative direction of v_θ is indicative that material moves from the equator to the polar axis of the disk). It is predicted that (Fig. 1-d) the mass density at first increases slightly and then decreases along θ direction towards the disk surface by increasing β_0 . Therefore we can conclude that mass density in areas close to the equator is more than areas close to the disk poles. Fig. 2 shows the effect of magnetic diffusivity parameter η_0 on the profiles of the physical variables. The dotted, dashed and solid lines correspond to $\eta_0 = 0.2$, $\eta_0 = 0.3$ and $\eta_0 = 0.4$ respectively. We supposed that $\alpha = 0.3$, $\beta_0 = 10^3$, $f = 1$, $\gamma = 5/3$ and $n = 1$. The radial and azimuthal velocities will slightly increase as η_0 increases (Figs 2-a, c). By moving towards disk surface and approaching the vertical axis, as η_0 increases v_θ , P and ρ decreases (Figs.2-b, d, e). Fig. (2-f) is dedicated to variations of the magnetic field. By looking at this Fig., we can conclude that variations of η_0 parameter have a main effect on the magnetic field strength. We adopt values of 0.15, 0.2 and 0.25 for viscosity parameter, α . According to Fig (3-a), by increasing the parameter α , radial velocity becomes more negative. This means that inflow increases. Therefore we achieve to the outflow in smaller angles. According to Fig (3-b) the magnetic field increases by increasing α and the magnetic field is stronger near the vertical axis. Gas pressure becomes smaller for larger values of α . In other words, by increasing the viscosity parameter, gas pressure decreases in the regions close to the disk surface, approximately (Fig. 3-c).

DISCUSSION

The main aim in this paper is to verify the effect of the magnetic diffusivity parameter on the structure of ADAFs along the θ direction. Since we also have considered $v_\theta \neq 0$, our solutions represent an inflow-outflow behavior. Here, we may also investigate the effects of the magnetic field parameters (β_0, η_0) and viscosity parameter (α), on the ADAFs structure. Our results show that inflow and outflow behaviors are not sensitive to η_0 changes but by increasing η_0 the velocity of winds driven from the surface of the disk and compression of the mass and pressure in approaching the vertical axis decreases. Therefore we observe that α and β_0 reduce the outflow in which it causes the reduction of the disk thickness.



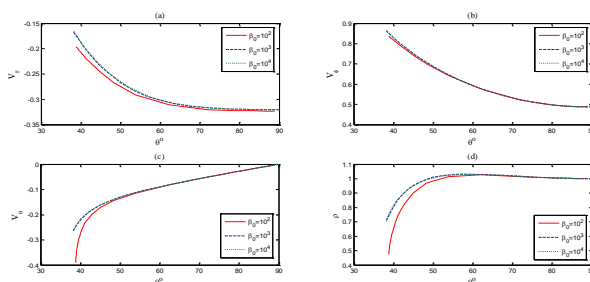


Figure 1. Profiles of the physical variables corresponding to ADAFs model along θ -direction for different values of the gas pressure to the magnetic pressure in midplane of the disk, β_0 . The dotted, dashed and solid lines denote $\beta_0 = 10^2, 10^3, 10^4$ respectively. Here $\alpha = 0.3, \gamma = 5/3, f = 1, \eta_0 = 0.3$ and $n = 1$.

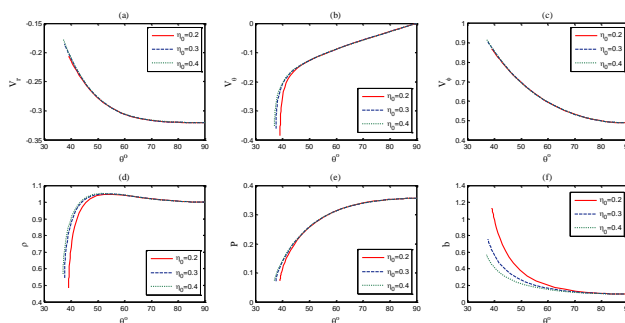


Figure 2. Profiles of the physical variables corresponding to ADAFs model along θ -direction for different values of magnetic diffusivity parameter, η_0 . The dotted, dashed and solid lines denote $\eta_0 = 0.2, 0.3$ and 0.4 respectively. Here $\alpha = 0.3, \beta_0 = 10^3, f = 1, \gamma = 5/3$ and $n = 1$.

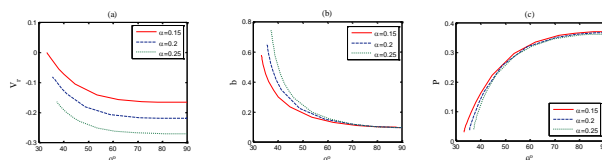


Figure 3. Profiles of the physical variables corresponding to ADAFs model along θ -direction for different values of viscosity parameter, α

REFERENCES

- Abbassi S., Ghanbari J., Najjar S., 2008, *MNRAS*, **388**, 663
 Abbassi, S., Ghanbari, J., Ghasemnezhad, M. 2010, *MNRAS*, **409**, 1113
 Balbus, S., & Hawley, J. F. 1998, *RvMP*, **70**, 1
 Bu, D., Yuan, F., & Xie, F. 2009, *MNRAS*, **392**, 325
 De Villiers, J.P., ierre, Hawley, J. F., Krolik, J. H., Hirose, S. 2005, *ApJ*, **620**, 878
 Ghanbari J., Salehi F., Abbassi S., 2007, *MNRAS*, **381**, 159
 Kaburaki, O. 2000, *ApJ*, **531**, 210
 Lynden-Bell, D., 1969, *Nat*, **223**, 690
 Meier, D. L. 2005, *Ap& SS*, **300**, 55
 Moffat H. K., 1978, Magnetic Field Generation in Electrically Conducting Fluids. *Cambridge univ. Press, Cambridge*
 Mosallanezhad A., Abbassi S., Beiranvand N., 2013, *MNRAS*, **437**.3112
 Narayan, R., & Yi, I. 1995a, *ApJ*, **444**, 238
 Narayan, R., & Yi, I. 1995b, *ApJ*, **452**, 710
 Ohsuga, K., Mineshige, S. 2011, *ApJ*, **736**, 2
 Shadmehri, M. 2004, *A&A*, **424**, 379
 Shadmehri, M., & Khajenabi, F. 2005, *MNRAS*, **361**, 719
 Shadmehri, M., & Khajenabi, F. 2006, *ApJ*, **637**, 439
 Shakura, N. I., Sunyaev, R. A., 1973, *A&A*, **24**, 337
 Stone, J. M., & Pringle, J. E. 2001, *MNRAS*, **322**, 461
 Tanaka, T., & Menou, K. 2006, *ApJ*, **649**, 345
 Yuan, F., Bu, D., & Wu, M. 2012, *ApJ*, **761**, 130 (II)
 Yuan, F., Wu, M., & Bu, D. 2012, *ApJ*, **761**, 129 (I)

