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Application of the Fuzzy Auto- Regressive Integrated Moving Average (FARIMA) Model for Iran's Steel Consumption Forecasting

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Abstract

The use of non-stochastic models such as fuzzy time series forecasting models for time series analysis has attracted the attention of researchers in recent years. Auto-Regressive Integrated Moving Average (ARIMA) models are one of the most important time series models used in financial market forecasting. Recent research activities in time series forecasting indicate that two basic limitations detract from their popularity for time series forecasting: (1) ARIMA models assume that future value of a time series have a linear relationship with current and past values as well as with white noise. (2) ARIMA models require a large amount of historical data in order to produce accurate results. Fuzzy autoregressive integrated moving average (FARIMA) models are the fuzzy improved version of the autoregressive integrated moving average (ARIMA) models, proposed in order to overcome limitations of the traditional ARIMA models; especially data limitation, and yield more accurate results. Empirical results of Iran's steel consumption forecasting indicate that the proposed model exhibit effectively improved forecasting accuracy, so it can be used as an alternative model to steel consumption forecasting, especially when the scrimpy data made available.

Keywords: Auto- Regressive Integrated Moving Average, Fuzzy models, Time series forecasting, crude Steel consumption.

1. Introduction

The steel industry is among the strategic industries and it plays an important role in the persistent economic growth of countries. Therefore, awareness of current and future conditions of this industry and identification of factors affecting them is important to economic analysis. The most important purposes of economic analysis are cited as identification of effective factors and accurate predictions. One of the main challenges faced by managers, is to provide logical approaches in order to create balance between supply and demand and control their associated parameters. In addition, one of the most important factors of industrial development is consumption management in all of the developed countries. As a result, efficient management and goals achievement need not only steel production management but also steel consumption management [1].

Hence, the main aim of this paper is to use scientific methods to manage consumption crude steel in the country using historical observations. However, the literature shows that yield accurate results in the prediction of consumption; especially in long-term horizon is relatively difficult. Researchers believe that main reason of this matter is scrimpy data and high level of complexity and uncertainty in financial markets.

Commonly, predictions accuracy is one of the most important factors affecting the quality of financial decision-making. In recent years, financial forecasting based on previously observed values has become an important issue in investment and financial decision-making and has drawn considerable attention as an active research area [2-3]. In the other hand time series forecasting is an important area of forecasting in which past observations of the same variable are analyzed in order to develop a model describing the underlying relationship. This modeling approach is particularly useful when little knowledge is available on the underlying data generating process or when there is no satisfactory explanatory model that relates the dependent variable to other explanatory variables. Several different approaches have been proposed to time series forecasting. One of the most popular and widely used time series models are autoregressive integrated moving average

(ARIMA) models that have enjoyed fruitful applications in forecasting problems. The popularity of the ARIMA model is due to its statistical properties as well as the well-known Box–Jenkins methodology [4], in the model building process. In addition, ARIMA models can implement various exponential smoothing models. Although ARIMA models have the advantage of accurate forecasting in a short time period and easy to implement, these models have some limitations that detract from their popularity for financial time series forecasting: *(1)* ARIMA models assume that future value of a time series have a linear relationship with current and past values as well as with white noise. (2) ARIMA models need large amount of historical data in order to yield desired results. According to data limitation, ARIMA models require at least fifty, or preferably one hundred and higher, data in order to yield desired results. However, in real situations, due to uncertainty resulting from the integral environment and rapid development of new technology, future situation must be forecasted using small data set over a short span of time. Efficient forecasting methods are, therefore, needed today that can achieve their objectives in situations with small quantities of historical data available. However, in our society today, due to factors of uncertainty from the integral environment and rapid development of new technology, we usually have to forecast future situations using little data in a short span of time.

Fuzzy forecasting methods are suitable under incomplete data conditions and require fewer observations than other forecasting models. Fuzzy theory [5-6] was originally developed to deal with problems involving linguistic terms and have been successfully applied to financial time series forecasting [7-9]. Tanaka et al. have suggested fuzzy regression to solve the fuzzy environment and to avoid a modeling error [10-12]. Song and Chissom [13–15] have presented the definition of fuzzy time series and outlined its modeling by means of fuzzy relational equations and approximate reasoning. Chen [16] presented a fuzzy Time series method based on the concept of Song and Chissom.

The fuzzy regression is an interval forecasting model that suitable for the condition of little attainable historical data. However, the performance of the fuzzy regression models is not always satisfactory. In addition, these models do not include the concepts of the Box-Jenkins models for time series forecasting.

In order to fulfill the limitations of the autoregressive integrated moving average models and also to yield more accurate results, the fuzzy autoregressive integrated moving average (FARIMA) is proposed by Tseng et al. [17]. This model is formulated based on the basic concepts of the ARIMA model and Tanaka fuzzy regression that combine the advantages of the fuzzy regression and ARIMA models. In FARIMA models, instead of using crisp parameters, fuzzy parameters, in the form of triangular fuzzy numbers are used. By using the fuzzy parameters, the requirement of historical data would be reduced.

In recent years, more hybrid forecasting models have been developed, integrating autoregressive integrated moving average (ARIMA) and fuzzy models together in order to improve the prediction accuracy and overcome the deficiencies of the single models.

Andres *et al.* [18] proposed a strategy for constructing a hybrid model, which combines the fuzzy clustering and the multivariate adaptive regression splines (MARS) in order to use their theoretical advantages of these models for bankruptcy forecasting, especially when the information applied for forecasting is drawn from company financial statements. Lin and Cobourn [19] combined Takagi–Sugeno fuzzy system and a nonlinear regression (NLR) model for daily ground-level ozone predictions. Chang *et al.* [20] developed a hybrid model by integrating fuzzy rule base (FRB), self-organization maps (SOMs), and Genetic Algorithms (GAs) to forecast the future sales of a printed circuit board factory. Teoh *et al.* [21] proposed a hybrid model based on multi-order fuzzy time series, which employs rough sets theory in order to mine fuzzy logical relationship from time series and an adaptive expectation model to adjust forecasting results, to improve forecasting accuracy.

Khashei *et al.* [22] based on the basic concepts of multilayer perceptrons (MLPs), proposed a new hybrid model for financial time series forecasting using fuzzy regression models in order to overcome the data limitation of the multilayer perceptrons and yield more accurate results, especially in incomplete data situations. Li and Su [23] introduced a hybrid model, integrating genetic algorithm and hierarchical adaptive network-based fuzzy inference system (HANFIS) in which GA optimizes the structure and number of fuzzy if-then rules in a hierarchical ANFIS by finding the best parameter values of a subtractive clustering method.

Pai [24] proposed the hybrid ellipsoidal fuzzy system (HEFST) model to forecast regional electricity loads in Taiwan. Azadeh *et al.* [25] presented a hybrid algorithm based on fuzzy linear regression (FLR) and fuzzy cognitive map (FCM) to deal with the problem of forecasting and optimization of housing market fluctuations. Huang *et al.* [26] presented a new forecasting model based on two computational methods, fuzzy time series and particle swarm optimization for academic enrollments.

Javedani *et al.* [27] in their research introduced a hybrid method combining Auto Regressive Fractional Integrated Moving Average models and Fuzzy Time Series for the forecast of long memory time series. The proposed method was developed as one algorithm consisting of two phases. The first phase is related to the autoregressive part of the model, while the second phase is related to the Moving Average part. Based on these ideas, the combined ARFIMA and FTS model is introduced and for the parameter estimation of the model, Particle Swarm Optimization

(PSO) method is selected, based on its performance on similar optimization problems.

Barak and Sadegh [28] forecasted the annual energy consumption in Iran using 3 patterns of ARIMA–ANFIS model. In the first pattern, ARIMA model was implemented on 4 input features, where its nonlinear residuals were forecasted by 6 different ANFIS structures including grid partitioning, sub clustering, and fuzzy c-means clustering (each with 2 training algorithms). In the second pattern, the forecasting of ARIMA in addition to 4 input features was assumed as input variables for ANFIS prediction. In the third pattern, the second pattern was applied with AdaBoost (Adaptive Boosting) data diversification model and a novel ensemble methodology was presented.

Su and Cheng [29] proposes a novel ANFIS (Adaptive Neuro Fuzzy Inference System) time series model based on integrated nonlinear feature selection (INFS) method for stock forecasting. In this study proposed an integrated nonlinear feature selection method to select the important technical indicators objectively. Secondly, it used ANFIS to build time series model and test forecast performance, then utilized adaptive expectation model to strengthen the forecasting performance. Bisht and Kumar [30] suggested a fuzzy time series forecasting method based on hesitant fuzzy sets for forecasting in the environment of hesitant information. The proposed method addresses the problem of establishing a common membership grade for the situation when multiple fuzzification methods are available to fuzzify time series data. Also, in this study an aggregation operator for aggregating hesitant information was proposed.

Garvalho and Costa [31] proposed a fuzzy forecasting methodology of time series, which was tested on two series. The method uses a triangular membership function in a fuzzification process, including an α-cut, and applies the extended autocorrelation function. The identification algorithm enables optimization of the number of fuzzy sets to be used, to determine the optimal order for the fuzzy prediction model and estimate its parameters with greater accuracy.

The rest of the paper is organized as follows. In the next section, the basic concepts of the fuzzy autoregressive integrated moving average (FARIMA) models are briefly reviewed. In section 3 the proposed model is applied to Iran's steel consumption forecasting and its performance is compared with ARIMA model. Conclusions will be the final section of the paper.

2. The Fuzzy Autoregressive Integrated Moving Average (FARIMA) model

For more than half a century, the autoregressive integrated moving average (ARIMA) models have dominated many areas of time series forecasting. In an ARIMA (*p*,*d*,*q*) model, the future value of a variable is assumed to be a linear function of several past observations and random errors [32]. That is, the underlying process that generates the time series with the mean μ has the form:

$$\varphi(B)\nabla^d\left(y_t-\mu\right)=\theta(B)a_t \tag{1}$$

where, y_t and a_t are the actual value and random error at time period *t*, respectively: $\varphi(B)=1-\sum_{i=1}^{p}\phi_i B^i$, $\theta(B)=1-\sum_{j=1}^{q}\theta_j B^j$ are polynomials in B of degree p and q, ϕ_i $(i=1,2,...,p)$ and θ_j $(j=1,2,...,q)$ are model parameters, $\nabla=(1-B)$, *B* is the backward shift operator, *p* and *q* are integers and often referred to as orders of the model, and *d* is an integer and often referred to as order of differencing. Random errors, a_t, are assumed to be independently and identically distributed with a mean of zero and a constant variance of. σ^2

However, the parameters of the autoregressive integrated moving average, $\varphi_1,\varphi_2,....,\varphi_p$ and $\theta_1,\theta_2,....,\theta_q$ are crisp. In the fuzzy autoregressive integrated moving average models [33], Instead of using these crisp parameters, fuzzy parameters, $\tilde{\varphi}_1,\tilde{\varphi}_2,....,\tilde{\varphi}_p$ and $\tilde{\theta}_1,\tilde{\theta}_2,....,\tilde{\theta}_q$, in the form of triangular fuzzy numbers are used. A fuzzy ARIMA model is described by a fuzzy function with a fuzzy parameter as follows:

$$\tilde{\Phi}_p(B)W_t=\tilde{\theta}_q(B)a_t \tag{2}$$

$$W_t=(1-B)^d\left(Z_t-\mu\right) \tag{3}$$

$$\begin{aligned}\tilde{W}_t &= \tilde{\phi}_1 W_{t-1}+\tilde{\phi}_2 W_{t-2}+....+\tilde{\phi}_p W_{t-p}\\ &+a_t-\tilde{\theta}_{p+1}a_{t-1}-\tilde{\theta}_{p+2}a_{t-2}-...-\tilde{\theta}_{p+q}a_{t-q}\end{aligned} \tag{4}$$

where $\{Z_t\}$ are observations,and $\tilde{\varphi}_1,\tilde{\varphi}_2,....,\tilde{\varphi}_p$:ed asfi) is modi4, are fuzzy numbers. Eq. ($\tilde{\theta}_1,\tilde{\theta}_2,....,\tilde{\theta}_q$

$$\begin{aligned}\tilde{W}_t &= \tilde{\beta}_1 W_{t-1}+\tilde{\beta}_2 W_{t-2}+....+\tilde{\beta}_p W_{t-p}\\ &+a_t-\tilde{\beta}_{p+1}a_{t-1}-\tilde{\beta}_{p+2}a_{t-2}-...-\tilde{\beta}_{p+q}a_{t-q}\end{aligned} \tag{5}$$

Fuzzy parameters in the form of triangular fuzzy numbers are used:

$$\mu_{\beta_i}(\beta_i)=\begin{cases}1-\dfrac{|\alpha_i-\beta_i|}{c_i} & if \quad \alpha_i-c_i\le\beta_i\le\alpha_i+c_i,\\ 0 & otherwise,\end{cases} \tag{6}$$

where $\mu_{\tilde{\beta}}(\beta_i)$ is the membership function of the fuzzy set that represents parameter β_i, α_i is the center of the fuzzy number, and c_i is the width or spread around the center of the fuzzy number. Using fuzzy parameters β_i in the form of triangular fuzzy numbers and applying the extension principle, the membership of *W* in Eq. (5) is given as:

$$\mu_{\tilde{w}}(W_t)=\begin{cases}1-\dfrac{\left|W_t-\sum_{i=1}^{p}\alpha_i W_{t-i}-a_t+\sum_{i=p+1}^{p+q}\alpha_i a_{t+p-i}\right|}{\sum_{i=1}^{p}c_i\left|W_{t-i}\right|+\sum_{i=p+1}^{p+q}c_i\left|a_{t+p-i}\right|} & \\ \quad for\quad W_t\neq 0,\quad a_t\neq 0 & \\ 0 \qquad\qquad\qquad Otherwise\end{cases}\tag{7}$$

Simultaneously, Z_t represents the *t*th observation, and *h*-level is the threshold value representing the degree to which the model should be satisfied by all the data points $y_1, y_2, \ldots, y_k$ to a certain *h*-level. A choice of the *h-level* value influences the widths *c* of the fuzzy parameters:

$$\mu_y(y_t)\geq h \qquad for\quad t=1,2,\ldots,k \tag{8}$$

The index *t* refers to the number of non-fuzzy data used for constructing the model. On the other hand, the fuzziness *S* included in the model is defined by:

$$S=\sum_{i=1}^{p}\sum_{t=1}^{k}c_i\left|\varphi_{ii}\right|\left|W_{t-i}\right|+\sum_{i=p+1}^{p+q}\sum_{t=1}^{k}c_i\left|\rho_{i-p}\right|\left|a_{t+p-i}\right| \tag{9}$$

where ρ_{i-p} is the autocorrelation coefficient of time lag *i-p*,icient of time ffis the partial autocorrelation coe φ_{ii} lag *i*.

The weight of c_i depends on the relation of time lag *i* and the present observation, where the *p* of AR (p) is derived by PACF and the *q* of MA (q) is derived by ACF. Next, the problem of finding the fuzzy ARIMA parameters was formulated as a linear programming problem:

$$\begin{aligned}&Minimize && S=\sum_{i=1}^{p}\sum_{t=1}^{k}c_i\left|\phi_{ii}\right|\left|W_{t-i}\right|+\sum_{i=p+1}^{p+q}\sum_{t=1}^{k}c_i\left|\rho_{i-p}\right|\left|a_{t+p-i}\right|\\ & && \sum_{i=1}^{p}\alpha_i W_{t-i}+a_t-\sum_{i=p+1}^{p+q}\alpha_i a_{t+p-i}+(1+h)\left(\sum_{i+1}^{p}c_i\left|W_{t-i}\right|+\sum_{i=p+1}^{p+q}c_i\left|a_{t+p-i}\right|\right)\geq W_t\ t=1,2,..,k\\ &subject.to && \sum_{i=1}^{p}\alpha_i W_{t-i}+a_t-\sum_{i=p+1}^{p+q}\alpha_i a_{t+p-i}+(1+h)\left(\sum_{i+1}^{p}c_i\left|W_{t-i}\right|+\sum_{i=p+1}^{p+q}c_i\left|a_{t+p-i}\right|\right)\leq W_t\ t=1,2,..,k\\ & && c_i\geq 0 \qquad for\ i=1,2,\ldots,p+q\end{aligned}\tag{10}$$

At last, according to the Ishibuchi and Tanaka [34] opinion, the data around the model's upper bound and lower bound is deleted when the fuzzy ARIMA model has outliers with wide spread, and then reformulating the fuzzy regression model.

3. Application of the FARIMA model to Iran's steel consumption forecasting

As regards the increase in consumption of steel products, estimate of future trends incorrectly leads to waste of financial and human resources. Therefore, it is necessary to use the forecasting methods with highest precision and efficiency. According to this, in order to demonstrate the effectiveness and superiority of the fuzzy auto- regressive integrated moving average model to traditional model, the following application in Iran's steel consumption forecasting have been considered. In this section, the process of the proposed model is illustrated. According to this, at first fitted the traditional ARIMA model to data, next, as discussed earlier the problem of finding the fuzzy ARIMA parameters are formulated as a linear programming problem. Finally, performances of two models were compared together. It should be noted that the information used in this investigation consists of 37 annual observations of the crude steel consumption of Iran from *1357 to 1393* that are shown in Fig. 1. In this way, 32 observations are first used to formulate models and the last five observations are used to evaluate the performances of models.

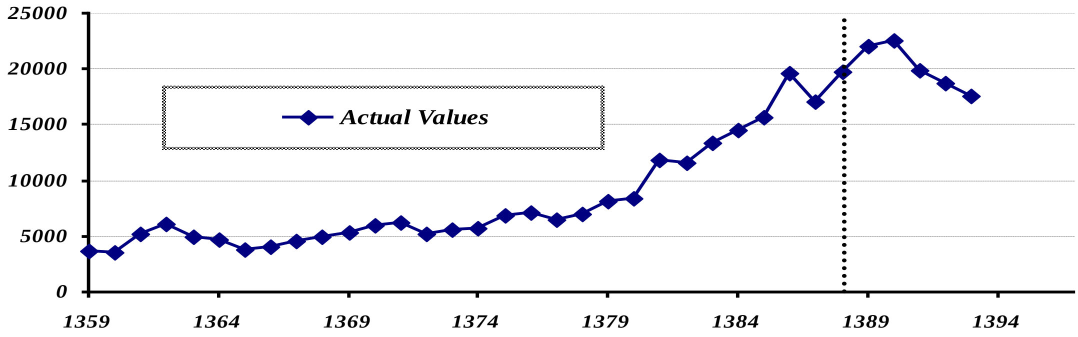

Figure 1- Iran's steel consumption (In thousand tones) from 1357 to 1393.

3.1. Fitting Auto- Regressive Integrated Moving Average (ARIMA) Model

Using the Eviews (9) package software, the best-fitted model is ARIMA (2, 1, 1). The fitted values by ARIMA model are plotted in Fig. 2.

$$y_t = 0.8496\, y_{t-1} + 0.1806\, y_{t-1} + 0.1374\, \varepsilon_{t-1} \qquad (11)$$

3.2. Fitting Fuzzy Auto- Regressive Integrated Moving Average (FARIMA) Model

Setting $(\alpha_0, \alpha_1, \alpha_2)$ = (*0.8496, 0.1806, 0.1374*), the fuzzy parameters are calculated using Eq. (10) (with $h=0$) as follows. The obtained upper and lower bounds in this stage are plotted in Fig. 3.

$$y_t = \langle 0.8496, 0.0396 \rangle\, y_{t-1} + \langle 0.1806, 0.00 \rangle\, y_{t-1} + \langle 0.1374, 0.00 \rangle\, \varepsilon_{t-1} \qquad (12)$$

It can be seen in fig. 3. that the actual values located in the fuzzy intervals, however, the thread of fuzzy intervals are too wide, especially when the macroeconomic environment is smooth. Therefore, the method of Ishibochi and Tanaka is applied in the next stage in order to resolve this problem and provide a narrow interval for the decision maker.

3.3. Deleting the outliers

It is known from the aforementioned results that the observation of *1381* is located at the upper bound (outlier); therefore, the LP constrained equation that is produced by this observation is first deleted and then the stage II is renewed, with $h=0$. The obtained upper and lower bounds in this stage are plotted in Fig. 4.

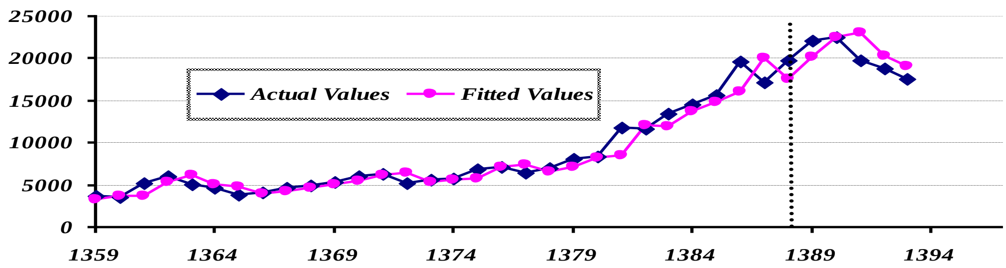

Figure 2- ARIMA fitted values for Iran's steel consumption.

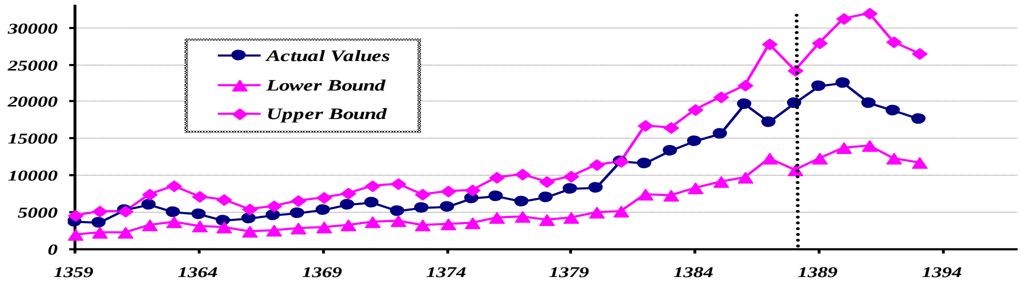

Figure 3- Upper and Lower bounds obtained for Iran's steel consumption..

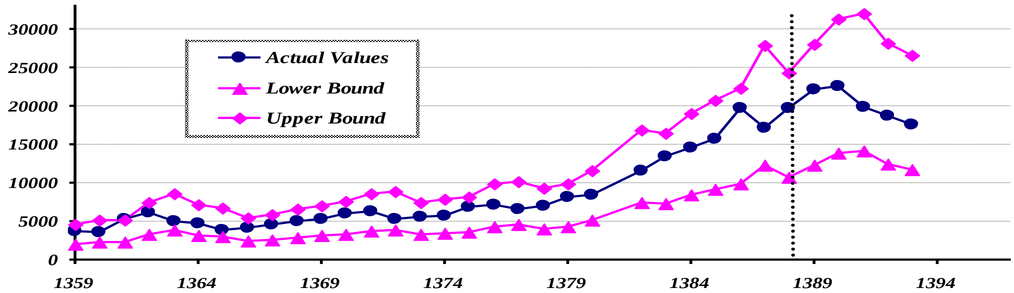

Figure 4- Upper and Lower bounds obtained for Iran's steel consumption (after deleting).

3.4. Comparison forecasting models

In this section, the predictive capability of fuzzy and nonfuzzy forecasting models both interval and point estimation cases is compared together, using Iran's crude steel consumption data sets. As mentioned previously the considered fuzzy and nonfuzzy interval forecasting models in this study are respectively including the fuzzy autoregressive integrated moving average (FARIMA) and traditional autoregressive integrated moving average (ARIMA). The width of the forecasted interval, and MAE (Mean Absolute Error) and RMSE (Root Mean Squared Error) are respectively employed as performance indicators in order to measure forecasting performance in the interval and point estimation cases. The MAE and MSE are respectively computed from the following equations:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|e_i| \qquad (13)$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(e_i)^2} \qquad (14)$$

Based on the results obtained from these case studied (Table 1), the predictive capabilities of the proposed model are rather encouraging and the possible interval by the proposed model with 100% confidence is narrower than the possible interval of the obtained interval by ARIMA (95% Confidence Interval) model. The width of the forecasted interval in the proposed model is 16298.19 in Iran's steel consumption forecasting case, indicating a 32.95% improvement upon the possible interval of the ARIMA (95% Confidence Interval).
In addition, according to the numerical results (Table 2 and 3), the MAE and RMSE of the proposed model are lower than the ARIMA for steel consumption case. For example in terms of RMSE, the percentage improvements of the proposed model over the ARIMA, are 8.46% in the steel consumption forecasting case.

Table 1- Comparison of the performance of forecasting models (interval estimation)

Model	Forecast Interval Width	Improvement Percentage	
		ARIMA (95% Confidence)	Fuzzy ARIMA
ARIMA	24310.02	0	….
Fuzzy ARIMA	16298.19	32.95%	0

Table 2- Comparison of the performance of forecasting models (point estimation)

Model	Performance of models		Improvement percentage of forecasting models	
	RMSE	MAE	RMSE (%)	MAE (%)
ARIMA	1403.146	990.6766	….	….
Fuzzy ARIMA	1284.440	828.7010	8.46	16.35

4. Conclusions

In today's world, quantitative methods have become important tools for financial markets forecasting and for improving decisions and investments. Time series analysis and forecasting has been an active research area for a last few decades. The accuracy of Time series forecasting is one of the most important factors in choosing the forecasting method. Despite the numerous time series models available, the research for improving the effectiveness of forecasting models to be continued. Several large-scale forecasting competitions with a large number of commonly used time series forecasting models conclude that combining several models or using hybrid models can be an effective way to improve forecasting performance. Additionally, because of the possible unstable or changing patterns in the data or scant data using the hybrid method can reduce the model uncertainty, which typically occurred in statistical inference and time series forecasting.

In this paper in order to modeling uncertainties in data sets and improve performance of time series forecasting, applied a method for time series forecasting is proposed drawing upon the basic concepts of ARIMA and fuzzy regression models. In this model the fuzzy numbers and fuzzy logic are applied in order to overcome the data limitation of the ARIMA models and provides a more flexible model for forecasting in less available data situations. Empirical results of Iran's crude steel consumption data sets indicate that the fuzzy ARIMA model can be used as an alternative forecasting tool for financial markets forecasting.

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