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A mathematical model for integrating cell formation with machine layout and cell layout in dynamic cellular manufacturing systems

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Abstract

The fundamental function of a Cellular Manufacturing System (CMS) is based on definition and recognition of a type of similarity among the parts that must be produced in a planning period. Cell Formation (CF) and cell machine layout design are two important steps in implementation of the CMS. This paper represents a new mathematical nonlinear programming model for dynamic cell formation that employs the rectilinear distance notion to determine the layout in the continuous space. In the presented model, the objective function accurately calculates the costs of inter and intra-cell movements of parts and calculates the cost of a cell restructure. Due to the problem complexity, the presented mathematical model is categorized in NP-hardness; thus, a Genetic Algorithm (GA) is used for solving this problem. Comparing the results from the GA and the results from solving a linear model through Lingo specifies that solving the presented model accurately for more than four machines in a logical time is impossible.

Keywords:
Cellular Manufacturing System (CMS), Cell Formation (CF), inter-cell layout, intra-cell layout, Genetic Algorithm (GA).

1. Introduction

In today's world, due to the enhancements in the customer's power of choice and extension of competitive markets, organizations are required to reform their structures. Group Technology (GT) is a production philosophy that aims to determine, categorize and assign parts to groups and part families and also it assigns machines to cells to produce these part families. This process is based on the parts characteristics and their similarity, called Similarity Coefficient (SC).

A Cellular Manufacturing System (CMS), which is the most important application of GT overcomes the inefficiency of traditional approaches through reduction in transportation time and distance. A flow shop layout has high efficiency in a mass production system, while a job shop is a very flexible system for producing various parts. In fact, each of these systems does not have other benefits. The CMS is an approach between these two manufacturing systems that aims to improve flexibility and efficiency to produce manufacturing groups in different sizes (see Figures 1 to 3). In the CMS, machines and parts assignment to cells must impose minimum cost to the system. After determining the assignment to machines and parts, machines locations must be determined. This issue is referred as Cell Layout (CL).

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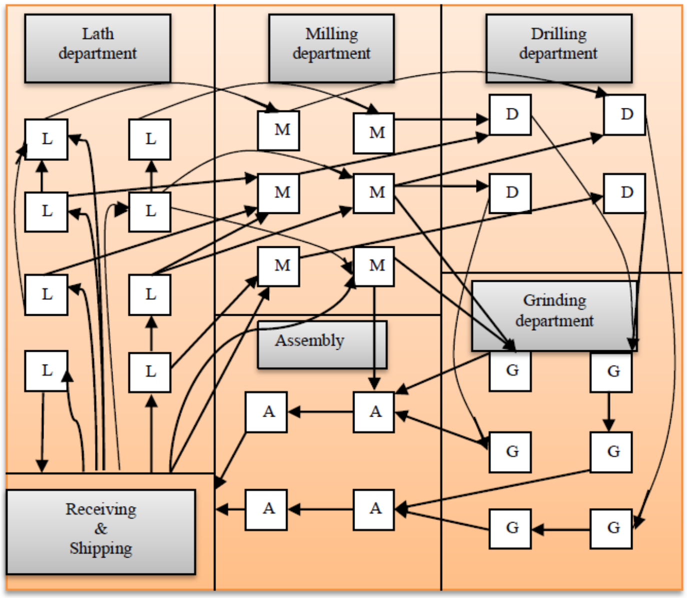

Figure (1). Job shop system

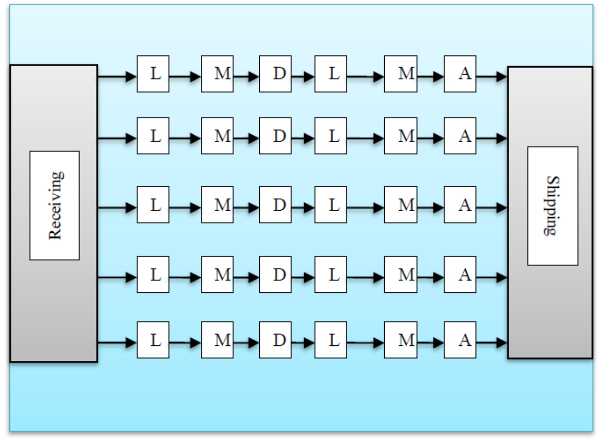

Figure (2). Flow shop system

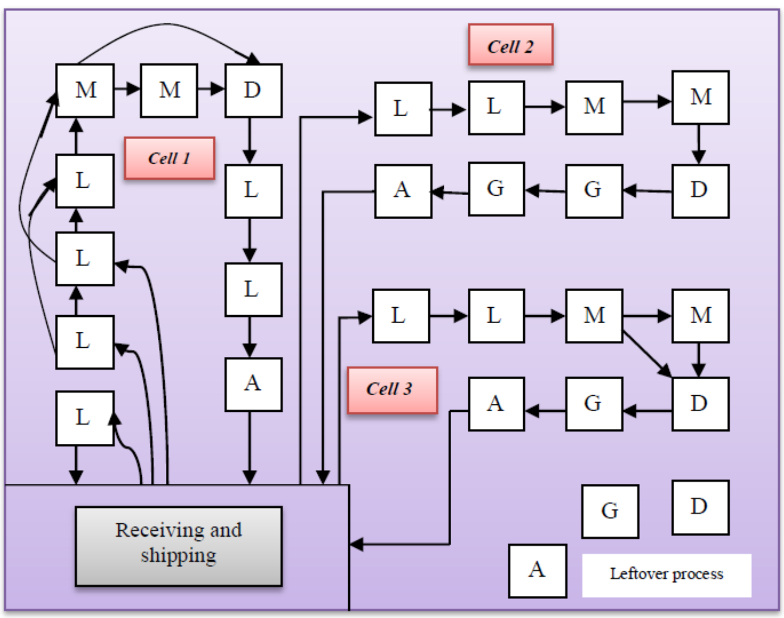

Figure (3). Cellular manufacturing system

It seems that with lower product lifecycle and shorter time distances for presenting the product, demand and product mix will continuously change [1]. Wemmerlov and Hyer [1] stated that a demand for the products, which are produced in manufacturing cells, are not so much predictable. As a result, planning horizon can be segmented to smaller periods. Therefore, each period has its own demand and product mix. In this situation, we are faced with dynamic manufacturing or dynamic environment requirements. It has to be said that in a dynamic situation, the demand and product mix in each period are different and certain.

For a prosperous design, planning must be done over time for all the periods with respect to the variations. Variations in cell structure within planning periods may include exchanging the machines between the cells, adding new machines to the cells, eliminating the machines from the cells and relocating the machines in the cells. In previous studies, for locating the machines in manufacturing cell space, line formed locations were the only consideration and the machines were assigned to these positions. It is obvious that if assigning number of machines to a cell cannot be line form, it turns into a U form that imposes additional costs to the system. With respect to this point, using the concept of distance in order to calculate amount of parts' relocations between two or more machines in a cell rather than considering positions for assigning the machines to manufacturing cells can make the problem more actual and it can ease the costs calculation.

2. Literature review

Studies conducted in dynamic cell formation can be categorized as follows:

- ✓ Studies that propose appropriate models in cellular manufacturing systems with respect to production information, and solve the model employing accurate or heuristic algorithms.
- ✓ Studies that attempt to propose new approaches including accurate or heuristic for existing models and comparing different solving methods.

2-1. Studies based on proposing models

The most popular studies conducted in designing Dynamic CMS (DCMS) are as follows. Kannan and Ghosh [3] examined a DCMS in terms of scheduling and illustrated that simpler scheduling can be achieved thorough DCMS. They also examined the impacts of the DCMS on preparation time, flow time and work-in-process. Balakrishnan and Cheng [4] proposed a flexible two-step method for solving the cell formation problem with respect to demand variations using dynamic programming and machine assignment. The first phase contains assigning machines to each period and the second phase of the method is to employ dynamic programming for designing within planning periods. Rheault et al. [5] proposed the idea of dynamic cell manufacturing to overcome the flexibility reduction in designing a CMS. They considered the demand and product mix in each period as a constant and certain value. Their objective function is to minimize the total inter-cell relocation costs and cell restructuring with constraints of capacity of cell space and common Quadratic Assignment Problem (QAP) constraints. Kioon et al. [6] proposed a new mathematical model for designing a DCMS. In addition to considering inter-cell relocation, cell restructuring and cell capacities, this model considers assumptions, such as level of inventory in each period, intra-cell relocation, maintenance costs and subcontracting. Schaller [7] proposed a mathematical model for a CF problem considering demand variations within the periods.
Wicks and Reasor [8] proposed a mathematical model for designing a dynamic manufacturing cell system based on part family and machine grouping. The objective function is to minimize the total inter-cell relocation costs, fix costs of machine purchasing and cell restructuring costs with constraints of machine capacity and lower bound of cells capacity. Chen [9] presented the DCMS model to minimize material relocation costs, cell restructure and fix machine cost with constraints of lower and upper bounds of cells capacity and constant number of cells. Mungwattana [10] examined the cell formation problem in his thesis. In his study, he considered a number of assumptions, such as multiple operational paths, batch movements of the parts, variable production costs, machine time capacity constraints, lower and upper bounds for cells capacity, number of cell constraint, demand variations, machine purchasing costs, cell restructuring cost, inter-cell relocating cost and multi-task machines. Safaei and Tavakkoli-Moghaddam [11] developed a new model for solving the dynamic cell formation problem considering subcontracting with some assumptions, such as batch movement of the parts, inter-cell relocating, demand variations in different periods, machine time capacity, maximum capacity of cells, existence of multitask machines and parts returns by purchasers. They solved the proposed model after linearization by a branch-and-bound (B&B) algorithm.

2-2. Studies based on solving method

Among the studies done in proposing different solving methods in a dynamic cell formation system design, we can mention to the following studies:
Safaei et al. [12] proposed nonlinear integer mathematical model for a CMS in a dynamic condition to overcome demand variations and product mix. The advantages of this model is to consider inter and intra-cell batch relocation, operational sequence, multiple paths and multiplicity in a machines type. The main constraints in this model are the upper bound of cells capacity and time capacity of machines. They solve their model by merging a Mean Field Annealing (MFA) algorithm with the Simulated Annealing (SA) algorithm, namely MFA-SA. They examined the obtained findings with SA and B&B methods and demonstrated that the MFA-SA algorithm achieves better results. Askin et al. [13] proposed a four-step algorithm for solving the CF problem considering demand variations and product mix. In the proposed model, existence of multiple operational paths is considered as assumption. Phase 1 is related to assigning the activities to specific types of machines. Phase 2 is about assigning parts activity to a specific machine. Phase 3 is related to candidate cell determination for locating the machine. Phase 4 is about cell design enhancement and completion.
Safaei et al. [14] solved the integer mathematical model related to the CF problem with dynamic and uncertain environment employing fuzzy programming. In this model, the demand and time capacity of machines are considered as fuzzy forms. The objective function of the proposed model is to minimize the total inter and intra-cell relocating costs, fix and variable costs of machines and cell restructuring costs. Tavakkoli-Moghaddam et al. [15] proposed multi-criteria linear-integer model that includes information, such as cell capacity constraints, inter-cell relocation, multi operational paths, machines

reestablishment in planning periods, existence of several single type machines and operation sequence. The objective function for their model is to minimize the machine purchasing costs, inter-cell relocation cost, part production cost and cell restructuring cost. They solved the model by SA and demonstrated that in sufficient time, more appropriate solutions is obtained by the B&B algorithm. Tavakkoli-Moghaddam et al [16] solved the Mungwattana's proposed model by meta-heuristic algorithms, such as SA, GA and Tabu Search (TS) and after comparing with the B&B method. They demonstrated that SA generates more appropriate solutions for solving this particular model. Bajestani et al. [17] proposed a multi-criteria programming model for a dynamic CF. They solved the proposed model employing a Scatter Search (SS) algorithm and demonstrated that for this problem, the SS algorithm outperforms multi-criteria GA. Saidi-Mehrabad and Safaei [18] developed the dynamic CF model considering number of variable cells for sequential planning periods and then solved the model by neural network in deterministic condition.

Reviewing the previous studies indicates that there is no paper on considering the simultaneous formation of cells and layout under a dynamic condition. In addition to the typical assumptions in a CMS issue, this paper attempts to examine cell formation problems and layout under a dynamic condition simultaneously. Also, using the notion of distance, a new establishment design in continues space for machine arrangement in manufacturing cells and cells layout in a job shop level is presented in this paper.

3. Proposed model

The presented model with a number of assumptions, parameters, decision variables, objective function and constraints are discussed below.

3-1. Model assumption

- Flow between machines in each period is determined. This number is obtained from the parts demand and parts operational paths and also batch size of parts transportation.
- Parts are moved between and in the cells in batches. Largeness of the batches per product is known and constant for all periods. Also, the size of the part batches for inter and intra-cell relocations is assumed the same.
- The inter-cell relocation cost is based on the unit of distance and it remains constant over time.
- The machine relocation cost during the periods is constant and known for a PER machine. This cost includes opening, transferring and resetting the machine.
- The number of cells is known and constant over time.
- There is one from each type of machine.
- The maximum capacity of cells is known and remains constant over time.
- The distance between two machines is calculated through a rectilinear distance.
- The sizes of all machines are equal with the dimensions of 1×1.
- Excess inventory between the periods is zero; delayed orders are not allowed and demands per period must be supplied in that period.
- The efficiency of machines and production is 100%.

3-2. Sets

$i, i' = \{1,2, \dots, , m\}$	Index set of machines
$j = \{1,2, \dots, , n\}$	Index of parts
$l, k, k' = \{1,2, \dots, c\}$	Index set of cells
$h = \{1,2, \dots, , H\}$	Index set for time periods

3-3. Model parameters:

D_{jh}	Demand for part type j in period h
B_j	Largeness of batch for transportation of part type j
C^j_{intra}	Cost of an intra-cell relocation for part type j based on the unit of distance
C^j_{inter}	Cost of an inter-cell relocation for part type j based on the unit of distance
C_i	Cost of relocation for machine i
R_{ij}	Number of operations on part type j by machine i
E	Horizontal length of the job shop (i.e., length of the job shop)
F	Vertical length of the job shop (width of the job shop)
SP	Set of (i,j) pairs when aij≥1 (set of elements for part-machine matrix that are not equal to zero)
NM	Maximum number of machines relocated per period.
α_j	Coefficient of cost (or penalty) based on existence of an exceptional part per period.
N	Very large number
$f_{ii'h}$	Number of travels between machines i and i' in period h.
N	Positive big number
A_{kl}, B_{kl}	$A_{kl}, B_{kl}, A_{ii'h}, B_{ii'h}$
$f^j_{ii'h}$	Number of travels for transferring part type j between machines i and i' in period h

$$f^j_{ii'h} = \begin{cases} \left[D_{jh} / B_j \right] & if \quad R_{i'j} - R_{ij} = 1 \\ 0 & if \quad R_{i'j} - R_{ij} \neq 1 \end{cases} \qquad (1)$$

3-4. Decision variables

$$X_{ikh} = \begin{cases} 1 \\ 0 \end{cases}$$ If machine *i* in period *h* is assigned to cell *k*,
Otherwise

$Y_{ikh} = \begin{cases}1 \\ 0\end{cases}$ If part j in period h is assigned to cell k, Otherwise

$Z_{ih} = \begin{cases}1 \\ 0\end{cases}$ If machine type i relocates in periods h and $(h+1)$ Otherwise

$U_{ijkh} = \begin{cases}1 \\ 0\end{cases}$ If $Y_{jkh} = 0$ and $X_{jkh} = 1$ Otherwise

$V_{ijkh} = \begin{cases}1 \\ 0\end{cases}$ If $Y_{jkh} = 1$ and $X_{jkh} = 0$ Otherwise

x_{ih} Horizontal component of machine type i in period h

y_{ih} Vertical component of machine i in period h

p^1_{kh} Left side horizontal component of cell k in period h

p^2_{kh} Right side horizontal component of cell k in period h

q^1_{kh} Bottom side vertical component of cell type k in period h

q^2_{kh} Top side vertical component of cell type k in period h

Therefore, the cost of relocation part j between machines i and i' in period h as follows:

If $X_{ikh}, X_{i'kh} > 0$ this cost equal to (2)

$$C^j_{ii'h} = (|x_{ih} - x_{i'h}| + |y_{ih} - y_{i'h}|)C^j_{intra} \tag{2}$$

If $X_{ikh}X_{i'kh} = 0$, $X_{ikh}X_{i'k'h} > 0$ this cost equal to (3)

$$C^j_{ii'h} = (|x_{ih} - x_{i'h}| + |y_{ih} - y_{i'h}|)C^j_{inter} \tag{3}$$

3-5. Mathematical formulation

With respect to input parameters and variables, the presented nonlinear model for this problem is as follows:

$$Minimize \sum_{h=1}^{H}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{i'=1}^{m} f^j_{ii'h}C^j_{ii'h} + \sum_{h=2}^{H}\sum_{i=1}^{m} C_i Z_{ih} + \sum_{h=1}^{H}\sum_{k=1}^{C}\sum_{(i,j)\in sp} \alpha_j.\frac{(U_{ijkh} + V_{ijkh})}{2} \tag{4}$$

$Subject\ To:$

$$\sum_{k=1}^{C} X_{ikh} = 1 \quad , \quad i = 1,2,\dots,m \quad , \quad \forall h \tag{5}$$

$$\sum_{k=1}^{C} Y_{jkh} = 1 \quad , \quad j = 1,2,\dots,n \quad , \quad \forall h \tag{6}$$

$$1 \le \sum_{i=1}^{m} X_{ikh} \le NM \quad , \quad k = 1,2,\dots,C \ , \ \forall h \tag{7}$$

$$NZ_{ih} \ge |x_{ih} - x_{i(h+1)}| + |y_{ih} - y_{i(h+1)}| \quad \forall i, h < H \tag{8}$$

$$|x_{ih} - x_{i'h}| + |y_{ih} - y_{i'h}| \ge 1 \tag{9}$$

$$\begin{cases} x_{ih} \ge p^1_{kh} - N(1 - X_{ikh}) \\ x_{ih} \le p^2_{kh} + N(1 - X_{ikh}) \\ y_{ih} \ge q^1_{kh} - N(1 - X_{ikh}) \\ y_{ih} \le q^2_{kh} + N(1 - X_{ikh}) \end{cases} \quad \forall i, k, h \tag{10}$$

$$\begin{cases} p^1_{kh} \ge 0 \\ q^1_{kh} \ge 0 \\ p^2_{kh} \le E \\ q^2_{kh} \le F \end{cases} \quad \forall k, h \tag{11}$$

$$\begin{cases} p^1_{kh} - p^2_{lh} + NA_{kl} + NB_{kl} \ge 0 \\ p^2_{kh} - p^1_{lh} - NA_{kl} - N(1 - B_{kl}) \le 0 \\ q^1_{kh} - q^2_{lh} + N(1 - A_{kl}) + NB_{kl} \ge 0 \\ q^2_{kh} - q^1_{lh} - N(1 - A_{kl}) - N(1 - B_{kl}) \le 0 \\ \qquad 0 \le k < l \le C \end{cases} \tag{12}$$

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The objective in the presented model is simultaneous decision making to specify cell for machines, part families and facility layout under a dynamic condition. The model is a nonlinear model that aims to minimize the inter and intra-cell relocation cost of the parts, the relocation costs of the machines during periods and the costs related to existence of exceptional parts. The last phrase in the objective function mentioned above attempts to minimize the number of exceptional parts. The quantity $^1/_2$ in this relationship is due to double calculation of ones.

The first constraint leads to the assignment of per machine in a single cell, in which there is one from each time of machine. The second constraint leads to the assignment of per part to a single part family. The third constraint demonstrates the capacity of per cell that puts number of machines in each cell in the quantity between 1 and *NM*. The forth constraint ensures that relocating machine type *i* during periods *h* and (1+*h*), variable Zih obtains the quantity one. The fifth constraint leads to no-overlap between the machines. In other words, this relationship ensures that the machines are not on each other, assume that dimensions of machines are 1×1. The set of relationship (10) causes relocating of per machine in space of its corresponding cell. The set of relationship (11) ensures that cells are in space of a job shop and the set of relationship (12) ensures that cells are not overlapped. At the end of the third section, in order to validate the model, we represent a numerical instance and we solve it by Lingo. After two hours of lunching the program the solution has been obtained. Also the design by model is illustrated.

The information of the problem is as follows:

Table1. Process of generating the parameters for the problem

B_j	U[10,30]
m	5
n	8
C^j_{intra}	10
C^j_{intra}	1
C	2
E	4
F	4
NM	3
C_i = [3312,2398,2201,1731,3141]	

Table2. Demand of parts in the first and second periods

Part number	Demand of period1 (D_{i1})	Demand of period 2 (D_{i2})
1	177	0
2	194	409
3	0	341
4	191	214
5	234	208
6	391	0
7	461	349
8	0	263

Part-machine matrix is as follow. The average of operation for this matrix is considered 3.

$$
\begin{array}{c|cccccccc|}
 & p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \\
\hline
M_1 & 0 & 0 & 1 & 3 & 0 & 1 & 2 & 0 \\
M_2 & 3 & 2 & 0 & 0 & 0 & 2 & 1 & 3 \\
M_3 & 2 & 0 & 2 & 0 & 2 & 3 & 0 & 0 \\
\hline
\end{array}
$$

Machine-part matrix for problem *5×8*

In the first period, machines 1, 2 and 5 and parts 1, 2, 3, 5 and 8 are relocated in cell 1 and machines 2 and 4 and parts 4, 6 and 7 are relocated in cell 2. In the second period, machines 2 and 4 are relocated in cell 1 and machines 1, 3, and 5 are relocated in cell 2. In this period, parts 7, 2 and 8 are relocated in cell 1 and parts 1, 3, 4, 5 and 6 form the part family of the cell 2. Layouts for machines are illustrated in figures 4 and 5. The special features of this algorithm make us unable to consider this algorithm as a simple random searcher.

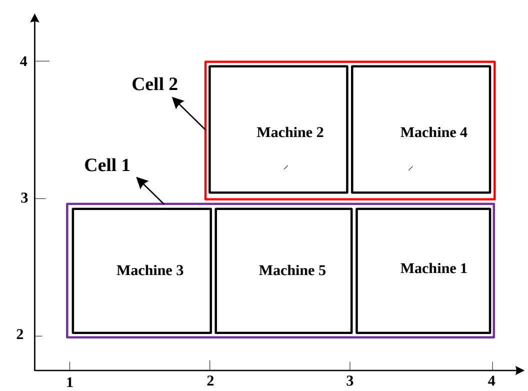

Figure 4 - Designed by solving the model in the first period

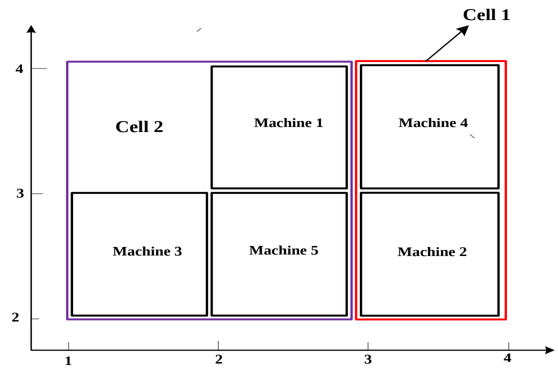

Figure 5 - Designed by solving the model in the second period

4. Proposed genetic algorithm

Several algorithms have been applied in the context of a DCMS design to approach the appropriate design. One of the most popular forms of these designs is the Genetic Algorithm (GA). This section attempts to examine some aspects of this algorithm and demonstrates its application in a DCMS design. The GA is known as the most popular meta-heuristic algorithm and is a component of evolutionary calculation and it is a subset of artificial intelligence. The primary idea of this algorithm is derived from the Darwin's evolutionary theory and its application is based on natural genetics.

In a way that the GA searches the solution space, not only the better quality solutions are acceptable, but also the solutions with lower fitness are acceptable, that leads the algorithm to escape from local optimum points. The GA varies in many ways with traditional optimization methods. In this algorithm, design space should be converted to genetic space. Therefore, we deal with series of coded variables. Another major difference between the GA and other optimization methods is that the GA works with a population or a set of points at a certain moment, while traditional optimization methods operate only in a particular point. A distinguishing feature of the GA is that principle of processing in this algorithm is random and it is guided to optimum place. Generally, the differences between the GA and other optimization methods can be expressed as follows:

✓ The GA does not search the solution in a single point and searches the solution in parallel.

✓ The GA does not use the deterministic rules and uses probabilistic rules.

✓ The GA is based on coded variables. Unless in cases which variables are illustrated as real numbers.

✓ The GA does not require backup information. It only determines the members of objective function and the fitness of path in search space.

Applying the GA, the following steps are necessary:

- Representing an appropriate solution structure.
- Obtaining appropriate initial solutions in a population size.
- Employing appropriate genetic operators (i.e., mutation and crossover) to obtain new solutions.
- Selecting population of the next generation from parent and offspring chromosomes.
- Chromosome evaluation measure (i.e., fitness function)
- Specifying the stopping criteria

The algorithm in two phases is used to solve the model of this problem. The first phase is related to machine assignment to manufacturing cells and layout determination, and the second phase is about determination of part assignment to part families.

4-1. Solution View

One chromosome coding is required for the GA, so that we can illustrate the solutions of a problem. The way that chromosomes are viewed determines how a problem is formulated in a form of an algorithm and what genetic operators are applied. Each chromosome is formed from genes that can be shown as binary and integer numbers or combination of characters that is a coded form of a feasible solution (appropriate or inappropriate) from the problem.

The considered chromosome for the first step of this problem includes a matrix with H rows and M columns that can be divided into the following sub-matrix.

- Sub-matrix of Z is related to assignment of machines to manufacturing cells. This sub-matrix consists of H (i.e., number of periods) rows and M (i.e., number of machines) columns. Each element of this matrix is a number between 1 and C (i.e., number of cells) and the element Z_{ih} represents the number of cell that includes machine type i in period h.
- Sub-matrix X is related to the horizontal component of machines' location. This sub-matrix also consists of H rows and M columns. With respect to the machines' dimension (1×1), one integer is sufficient to familiarize per horizontal and vertical components of the machines. Per element of this matrix is a number between 1 and E (i.e., length of the job shop) and the element x_{ih} represents the horizontal component of location that includes machine i in period h.
- Sub-matrix Y that is related to the vertical component of machines' location. This sub-matrix also consists of H rows and M columns. Each element of this matrix is a number between 1 and F (i.e., width of the job shop)

and the element y_{ih} represents the vertical component of location that includes machine *i* in period *h*.

Figures 6 and 7 illustrate the general and detailed views of the chromosome structure related to machines alignment to manufacturing cells, respectively.

$$[[Z] \quad [X] \quad [Y]]$$

Figure 6 - General view of the chromosome structure

.

$$\left[\left[\begin{matrix} z_{11} z_{12} & \dots & z_{1M} \\ \vdots \quad \vdots & & \vdots \\ z_{h1} z_{h2} & & z_{hM} \end{matrix}\right| \begin{matrix} x_{11} x_{12} & \dots & x_{1M} \\ \vdots \quad \vdots & & \vdots \\ x_{h1} x_{h2} & & x_{hM} \end{matrix}\left| \begin{matrix} y_{11} y_{12} & \dots & y_{1M} \\ \vdots \quad \vdots & & \vdots \\ y_{h1} y_{h2} & & y_{hM} \end{matrix}\right]\right]$$

Figure 7 - Detailed view of the chromosome structure.

The considered chromosome for the first second of this problem includes a matrix with H rows and N columns. Its detailed structure related to parts alignment to part families is shown in Figure 8.

$$\begin{bmatrix} z'_{11} z'_{12} & \dots & z'_{1N} \\ \vdots \quad \vdots & & \vdots \\ z'_{h1} z'_{h1} & & z'_{HN} \end{bmatrix}$$

Figure 8 - Detailed view of the chromosome structure

4-2. obtaining the primary solutions

The elements of matrix Z is obtained randomly from within the numbers of 1 to C. The elements of matrices X and Y are selected in a way that machines are not overlapped; in a way that numbers of a column from X and Y are not simultaneously equal to another column of X and Y. The elements of matrix z' are also obtained randomly from within the numbers of 1 to C.

4-3. Application of genetic operators

A genetic operator is used to produce a new generation of offspring.

4-3-1. Mutation

In the proposed GA, in sections of machine assignment to manufacturing cells, five types of the operator are used simultaneously. These operators are as follow:

- *Machine relocation in cells:* This operator is on matrix *Z* and that is substituting two numbers from two columns in a row of matrix *Z*.
- *Relocation of two machines:* This operator is on matrix X and Y simultaneously. Selecting two columns from a row of matrices X and Y and substituting the numbers of these columns in the same row, the location of those machines in considered period will change.
- *Relocation and cell change of two machines:* This operator is on matrix *Z*, *X* and that is two previous operators simultaneously.
- *Approaching one machine to another:* This operator is on matrix *X* or *Y*. One of the columns from matrix *X* or *Y* is selected and it changes to the numbers of another column from the same matrix with one unit difference.
- *Assigning the machines with more flow to a single cell:* This operator is on matrix *Z*. With respect to the numbers of flow matrix, machines with more relations are assigned to a single cell.

In the section about part assignment to part families in this algorithm, we use substitution of two numbers from two columns of a row in matrixZ'.

4-3-2. Crossover

In both parts of the solution view in this algorithm, the selected crossover means substitution of a part of a row from parent with the same part of the same row of another parent and generating two offspring similar to the two parents.

4-4. Selecting the next generation

Selecting the parents for producing next generation plays a major rule in the genetic algorithm. The aim is to select the best chromosomes (i.e., the solutions that are better than others) for entering to the next generation or producing new generation. Generally, each chromosome with a particular probability has an opportunity to produce or enter to the next generation. Therefore, chromosomes with high fitness should have more probability to be selected. Several methods have been proposed for selecting the next generation that are in two categories: probabilistic and non-probabilistic. Probabilistic methods contain roulette wheel selection, scaling and grouping. Non-probabilistic methods include competitive selection and elite models. Roulette wheel mechanism is used in this study. In this method, members are selected based on their relative consistency. In other words, a roulette wheel method selects the next generation's members to the number of population, giving more probabilities to more appropriate chromosomes and generating the random number between zero and one.

4-5. Criterion for evaluating chromosomes (Fitness function)

The fitness function is an implication of the objective function. In the GA, the fitness value for each chromosome is equivalent to the value of the objective function for a solution. For instance, if the objective in a cellular manufacturing problem is to minimize the sum of costs according to the problem model, an offspring will be acceptable when it more minimizes the cost function relative to its parents. Also in this problem, fitness for each chromosome is calculated based on an objective function.

4-7. Stopping condition

To stop the GA and present a final solution, stopping criterion should be considered. The stopping criteria that are mainly used are as follows:

- The maximum specified numbers of generation: if number of generations passes the maximum specified numbers of generation, algorithm will end.
- Convergence of population: in broad terms, GA attempts to converge the population to a single population. If the current population converges to a single solution, algorithm will end.
- Reaching to specified solving time.

The criterion of the maximum specified numbers of the generation is used here.

5. Comparing the results

The proposed GA is coded by Visual Basic (VB) and run by a Pentium 4 personal computer (PC). The number of populations, number of replications for generations, mutation rate and crossover rate for the proposed GA are considered 20, 100, 0.25 and 0.85, respectively. In order to validate the proposed model and verify its quality, a linear model of dynamic cell formation for problems is solved by Lingo. Two hours are considered as the maximum elapse time to solve the problem. After two hours, the GA is stopped and the best obtained solution is reported. The solved example in Section 2 is resolved by the proposed GA and compared with the results obtained in Section 2. After 10 times running the program with VB, the optimum solution imposes a cost of 84253 on the system. The solution obtained from this process is exactly similar to the obtained solution in Section 2. The results obtained from 11 problems with various dimensions that are solved with the GA and also using Lingo is shown in table 3. The number of periods is 2, and the number of cells for 3 to 7 and 8 to 12 machines is considered 2 and 3, respectively.

A new mathematical model for the problem of cell formation and cell layout in a CMS in a dynamic environment under dynamic conditions is represented in this paper. The presented model with the notion of the distance attempts to calculate the costs in a more realistic way. And also the problem of layout is considered in its actual concept in this model. After linearizing the presented model using the proposed methods, a numerical instance is provided for the linearized model and the obtained results are shown schematically. After a brief introduction of the GA, it is explained with respect to the proposed model, and the obtained results from solving the problem with the GA are compared to the results obtained from Lingo. With respect to the values presented in Table 4, it can be concluded that, in shorter time, the results of the GA is more appropriate than the branch-and-bound (B&B) algorithm applied in Lingo. Additionally, comparison of results reflects that accurate solving in the presented model for more than four machines is impossible.

Table3. Comparing the obtained results from GA and Lingo

Dimensions of problem	Average of 10 genetics	Optimum genetics	Time of genetic	Solution of Lingo	Lower bound	Time of B&B
3	24283	24283	12	24283	24283	42
4*6	28641	28641	17	28641	28641	6180
5*8	85637	84180	22	84253	1117	7200
6*9	120903	116046	32	121203	0	7200
7*11	224095	215470	42	248907	0	7200
8*13	497882	456124	65	706036	0	7200
10*12	706813	664902	79	771098	0	7200

*Time is based on seconds

6. Future research

The following areas can be attractive for future research, and the present study can provide the necessary background for researchers who want to work in these areas.

- Considering multi-operational paths; in the presented model, the operational paths are considered constant. Considering multi-operational paths in the presented model can provide a model close to the real situation of job shop.
- Considering unequal dimensions for machines in the proposed model; dimensions of machines are considered equal (all of them equal to 1) in the proposed model. In order to obtain more appropriate schema from the space of a job shop,

dimensions of machines can be considered as input parameters.

- Developing probabilistic models and fuzzy models; factors (e.g., available machines, operation time, costs, transportation time and demand for each part) can be considered as fuzzy or probabilistic.
- Considering reliability for manufacturing machines and in consequence calculating the reliability of a cell and ultimately calculating reliability of manufacturing system and incorporating them with dynamic conditions can be interesting subjects for future research.

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