Investigation of non-Canonical Intermediate Inflation in light of Planck 2015

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Intermediate inflation is considered in a non-canonical scalar field model, in which the kinetic term of scalar field is taken as a power-law function. The free parameters of the model are constrained by using the most recent observational data related to scalar spectra index, tensor-to-scalar ratio, and scalar perturbation amplitude. The results are used to depict the potential behavior of the model and estimate the initial and final time of inflation.

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I. INTRODUCTION

Inflationary scenario is known as the best candidate for describing very early evolution of the Universe. The scenario was first introduced by Alan Guth , as a possible solution to the Big Bang model problems. Since there, various models of inflation have been introduced, based on canonical scalar field dynamics which dominates the Universe and has a negligible interaction with the other components of matter. Inflationary scenario properly solves the hot Big Bang model problems, not to mention the fact that the scenario predicts an interesting feature as quantum perturbations in the early times of the Universe evolution, that has received huge interest. The perturbations are separated to scalar, vector and tensor perturbation [1]. Amongst them, scalar perturbation, seeds for large scale structure of the Universe, and tensor perturbation, known as gravitational wave too, are the most important ones. The latest observational data comes from Planck data which released on February 2015 [2]. This states that the amplitude of scalar perturbation is about $\ln\left(10^{10}A_s^2\right) = 3.094$, and the scalar spectra index is about $n_s = 0.9645$. In contrast with scalar perturbation, Planck does not give an exact value for tensor-to-scalar ratio r. It only specifies an upper bound for this parameter as $r < 0.10$ [2].
One of inflationary scenarios is "intermediate inflation" [3], where the scale factor gets an exponential function of time as $a(t) = \exp\left(At^\alpha\right)$, $A > 0$ and $0 < \alpha < 1$. This can be acquired from a potential asymptotically looks like negative power but not exactly [4]. The scenario indicates on a expansion faster than power-law inflation ($a(t) = t^p, p > 1$), and slower than de-Sitter inflation ($a(t) = \exp(Ht), H = cte$). Intermediate inflation in Einstein gravity creates a scale invariant perturbation when $\alpha = 2/3$ [3]. The scenario is able to satisfy the bound on scalar spectra index n_s and tensor-to-scalar ratio r, measured by observation on CMB [5].

Recently the cosmological models of scalar field including non-canonical kinetic term have played a significant role in cosmological studies. The general form of its action is expressed by $\mathcal{L}_\phi = \{(\phi)\mathcal{F}(\mathcal{X})\mathcal{V}(\phi)$, where $X = (\nabla_\mu\phi\nabla^\mu\phi)/2$ [6]. The case with $V(\phi) = 0$ leads to a well-known model as k-essence. The main idea of k-essence comes from Born-Infold action of string theory [7]. The model is able to give some interesting results about dark energy [8]. In [9], the model is applied as a possible way for inflation and describing early time evolution of the universe.
In the present work, we are going to take $f(\phi) = 1$, and $V(\phi)$ as scalar field potential; in other word, we take a pure kinetic k-essence plus a potential term. This kind of model is known as non-canonical scalar field [10]. This case is another class of the general form which could be as important and interesting as k-essence model. The cosmological solution of the model has been studied in [10], where it was shown that it is possible to build up a unified model of dark matter and dark energy for a simple form of non-canonical kinetic term $F(X)$. The same case has been considered in [11] where the authors found that producing a unified model of dark matter and dark energy for a pure kinetic k-essence is very difficult. It sounds that the non-canonical scalar field model is capable to be an appropriate model of the universe evolution and has merit for considering in more detail. Then, we motivated to use non-canonical scalar field in intermediate inflation scenario as a possible model for describing one of earliest universe evolution. In this regards, the kinetic term is taken as a power-law function of X, and the general form of evolution equation are obtained. Consequently, the model contains some free parameters, that are aimed to be determined by using the newest observational data. Therefore, it is absolutely necessary to discuss perturbation of the model and derive related parameters. The perturbation parameters which are being to use, are scalar spectra index, tensor-to-scalar ratio and scalar perturbation amplitude. The result indicates on the presence of a non-canonical term for kinetic energy term, and $\alpha > 2/3$.

II. NON-CANONICAL SCALAR FIELD MODEL

The general form of Lagrangian could be read as

$$\mathcal{S} = \int \lceil^{\triangle}\S\sqrt{-\}}\left(\frac{\mathcal{M}^{\in}_{\surd}}{\in}\mathcal{R} + \mathcal{L}_{\mathcal{N}}\right) \tag{1}$$

where $\mathcal{L}_{\mathcal{N}}$ is the Lagrangian of non-canonical scalar field which is defined as $\mathcal{L}_{\mathcal{N}} = \mathcal{F}(\mathcal{X}) - \mathcal{V}(\phi)$. The kinetic term of non-canonical scalar field is denoted by $F(X)$, which it is an arbitrary function of X (where $X = -g^{\mu\nu}\nabla_{\mu}\phi\nabla_{\nu}\phi$). At the rest of the work, the kinetic term is taken as a power-law function of X as $F(X) = F_0 X^n$.
Variation of action with respect to independent variables $g_{\mu\nu}$ and ϕ, and substituting the spatially flat FRW metric comes to the Friedmann evolution equations. In addition, taking variation of the action with respect to the scalar field results in the wave equation, which is a restate of energy conservation relation. Surely the case for $n = 1$ and $F_0 = 1$ comes to usual canonical scalar field model.

Intermediate inflation scenario illustrates an expansion phase that stays between power-law inflation and de-Sitter expansion in very early times. In this scenario, the scale factor of the Universe is given by an exponential function of time as $a(t) = \exp\left(At^{\alpha}\right)$, in which α stands in the range $0 < \alpha < 1$, and A is a constant to justify the dimension.
Applying the assumption on Friedmann eqiations, the scalar field is derived as a function of cosmic time, and in turn the potential could be depicted as a function of scalar field.
In order to have a quasi-de Sitter expansion during inflation, the time rate of the Hubble parameter during a Hubble time should be smaller than unity [12]. The same behavior is usually assumed for time derivative of scalar field [12]. These conditions are known as slow-roll approximations, and based on them we defined the main slow-roll parameters as following $\epsilon_H = -\dot{H}/H^2$ and $\eta_H = -\ddot{\phi}/H\dot{\phi}$ [12]. As a first condition for inflation, there must be positive acceleration for the Universe, that is satisfied when the slow-roll parameter ϵ_H is smaller than unity. Amount of inflation during inflation time period is measured by number of e-folds parameter defining as $N \equiv \ln\left(a(t_e)/a(t_i)\right)$. Then, the initial and final value of the scalar field during inflation are read as

$$\phi_i^{\frac{2n\alpha}{z}} = \frac{(1-\alpha)\mathcal{F}^{\frac{\alpha}{\ddagger}}}{\mathcal{A}\alpha\xi^{\frac{\in\backslash\alpha}{\ddagger}}};\quad \phi_e^{\frac{2n\alpha}{z}} = \mathcal{A}\phi_{\rangle}^{\frac{\in\backslash\alpha}{\ddagger}};\quad \mathcal{A}\equiv\infty+\frac{\alpha\mathcal{N}}{\infty-\alpha}. \tag{2}$$

The constant parameters are aimed to be specified using the latest observational data. In this regards, studying perturbation is absolutely necessary. Inflationary models predict three kind of perturbations as scalar, vector and tensor perturbation. One of the most important metric perturbation is scalar perturbation. Scalar fluctuations become seeds for cosmic microwave background (CMB) anisotropies, or for large scale structure (LSS) formation. The amplitude of scalar perturbation could be derived doing some calculation, and it is given by [13]

$$\mathcal{P}_{\int} = \frac{\mathcal{H}^{\triangle}}{\triangle\pi^{\in}\rfloor_{\int}(\rho + {}_{\surd})} \tag{3}$$

where c_A is sound speed, that is constant, equal to $c_A^2 = (2n-1)^{-1}$ (reader could refer to [13] for more detail). Dependence of scalar perturbation on wavenumber k is described by n_s known as scalar spectra index. The parameter is given by $n_s - 1 = d\ln(\mathcal{P}_{\int})/\lceil\ln(\|) = -\triangle\epsilon_{\mathcal{H}} + \in\backslash\eta_{\mathcal{H}}$.
In addition to scalar fluctuation, the inflationary scenario predicts tensor fluctuation, which is known as a gravitational wave, too. It has been found out that the tensor fluctuations play a significant role, and they should be more attended for determining best-fit values of the cosmological parameters from the CMB and LSS spectra. The amplitude of tensor perturbation is obtained as [13]

$$\mathcal{P}_{\mathcal{T}} = \frac{\forall}{\mathcal{M}^{\triangle}_{\surd}}\frac{\mathcal{H}^{\in}}{\triangle\pi^{\in}} \tag{4}$$

The tensor spectra index is defined in a similar way, given by $n_T = d\ln(\mathcal{P}_{\mathcal{T}})/\lceil\ln(\|) = -\in\epsilon_{\mathcal{H}}$.
The imprint of tensor fluctuation on the CMB brings the idea to indirectly determine its contribution to power spectra by measuring CMB polarization. Such a contribution could be exhibited by the r quantity, which is known as tensor-to-scalar ratio and represents the relative amplitude of tensor-to-scalar fluctuation,

$$r = \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\int}} = \frac{16\epsilon_H}{\sqrt{2n-1}}. \tag{5}$$

Therefore, constraining r is one of the main goals of the modern CMB survey. According to the current accuracy of observations, it is only possible to place an upper bound on the allowed range of r [14]. The latest data about the quantity comes from Planck collaboration on February 2015. Planck full mission data for ΛCDM+r model resulted in a new constraint on the quantity r as $r < 0.10$ (Planck TT,TE,EE+lowP), < 0.11 (Planck TT+lowP+lensing) at 95% C.L. Note that, as we concentrate on Planck-2015 data about the quantity r, we realize that the previous mentioned constraint could rise in some cases. For instance, according to [2], for $\Lambda CDM + r + d\ln n_s/d\ln k$ model, there is $r < 0.176$ (Planck TT+lowP+lensing)

III. OBSERVATIONAL CONSTRAINT

The provided argument simplifies our way to determine the free parameters of the model by utilizing the latest observational data. In this regards, the perturbation parameters are being computed at the end of inflation. In the first step, we turn our attention to scalar spectra index, tensor-to-ratio parameter, and amplitude of scalar perturbation. The final result has been prepared in the following table for different values of n_s, r, and N.

	n_s	0.9625			0.9635		
	r	0.08	0.10	0.12	0.08	0.10	0.12
$N = 60$	α	0.685	0.685	0.685	0.697	0.697	0.697
	n	1.646	1.233	1.009	1.528	1.158	0.957
	n_T	−0.015	−0.015	−0.015	−0.014	−0.014	−0.014
	$\bar{A}$	1.309	1.413	1.504	1.748	1.890	2.014
	$\bar{t}_i$	2.000	1.788	1.633	2.075	1.856	1.694
	$\bar{t}_e$	2.470	2.209	2.016	2.456	2.200	2.008
$N = 65$	α	0.651	0.651	0.651	0.662	0.662	0.662
	n	1.836	1.355	1.093	1.706	1.271	1.036
	n_T	−0.016	−0.016	−0.016	−0.015	−0.015	−0.015
	$\bar{A}$	0.543	0.582	0.620	0.726	0.781	0.830
	$\bar{t}_i$	1.804	1.614	1.473	1.884	1.685	1.538
	$\bar{t}_e$	2.898	2.592	2.366	2.873	2.570	2.346

TABLE I. Constraint on the parameters n and α based on Planck data about scalar and tensor spectra indices. The quantity $\bar{A}$ is defined as $\bar{A} = 10^{-9}A$. The initial and final time of inflation is denoted as $\bar{t}_i = 10^5 t_p$, and $\bar{t}_e = 10^8 t_p$, where the variable t_p indicates the Planck time $\approx 10^{-43}$.

The observational constraint values for free parameters of the model are expressed in Table.I. In order to consider the general behavior of the potential during inflation, the potential is plotted for different values of tensor-to-scalar ratio, scalar spectra index, and number of e-folds. It is shown that the potential is always smaller than Planck energy, and reduces by passing time.
Fig.?? illustrates the potential behavior in term of scalar field for three different values of tensor-to-scalar ratio r, by taking $n_s = 0.9635$, and $N = 60$. For $r = 0.08$, related to Fig.1, the potential has a similar behavior as [15], it increase at first and then start decreasing. During inflation period, scalar field gets larger when a smaller value for r is picked out, not to mention the fact that the situation is opposite for potential so that it goes up by elevation of r. Additionally, the difference between initial and final scalar field becomes bigger by reduction of r. Fig.?? is devoted to depict the potential behavior versus scalar field for different values of scalar spectra index n_s and number of e-folds N. Fig.4 portrays the potential behavior for $r = 0.12$, $N = 60$, and three different values of scalar spectra index. It is found that as well as growth of the potential by downturn of n_s, the gap between ϕ_i and ϕ_e rises. Effect of number of e-folds on the potential behavior is shown in Fig.5. Larger value of N, which indicates more expansion of the Universe, causes bigger difference between initial and final value of scalar field. It could be seen when N grows, inflation happens for bigger potential, and inflation ends for bigger value of scalar field as well. In other word, inflation lasts more in order to produce higher amount of inflation.
To sum up briefly, it could be said that, the general behavior of the potential is same during inflation: it decreases by enhancement of scalar field, or by passing time, and there is almost the same order of initial and final value of the potential for each cases. It also should be noticed that the scalar field in all cases above sounds to be bigger than Planck mass, however its magnitude is the same order of Planck mass. From Fig.??, it could be demonstrated that bigger values of r results in smaller values of ϕ during inflation. Consequently, the scalar field decreases and it could goes below Planck mass if bigger value of r is chosen.

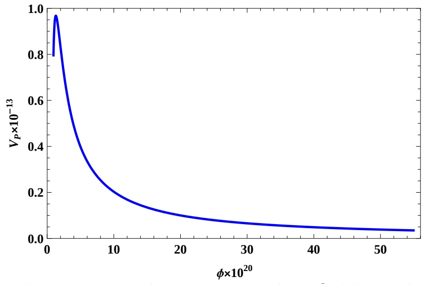

FIG. 1. The potential versus scalar field is displayed for $n_s = 0.9635$, $N = 60$ and $r = 0.08$. Here the variable V_P is defined by $V_P = V(\phi)/M_p^4$.

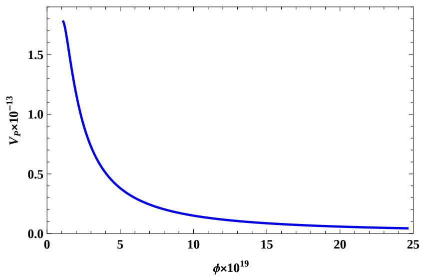

FIG. 2. The potential versus scalar field is displayed for $n_s = 0.9635$, $N = 60$ and $r = 0.10$. Here the variable V_P is defined by $V_P = V(\phi)/M_p^4$.

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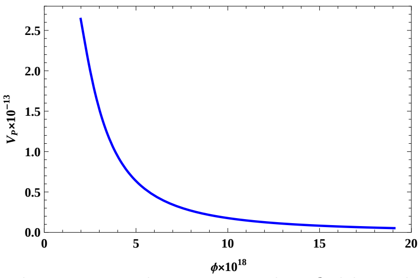

FIG. 3. The potential versus scalar field is displayed for $n_s = 0.9635$, $N = 60$ and $r = 0.11$. Here the variable V_P is defined by $V_P = V(\phi)/M_p^4$.

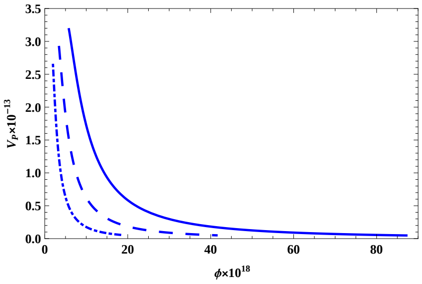

FIG. 4. The potential behavior versus scalar field is plotted for $r = 0.12$, $N = 60$, and three different values of n_s as $n_s = 0.9615$(solid line), 0.9625(dashed line), and 0.9635(dotted line).

IV. CONCLUSION

Intermediate inflation was studied in a non-canonical scalar field model. This type of inflation model takes an exponential function of time for the universe scale factor as $a(t) = \exp\left(At^{\alpha}\right)$, where $0 < \alpha < 1$, to describe an accelerated expansion between power-law and de-Sitter expansion. Non-canonical scalar field model contains a modified term of kinetic energy in action, and in this paper it was picked out as a power-law function.

Constraining the free parameters of the model by using the latest observational data is the main goal of the work. Doing so, we used observational data about scalar spectra index, tensor-to-scalar ratio, and scalar perturbation amplitude. The constraint value for α showed that this parameter is about $\alpha \approx 0.6, 0.65, 0.7$. On the other hand, the obtained constraint for power of kinetic term gives $n \approx 1, 1.2, 1.5, 2$, which indicates on the presence of a non-canonical kinetic energy. By using these constraint values, tensor spectra index was determined, and the results shows that the model prediction about tensor spectra index is in good agreement with observational data. The other free parameter was A, which was estimated of order 10^9, resulted from scalar perturbation amplitude. The potential behavior and inflation time were other topic which investigated at next stage. The potential was plotted for different values of n_s, r, and number of e-folds N. It was shown that the general behavior of the potential is same so that the model

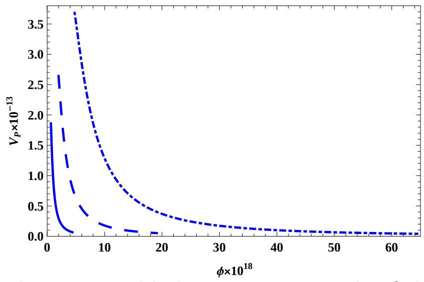

FIG. 5. The potential behavior versus scalar field is plotted for $r = 0.12$,$n_s = 0.9635$ and three different value of N as $N = 55$(solid line), 60(dashed line), and 65(dotted line). Here the variable V_P is defined by $V_P = V(\phi)/M_p^4$.

produces a potential that reduces by passing time or increasing scalar field. Studying inflation time came into this result that inflation starts at about 10^{-38} second after big bang, and it ends at about 10^{-35} second after big bang.

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