# A pricing model based on quantity discount in a multi-echelon closed-loop supply chain with different production technologies

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## Abstract

*In this paper, a novel multi-echelon closed-loop location-allocation-inventory problem (MCLIP) is addressed that optimizes strategic and tactical decisions, simultaneously. In order to represent the purchasing cost of raw material from the supplier, a pricing model under quantity discounts is applied in the closed-loop supply chain (CLSC). Furthermore, environmental impacts of different production technologies at manufacturing centers are investigated which can be identified by calculating the total costs of the proposed model. Considering the capability of returning the reworked products to the forward logistics that can affect the ordering patterns of distribution centers (DCs) is another significant difference of this study from similar related researches. The proposed problem is formulated based on a mixed integer non-linear programming (MINLP). In the following, the computational results and sensitivity analyses are presented using GAMS software to reveal the applicability of the proposed model and the results are analyzed in depth to provide some managerial insights.*

**Keywords:**
Closed-loop supply chain, Location-allocation-inventory, Price discount, Production technology, Returned products

## Introduction

In last few days, green supply chains tend to invest on integrating some business operations to minimize the side effects such as natural source reduction, water and air pollution, etc. [1]. In practice, product recovery is one of the most prevalent methods for making a green supply chain which is one of the main requirements of making a closed-loop supply chain. In other words, implementing a reverse logistics in a specific channel is one of the vital needs to achieve a CLSC. In particular, the reverse logistics can be defined as some efforts done to return or properly dispose the unsold, damaged, and end-of-life (EOL) products. Such efforts are reworking, repairing, disposing, recycling, and remanufacturing. In a broader sense, CLSC management is a combination of forward and reverse logistics as traditional and modern processes, respectively. It is noteworthy that the prevalent assumption in the related literature that indicates the creation of new spare parts by combining the returned products with subassemblies to transfer to the secondary market is not an acceptable assumption in real world [2]. With this in mind, we decide to address a new approach in which the returned products from the customers can go back to the remanufacturing centers with the aim of re-entering to the forward logistics. Regarding this point, DCs may change their ordering patterns and on the other side, it is expected that the retailers' demands influenced by applying this strategy.

On the other hand, in most supply networks, the procurement of components and raw material from the suppliers incurred the most amounts of expenses on the downstream partners. For instance, 40-60% of production costs in most US manufacturers are related to the process of purchasing raw material from the supplier [3]. In this regard, an effective way to properly handle the purchasing costs is to reduce the operational costs. Regarding these points, we address a discount method with the aim of realizing the optimal purchasing decisions. In this regard, a pricing model under quantity discounts is applied in the proposed CLSC to represent the purchasing cost of raw material from the supplier.

In this paper, a multi-echelon closed-loop location-allocation-inventory problem (MCLIP) is presented in which some strategic decisions including facility location are investigated simultaneously with tactical ones like allocation and inventory decisions. Moreover, we survey the environmental impacts of employing different production technologies at manufacturing centers in cost forms. Furthermore, a pricing model based on quantity discount is proposed to illustrate the procurement cost of components and raw material from supplier. According to the mentioned importance for both location-allocation-inventory closed-loop supply chains and discount models, it seems that our attempt to consider the CLSC, discount, production technology, and returned products all together, would be a significant step to solve the problem have special position in the literature.

The main contributions of the existing study that distinguish it from the other similar works are:

- Presenting a novel multi-echelon CLSC
- Considering the environmental impacts of using different technologies in production process (i.e., $CO_2$ emission from the production process)
- Considering both the incremental price breaks (i.e., price levels) for the raw material delivered by the supplier and failure rate of raw material
- Investigating the effects of the fraction of the products returned from the reverse logistics on channel's ordering pattern and chain's demand quantity in the forward logistics
- Applying CCP (chance constraint programming) method

The rest of this paper is organized as follows. Section 2 addresses a brief literature review of the previous related researches. In Section 3, the problem description is presented. Section 4, contains the mathematical formulation of the proposed problem. Section 5 addresses the solution of the model via GAMS software which additionally includes some model evaluations and sensitivity analyses. Finally, conclusions and future research suggestions are discussed in Section 6.

## Literature Review

Closely related to the recent studies, the joint location-inventory problem is becoming an increasingly important issue, which was introduced by Baumol and Wolfe [4] for the first time. After a while, Teo, et al. [5] used an analytical modeling approach to study the impact of consolidated DCs into a central DC on the facility investments and inventory costs, and presentation with an example that for stochastic demand, the total investment costs of the facility and an integrated system can be worse a decentralized system. Nozick and Turnquist [6] considered a more completely model for individual products in a multi-product and two-echelon inventory system, that presents a method for optimizing the trade-off among customer service and cost, With developing concerns about dynamic environments. Freling, et al. [7] surveyed a single sourcing model with transportation and inventory costs in a dynamic environment. Shu, et al. [8] presented the stochastic inventory-transportation network design problem consisting of one supplier and multiple retailers in an uncertain environment. In addition, Miranda and Garrido [9] simulated the inventory-location decisions by the means of a non-linear mixed-integer model.

Recently, a new approach in the location-inventory problems was adopted by Daskin, et al. [10] and Shen, et al. [11], in which *(Q,r)* inventory policy was considered. Afterwards, Qi and Shen [12] added the routing decision to the inventory-location problem and showed effects of uncertainty on supply chain decisions. Javid and Azad [13] developed a novel model in a stochastic supply chain to optimize location, allocation, capacity, inventory, and routing decisions simultaneously. Berman, et al. [14] developed the literature by studying a coordinated location-inventory model, where DCs follow a periodic review *(R,S)* inventory policy. To this end, they introduced two types of coordination mechanisms: 1) partial coordination, in which each DC may choose its own review interval from the menu, and 2) full coordination, where all the DCs have an identical review interval. To expand the related models, Tancrez, et al. [15] studied the integrated location-inventory problem for three levels supply chain networks, including suppliers, DCs and retailers. Sadjadi, et al. [16] developed a three-level supply chain network with uncertain demand and lead time, which includes a single supplier, multiple DCs and retailers to optimize the facility location-allocation, retailers' demands, and inventory replenishment decisions, simultaneously. With the aim of focusing on the environmental considerations, Kumar, et al. [17] considered production and pollution routing problem with time window in a vehicle routing model, where the location and inventory decisions are integrated.

CLSC is the most important stream of related research efforts to our work. As a preliminary investigation, Chung, et al. [18] investigated an inventory system for traditional forward-oriented material flow also a reverse material flow, and analyzed a remanufacturing capability in a multi-echelon closed-loop model. Most recently, Abdallah, et al. [19], Kannan, et al. [20], and Diabat, et al. [21] studied the effects of forward and reverse logistics on carbon emissions in the closed-loop supply chains. In order to investigate a wide spectrum of decisions together, Nekooghadirli, et al. [22] studied a new bi-objective location-routing-inventory closed-loop problem which has been solved by four multi-objective meta-heuristic algorithms. In a broader case, Asl-Najafi, et al. [23], added the disruption problem to the dynamic location-allocation problem with two cost and time minimization objective functions solved by a new hybrid meta-heuristic algorithm based on MOPSO and NSGA-II. Kaya and Urek [24] developed the location, inventory and pricing decisions by MINLP models, which has been solved by meta-heuristic algorithms. Al-Salem, et al. [25] formulated a closed-loop inventory-location problem as an MINLP. They transformed the existing problem to an MIP by using the problem reformulated and piecewise linearization which is solvable exactly using CPLEX. In a new stream, Zhang and Unnikrishnan [26] solved a coordinated inventory-location problem in a closed-loop supply chain. It is noteworthy that some recent researches have widely studied the pricing, advertising, and coordination problems using game theory

Table 1 - Findings of the literature survey

| Reference | Closed-loop | Location | Inventory | Price discounts | Returned products | Multi-period | Production technology |
|---|---|---|---|---|---|---|---|
| [6] |  | × | × |  |  |  |  |
| [10] |  | × | × |  |  |  |  |
| [7] |  |  | × |  |  | × |  |
| [11] |  | × | × |  |  |  |  |
| [9] |  | × | × |  |  |  |  |
| [12] |  | × | × |  |  | × |  |
| [18] | × |  | × |  | × |  |  |
| [1] | × | × | × |  | × |  |  |
| [22] |  | × | × |  |  | × |  |
| [2] | × | × | × |  | × |  |  |
| [31] |  | × | × |  |  | × |  |
| Our work | × | × | × | × | × |  | × |

approach (i.e., Gao, et al. [27], De Giovanni, et al. [28], Bazan, et al, [29], and Xie, et al. [30]).

As can be seen in Table 1, there are few studies that address the main issues of this research all together. To be specific, studying a CLSC with considering environmental impacts of different production technologies under a quantity discount model would be an interesting topic in the existing literature.

## Problem description and assumptions

Figure 1 displays a detailed schematic view of the proposed CLSC. As depicted in Figure 1, first, the forward logistics supplier provides the required raw material for the manufacturers. Then, different kinds of products would be produced in the manufacturing centers using different production technologies. In the next step, DCs, retailers, and customers register their demands from the manufacturers, DCs, and retailers, respectively. We assume that the location of the retailers and both forward and reverse suppliers are known. Furthermore, the supplier and DCs are assumed un-capacitated. After shipping the products to the retailers, they separate the fraction of products, which are not qualified for customer's usage, and send back $\delta$ percent to the RCs so that $1-\delta$ percent of products can be used by customers. In the meantime, the RCs purchase raw material from the second supplier in the reverse channel with incremental price breaks (i.e., price level) while considering failure rate of raw material. With the aid of inspecting the returned products, they separate $\eta$ percent which are unrepairable and dispose them with a specific disposal cost. Therefore, the rest of returned products including $(1-\eta)$ percent will be repaired by the RCs spending reworking cost to prepare them for the forward logistics usage.

Due to the uncertain demands of the retailers, DCs keep safety stock to overcome demands fluctuations. In this paper, single sourcing allocation has been used in which a retailer can be linked to a single DC in the forward logistics as well as the retailers that would be able to transfer the returned products to a single RC. Moreover, a customer, supplier, and RC can be served by a single retailer, manufacturer, and supplier, respectively. In order to reduce transportation costs, we assume that each RC can only be located near DCs. The demands and returns for product $p$ at retailer $i$ are assumed to be independent and normally distributed, i.e. $N(\mu_{ip},\sigma_{ip}^2)$ and $N(\lambda_{ip},\rho_{ip}^2)$ where $\lambda_{ip}=\delta_p\mu_{ip}$, and $\delta\in[0,1]$.

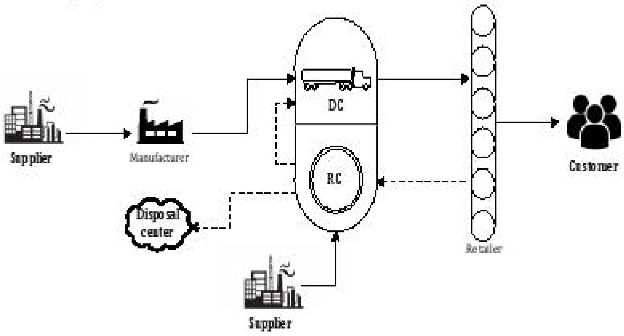

Figure 1 - Graphical model of investigation

## Problem formulation

In this section, in order to formulate the proposed problem, we first define the notations as follows:

*Sets*

|  |  |
|---|---|
| $I$ | set of retailers, indexed by $i$ |
| $J$ | set of potential regional distribution center sites, indexed by $j$ |
| $K$ | set of potential regional reworking center sites, indexed by $k$ ($j$ and $k$ are aliases) |
| $P$ | set of products, indexed by $p$ |
| $L$ | set of customers, indexed by $l$ |
| $M$ | set of potential regional disposal center sites, indexed by $m$ |
| $O$ | set of potential supplier sites in the forward logistics, indexed by $o$ |
| $R$ | set of potential supplier sites in the reverse logistics, indexed by $r$ |

| | |
|---|---|
| $S$ | set of required raw material for reworking operations, indexed by $s$ |
| $U$ | set of potential regional manufacturing center sites, indexed by $u$ |
| $T$ | set of different production technologies, indexed by $t$ |
| $V$ | set of incremental price breaks, indexed by $v$ |

*Parameters*

| | |
|---|---|
| $F_j$ | fixed cost of locating a DC at site $j$ |
| $F'_k$ | fixed cost of locating a RC at site $k$ |
| $G'_m$ | fixed cost of locating a disposal center at site $m$ |
| $G_u$ | fixed cost of locating a manufacturing center at site $u$ |
| $d_{ijp}$ | shipping cost of per unit of product $p$ from DC $j$ to retailer $i$ |
| $d_{ikp}$ | shipping cost of per unit of product $p$ from retailer $i$ to RC $k$ |
| $d'_{lip}$ | shipping cost of per unit of product $p$ from retailer $i$ to customer $l$ |
| $d''_{kmp}$ | shipping cost of per unit of product $p$ from RC $k$ to disposal center $m$ |
| $dd_{kjp}$ | shipping cost of per unit of product $p$ from RC $k$ to DC $j$ |
| $b_{kp}$ | unit reworking cost of product $p$ at RC $k$ |
| $c_{mp}$ | unit disposing cost of product $p$ at disposal center $m$ |
| $\beta$ | weight factor associated with transportation costs |
| $\theta$ | weight factor associated with inventory costs |
| $\gamma$ | weight factor associated with reworking cost |
| $\delta_p$ | fraction of returned product $p$ from forward logistics |
| $\eta_p$ | fraction of unrepairable product $p$ |
| $A_{jup}$ | fixed ordering cost of product $p$ from DC $j$ to the manufacturing center $u$ |
| $\lambda_{rs}$ | failure rate of raw material $s$ provided by supplier $r$ |
| $Z_\alpha$ | standard normal deviate with $P(Z \le Z_\alpha)$ |
| $g_{ujp}$ | fixed shipping cost of product $p$ from manufacturing center $u$ to DC $j$ |
| $g'_{ikp}$ | fixed shipping cost of product $p$ from retailer $i$ to RC $k$ |
| $a_{ujp}$ | variable shipping cost of product $p$ from manufacturing center $u$ to DC $j$ |
| $a'_{ikp}$ | variable shipping cost of product $p$ from retailer $i$ to RC $k$ |
| $LT_p$ | lead time for product $p$ from manufacturing center to DC $j$ |
| $h_p$ | unit holding cost of product $p$ at DCs and RCs |
| $q_{lp}$ | demand quantity of customer $l$ for product $p$ |
| $c'_{rks}$ | all shipping and displacement costs per unit of raw material $s$ from supplier $r$ to RC $k$ |
| $c''_{oup}$ | all shipping and displacement costs per unit of product $p$ from supplier $o$ to manufacturing center $u$ |
| $\tau_t$ | cost of environmental impacts for the production process using technology $t$ |
| $Cap_{ks}$ | capacity of RC $k$ for raw material $s$ |
| $Cap'_{rs}$ | capacity of supplier $r$ for raw material $s$ |
| $q'_{kps}$ | demand quantity of RC $k$ for raw material $s$ to repair the product $p$ |
| $\phi_{skrv}$ | purchasing cost of per unit of raw material $s$ from supplier $r$ to RC $k$ at price level $v$ |
| $\phi'_{skrv}$ | incremental price breaks occurs for raw material $s$ shipped from supplier $r$ to RC $k$ at price level $v$ |
| $\phi''_{skrv}$ | the existing price level $v$ for raw material $s$ from supplier $r$ to RC $k$ |

*Decision variables*

| | |
|---|---|
| $X_j$ | 1; if a DC is located at site $j$; and 0, otherwise |
| $Y_{ijp}$ | 1; if retailer $i$ is served by DC $j$ for product $p$; and 0, otherwise |
| $W_k$ | 1; if a RC is located at site $k$; and 0, otherwise |
| $Z_{ikp}$ | 1; if the returned product $p$ from retailer $i$ is collected by RC $k$; and 0, otherwise |
| $V'_{lip}$ | 1; if customer $l$ is served by retailer $i$ for product $p$; and 0, otherwise |
| $H_m$ | 1; if a disposal center is located at site $m$; and 0, otherwise |
| $O'_{kmp}$ | 1; if reworking center $k$ is served by disposal center located at $m$ for product $p$; and 0, otherwise |
| $W'_u$ | 1; if a manufacturing center is located at site $u$; and 0, otherwise |
| $X'_{rksv}$ | 1; if price level $v$ is used for raw material $s$ shipped from supplier $r$ to RC $k$, and 0, otherwise |
| $X''_{oup}$ | 1; if product $p$ is shipped from supplier $o$ to manufacturing center $u$; and 0, otherwise |
| $U'_{kjp}$ | 1; if product $p$ is shipped from RC $k$ to DC $j$; and 0, otherwise |
| $XP_{upt}$ | 1; if product $p$ is produced at manufacturing center $u$ using production technology $t$; and 0, otherwise |
| $XP'_{rksv}$ | quantity of raw material $s$ from the supplier to RC $k$ at price level $v$ |

Now, the proposed problem can be formulated as:

$$
\begin{aligned}
\min & \sum_{j\in J} f_j X_j + \beta \sum_{j\in J}\sum_{i\in I}\sum_{p} \mu_{ip} d_{ijp} Y_{ijp} \\
&+ \Big(\sum_{j\in J}\sqrt{\sum_{p}\sum_{u} 2\theta h_p (A_{jup} + \beta g_{ujp})\Big(\sum_{i\in I}\mu_{ip}Y_{ijp} - \sum_{k\in K}(1-\eta_p)\delta_p\mu_{ip}U'_{kjp}\Big)} \\
&+ \beta\sum_{p}\sum_{o}\sum_{j\in J}\sum_{i\in I} a_{ujp}\Big(\mu_{ip}Y_{ijp} - \sum_{k\in K}(1-\eta_p)\delta_p\mu_{ip}U'_{kjp}\Big)\Big) \\
&+ \beta\sum_{k\in K}\sum_{j\in J}\sum_{p} U'_{kjp}DD_{kjp} + \theta\sum_{p}\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} h_p(1-\eta_p)\delta_p\mu_{ip}U'_{kjp} \\
&+ \Big(\theta Z_\alpha h_p \sum_{j\in J}\sqrt{\sum_{p} LT_p \sum_{i\in I}\sigma_{ip}^2 Y_{ijp}}\Big) \\
&+ \sum_{k} f'_k W_k + \Big(\beta\sum_{p}\sum_{i\in I}\sum_{k\in K}\delta_p\mu_{ip}d_{ikp}Z_{ikp}\Big) \\
&+ \Big(\sum_{k\in K}\sqrt{\sum_{p}\sum_{i\in I} 2\theta h_p\delta_p\beta g'_{ikp}\mu_{ip}Z_{ikp}} + \beta\sum_{p}\sum_{k\in K}\sum_{i\in I} a'_{ikp}\delta_p\mu_{ip}Z_{ikp}\Big) \\
&+ \Big(\theta Z_\alpha h_p \sum_{k\in K}\sqrt{\sum_{p} LT_p \sum_{i\in I}\rho_{ip}^2 Z_{ikp}}\Big) \qquad (1) \\
&+ \sum_{i\in I}\sum_{p}\sum_{l} d'_{lip}V'_{lip} + \theta\sum_{p}\sum_{i\in I}\sum_{j\in J}(1-\delta_p)\mu_{ip}Y_{ijp} \\
&+ \sum_{m} H_m G'_m + \beta\sum_{p}\sum_{k\in K}\sum_{m}\sum_{i\in I}\eta_p\delta_p\mu_{ip}Y_{ikp}d''_{kmp}O'_{kmp} \\
&+ \sum_{p}\sum_{k\in K}\sum_{m}\sum_{i\in I} c_{mp}\eta_p\delta_p\mu_{ip}Y_{ikp}O'_{kmp} \\
&+ \gamma\sum_{p} b_{kp}(1-\eta_p)\delta_p\sum_{i\in I}\sum_{k\in K}\mu_{ip}Z_{ikp} + \sum_{u} W'_u G_u + \beta\sum_{p}\sum_{o}\sum_{u} c''_{oup}X''_{oup} \\
&+ \sum_{u}\sum_{p}\sum_{t} XP_{upt}\tau_t + \beta\sum_{r}\sum_{s}\sum_{k}\sum_{v} c'_{rks}XP'_{rksv}
\end{aligned}
$$

$$+\sum_{r}\sum_{s}\sum_{k}\sum_{v} XP'_{rksv}\Phi_{skrv}\Phi''_{skrv}+\sum_{s}\sum_{r}\sum_{k}\sum_{v} XP'_{rksv}\lambda_{rs}$$

$s.t.$

$$\sum_{j\in J} Y_{ijp}=1 \quad \forall i\in I, p \tag{2}$$

$$Y_{ijp}-X_j\leq \mathbf{0} \quad \forall i\in I, j\in J, p \tag{3}$$

$$\sum_{k\in K} Z_{ikp}=1 \quad \forall i\in I, p \tag{4}$$

$$Z_{ikp}-W_k\leq \mathbf{0} \quad \forall i\in I, k\in K, p \tag{5}$$

$$W_k-\sum_{i\in I} Y_{ikp}\leq 0 \quad \forall k\in K, p \tag{6}$$

$$\sum_{i\in I} V'_{lip}=1 \quad \forall p, l \tag{7}$$

$$\sum_{j\in J}(1-\delta_p)\mu_{ip}Y_{ijp}\geq \sum_{l} q_{lp}V'_{lip} \quad \forall i\in I, p \tag{8}$$

$$\sum_{m} O'_{kmp}=1 \quad \forall k\in K, p \tag{9}$$

$$O'_{kmp}-H_m\leq \mathbf{0} \quad \forall k\in K, m, p \tag{10}$$

$$\begin{aligned}&\sum_{l} q_{lp}V'_{lip}-(1-\delta_p)\mu_{ip}\sum_{j\in J} Y_{ijp}\\ &\leq \varphi^{-1}(\alpha)(1-\delta_p)\sum_{j\in J} Y_{ijp}\sigma_{ip}\end{aligned} \quad \forall i\in I, p \tag{11}$$

$$\sum_{o} \mathrm{X}''_{\mathrm{oup}}=1 \quad \forall p, u \tag{12}$$

$$\mathrm{X}''_{\mathrm{oup}}-\mathrm{W}'_{\mathrm{u}}\leq 0 \quad \forall p, o, u \tag{13}$$

$$\sum_{j\in J} U'_{kjp}=W_k \quad \forall p, k\in K \tag{14}$$

$$Z_{ikp}-\sum_{j\in J} U'_{kjp}\leq 0 \quad \forall i\in I, p, k\in K \tag{15}$$

$$\sum_{r}\sum_{s}\sum_{v} XP'_{\mathrm{rksv}}\leq \sum_{s} W_k Cap_{ks} \quad \forall k\in K \tag{16}$$

$$\sum_{\mathrm{k}}\sum_{v} XP'_{\mathrm{rksv}}\leq Cap'_{rs} \quad \forall r, s \tag{17}$$

$$\sum_{s}\sum_{v}\sum_{r} XP'_{\mathrm{rksv}}\geq \sum_{p}\sum_{s} q'_{kps}W_k \quad \forall k\in K \tag{18}$$

$$XP'_{\mathrm{rksv}}\leq (\phi'_{rksv}-\phi'_{rks(v-1)})X'_{rksv} \quad \forall k, r, s, v \tag{19}$$

$$XP'_{\mathrm{rksv}}\geq (\phi'_{rksv}-\phi'_{rks(v-1)})X'_{rks(v+1)} \quad \forall k, r, s, v \tag{20}$$

$$\begin{aligned}&W_k, Y_{ikp}, Z_{ikp}, X_j, V'_{lip}, H_m, O'_{kmp}, \mathrm{W}'_u, X'_{\mathrm{rksv}},\\ &X''_{\mathrm{oup}}, XP_{\mathrm{utp}}, U'_{kjp}\in\{0,1\}, XP'_{rksv}\geq 0\\ &\forall i, k, j, p, l, m, u, v, r, t, s, o\end{aligned} \tag{21}$$

The objective function (1) minimizes the total cost of the CLSC. The first term of objective function (1) indicates the fixed location cost of the DCs. The second term represents the delivery cost to the retailers from the assigned DCs. The third term indicates three main costs including holding cost at DCs, ordering cost of the DCs from manufacturers, shipping cost from manufacturers to DCs. The fourth term shows the delivery cost to the DCs from the assigned RCs. The fifth term is the holding cost of the reworked products which are kept at RCs and would be transferred to the DCs as soon as they need. The sixth term represents the total expected safety stock inventory cost based on risk pooling of the uncertainty in demand. The seventh term represents the fixed location cost of the remanufacturing centers. The eighth term represents the delivery cost from the retailers to the assigned RC. The Ninth term represents the total expected working inventory at the RC relative to the assigned returns. The Tenth term represents the total expected safety stock inventory cost. The Eleventh term represents the delivery cost from the retailers to the assigned customer. The twelfth term represents the percent of cost from the retailer to the customer's demand. The Thirteenth term represents the fixed location cost of the disposal centers. The Fourteenth term represents the delivery cost from the RC to the assigned disposal center. The Fifteenth term represents disposing cost of product at disposal center. The Sixteenth term represents reworking cost of product at RC. The Seventeenth term represents the fixed location cost of the manufacturing centers. The Eighteenth term represents the cost of sending products from supplier to the manufacturing centers. The Nineteenth term represents environmental impact of production at the manufacturing center with using a special technology. The Twentieth term represents the cost of sending raw material from supplier to RC. The two last terms represent the cost of buying raw material from supplier and the failure rate of raw material provided by suppliers.

Constraints (2) state that each retailer can only purchase from one and only the one DC. Constraints (3) denote that serving by a DC is not possible unless the corresponding DC is opened. Constraints (4) indicate that each retailer can return products just to one RC. Constraints (5) state that returns can only be made to open RCs. Constraints (6) indicate that a RC cannot be located unless a DC is opened at the same site and the retailers are assigned to this DC. Constraints (6) ensure that the proposed SC is closed-loop. Constraints (7) are similar to constraints (2). Constraints (8) ensure that all the customers' demands are satisfied. Constraints (9) indicate that each RC can deliver the products just to one disposal center. Constraints (10) denote that shipping to a disposal center is not possible unless the corresponding disposal center is opened. Constraints (11) are used for CCP implementation. Constraints (12) state that each manufacturer can only be supplied by one and only the one supplier. Constraints (13) denote that serving by a manufacturer is not possible unless the corresponding manufacturing center is opened. Constraints (14) state that if a RC is selected, then the entered reverse products must be shipped to a DC; otherwise, there should not be any assignment. Constraints (15) show that the possibility of having a reverse flow to the specific RC depends on the selection of that RC. Constraints (16) indicate that if a RC is selected, then it can just receive an amount of raw material up to his/her own capacity size from the supplier. Constraints (17) state that suppliers are capacitated. Constraints (18) ensure that RCs' demand for raw material are satisfied. Constraints (19) and (20) force quantities in the discount range for a vendor to be incremental. Because the "quantity" is incremental, if the order quantity lies in discount interval $V$, namely, $X'_{rksv} = 1$, then the quantities in interval 1 to $V-1$, should be at the maximum of those ranges [3]. Constraints (20) assure that a quantity in any range is no greater than the width of the range. Finally, constraints (21) are the standard integrality constraints.

## Solution via GAMS

In this section, we discuss the application of the discussed model in a real case study provided by Asl-Najafi, et al. [23]. Based on this case study, the product flow would be started by Tehran as a supplier in the forward logistics and continued through the DCs to the customers. In this case study, 14 major zones have been considered in Iran as DCs. In order to solve the presented problem in a short time via GAMS, only one kind of product $(p=1)$ and one kind of raw material. The proposed MINLP are considered $(s=1)$ model is solved by GAMS software on a Windows 8 with Intel® Core™ i7 CPU 2.50 GHz 2.49 GHz processor with 12.0 GB of RAM. Remind that the number of data has been used from a case study presented in [23] and the rest are generated randomly.

Table 2 - Parameter Values

| Parameters | $\beta$ | $\theta$ | $\gamma$ | $Z_\alpha$ | $\alpha$ | $\eta_p$ | $\delta_p$ |
|---|---|---|---|---|---|---|---|
| Values | 1 | 0.8 | 0.5 | 1.96 | 0.05 | 0.15 | 0.05 |

Table 3 - Retailer parameters

| No. | Retailer | $\mu_i$ | $\rho_i^2$ | $\sigma_i^2$ |
|---|---|---|---|---|
| 1 | Shiraz | 60 | 0.05 | 0.1 |
| 2 | Karaj | 75 | 0.05 | 0.2 |
| 3 | Rasht | 88 | 0.08 | 0.1 |
| 4 | Kermanshah | 45 | 0.05 | 0.12 |
| 5 | Esfahan | 215 | 0.07 | 0.2 |
| 6 | Tabriz | 120 | 0.05 | 0.1 |
| 7 | Ahvaz | 100 | 0.06 | 0.2 |
| 8 | Zanjan | 215 | 0.1 | 0.2 |
| 9 | Yazd | 185 | 0.05 | 0.15 |
| 10 | Arak | 211 | 0.07 | 0.15 |
| 11 | Qom | 180 | 0.08 | 0.1 |
| 12 | Kerman | 88 | 0.08 | 0.2 |
| 13 | Ardebil | 55 | 0.1 | 0.15 |
| 14 | Golestan | 300 | 0.05 | 0.12 |

Table 4 - Values of important parameters for DCs

| $j=k$ | $F_j$ | $g_j$ | $A_j$ | $a_j$ |
|---|---|---|---|---|
| DC1: Qom | 10000 | 10 | 11 | 5 |
| DC2: Esfahan | 7500 | 20 | 20 | 12 |
| DC3: Tabriz | 9500 | 21 | 23 | 15 |
| DC4: Arak | 8500 | 40 | 41 | 5 |
| DC5: Yazd | 1100 | 35 | 32 | 8 |
| DC6: Zanjan | 1300 | 41 | 35 | 12 |

**Table 5 – Customer $l$ is served by retailer $i$**

| | | | | |
|---|---|---|---|---|
| $L_1.i_{14}$ | $L_2.i_{12}$ | $L_3.i_{12}$ | $L_4.i_{11}$ | $L_5.i_{12}$ |
| 1 | 1 | 1 | 1 | 1 |
| $L_6.i_8$ | $L_7.i_7$ | $L_8.i_7$ | $L_9.i_7$ | $L_{10}.i_7$ |
| 1 | 1 | 1 | 1 | 1 |
| $L_{11}.i_8$ | $L_{12}.i_7$ | $L_{13}.i_{12}$ | $L_{14}.i_7$ | |
| 1 | 1 | 1 | 1 | |

Part of results can be stated as follows:

- ✓ $DC_1, DC_2 = 1$: located as a DC
- ✓ All of the retailers are served by DCs located at $j = 1, 2$.
- ✓ Retailer $i$ from 1 to 11 and 13,14 are served by $DC_1$ and retailer 12 is served by $DC_2$ .
- ✓ $DC_2 = 1$ is located as an RC in the reverse logistics.
- ✓ $DSC_4 = 1$ is located as a disposal center.
- ✓ $DC_j$ for $j$=1 to 6 is served by disposal center located at $DSC_4$ .
- ✓ Returned products from retailer $i$ (=1 to 14) are collected by the reworking center located at $DC_2$ .
- ✓ $U_3 = 1$ is located as a manufacturing center.
- ✓ $U_3.P_1.T_1 = 1$ means that product 1 is produced in manufacturing center 3 using technology 1 .
- ✓ Number of units of raw material shipped from supplier 1 to $RC_2$ at price levels 1 and 3: $RC_2.V_1 = 12$ and $RC_2.V_3 = 40$
- ✓ Total cost $Z^* = 105718.616$ .

## Sensitivity analyses

In this section, in order to gain more managerial insights, some sensitivity analyses based on the parameter values of Table 2 have been conducted and discussed in-depth.

In the direction of conducting sensitivity analysis, Figure 2 illustrates the change of TCSC (total costs of the supply chain) vs. changes of $\gamma$ as the reworking cost weights $with\,(\beta = 1, \theta = 0.8)$.

Figure 3 illustrates the change of TCSC vs. changes of transportation cost weight $(\beta), with\,(\gamma = 0.5, \theta = 0.8)$.

Figure 4 illustrates the change of TCSC vs. changes of $\theta$ as the inventory cost weight $with\,(\gamma = 0.5, \beta = 1)$.

With respect to Figure 2, a considerable incremental behavior can be seen in TCSC by increasing the reworking cost weight. On the opposite, as the amount of transportation cost weight increases, a non-linear behavior in the TCSC appears which can be seen in Figure 3. It is hence important for decision makers to choose an appropriate value for $\beta$. Moreover, based on Figure 4, as the inventory cost weight increases, an ascending behavior can be seen in the TCSC. More importantly, the model is more sensitive in low values of the transportation cost weight compared to the higher values of $\beta$, which indicated another insightful managerial implication about the transportation cost weight. As a remark, TCSC's changes under the variation of the transportation cost weight has more incremental slope than the $\theta$ and $\gamma$ variations. In light of this, a variation in $\beta$ has more impact in TCSC relative to an equal change in $\theta$ and $\gamma$.

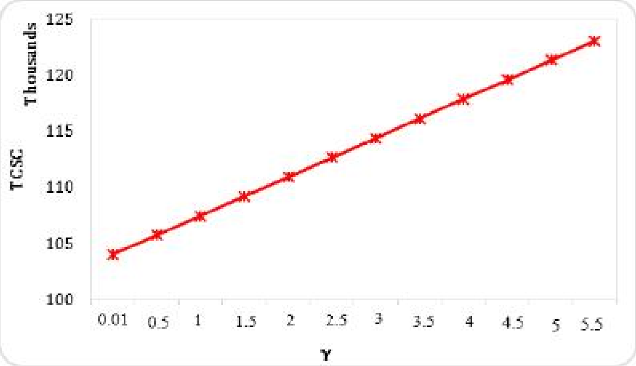

Figure 2 - Sensitivity analysis for $\boldsymbol{\gamma}$

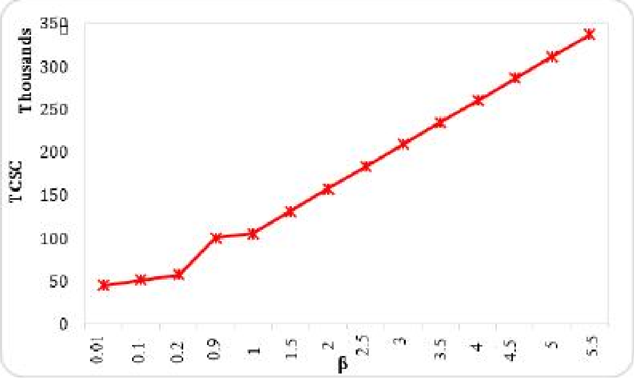

Figure 3 - Sensitivity analysis for $\boldsymbol{\beta}$

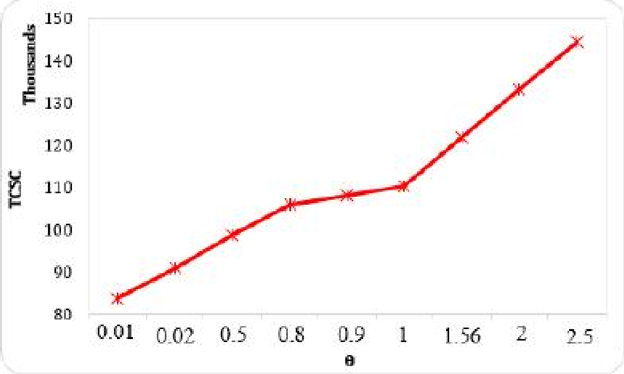

Figure 4 - Sensitivity analysis for $\boldsymbol{\theta}$

Table 6, demonstrates the effect of transportation cost weight on the number of facilities opened in the forward SC. It can be concluded that as $\beta$ increases, the number of opened DCs increases subsequently.

Table 6 – Analysis on different values of parameter $\beta$

| $\beta$ | 0.1 | 0.2 | 0.9 | 2.5 | 3 | 3.5 |
|---|---|---|---|---|---|---|
| DCs | 2 | 2 | 1, 2 | 1, 2 | 1, 2 | 1, 2 |

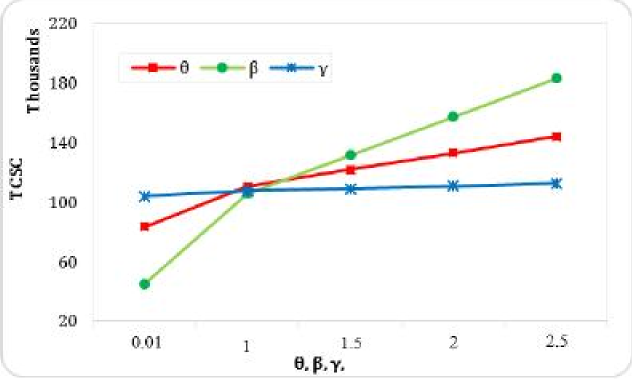

Figure 5 - Alteration of $\gamma, \theta, \beta$ *.vs.TCSC*

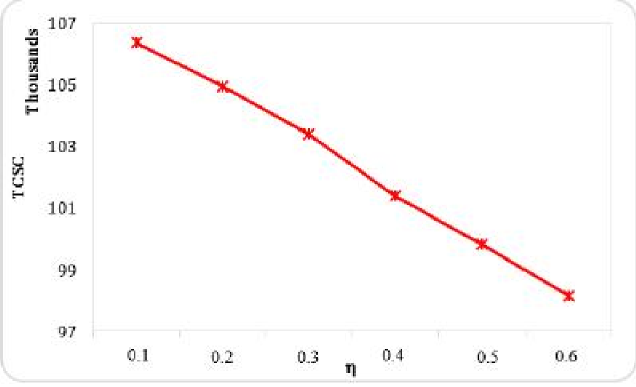

Figure 6 - Sensitivity analysis for $\eta$ *.vs.TCSC*

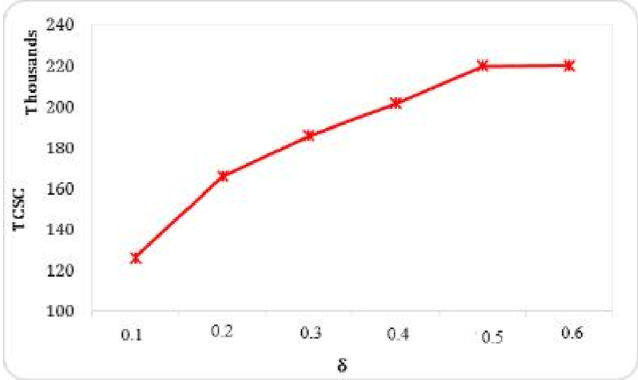

Figure 7 - Sensitivity analysis for $\delta$ *.vs.TCSC*

The significant impact of $\beta$ on the slope of the TCSC's graph compare to $\theta$ and $\gamma$ can be seen in Figure 5. It can be concluded that transportation decisions make the most important part of the proposed network problem and have considerable effect on the decisions of the whole supply chain relative to the other similar parameter values.

Remind that in the proposed model, two kinds of events can be happened for the returned products: disposing or reworking. Regard to our assumption that the disposal cost of per returned product is much lower than the reworking cost i.e., $(c_{mp} < b_{kp})$, the model expectedly prefers to dispose the most fraction of the returned products instead of sending back them to the forward logistics. In this regard, as $\eta$ (fraction of unrepairable products) increases, the TCSC decreases that has been illustrated in Figure 6. Due to the high disposal costs of some products, it is more affordable to have reworking operations on them with the aim of returning to the forward logistics.

As expected, by investigating Figure 7, it can be concluded that as fraction of returns from forward logistics increases, the respective costs of reverse logistics increases which is the sufficient reason for increasing the TCSC. By solving the proposed model on a large scale problem consisting of 14 demand zones and 6 DCs, and setting the parameters based on afore-mentioned calculations, Esfahan is selected as a DC and RC in the forward and reverse channel. Note that in the large scale problem, we fixed the supplier location on Tehran.

## Conclusion

In this paper, a multi-echelon closed-loop location-allocation-inventory problem (MCLIP) is presented in which some strategic and tactical decisions are analyzed in depth. In order to investigate the green part of the proposed SC, environmental impacts of different production technologies employed by manufacturers are considered in cost forms. Furthermore, a pricing model based on quantity discount is proposed to illustrate the purchasing cost of raw material from supplier in the reverse channel. The capability of returning the reworked products to the forward logistics that can affect the ordering patterns of DCs is another significant issue has been surveyed. The problem was formulated as a mixed integer nonlinear location-allocation model. Finally, several sensitivity analyses were conducted to provide some managerial insights. Some extensions may be valuable for future researches, for example, considering the routing decisions beside the location-inventory problem. Furthermore, considering the total time of the SC (i.e., transportation time) as the second objective function along with the total cost can be a practical extension for the proposed model.

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