

A time-dependent vehicle routing problem for disaster response phase in multi-graph-based network

Mostafa Setak^{a*}, Shabnam Izadi^a and Hamid Tikani^a

^aDepartment of Industrial Engineering
K.N.Toosi University of Technology, Iran, Tehran
Tel: (+98) 21 84063373, Fax: (+98) 21 88674858, E-mail: setak@kntu.ac.ir

Abstract

Logistics planning in disaster response phase involves dispatching commodities such as medical materials, personnel, food, etc. to affected areas as soon as possible to accelerate the relief operations. Since, transportation vehicles in disaster situations can be considered as scarce resources, thus, the efficient usage of them is substantially important. In this study, we provide a dynamic vehicle routing model for emergency logistics operations in the occurrence of natural disasters. The aim of the model is to find optimal routes for a fleet of vehicles to give emergency commodities to a set of affected areas by considering the existence of more than one arc between each two nodes in the network (multi-graph network). Proposed model considers FIFO property and focused on minimization of waiting time and total number of vehicles. Various problem instances have been provided to indicate the efficiency of the model. Finally, a brief sensitivity analysis is presented to investigate the impact of different parameters on the obtained solutions.

Keywords:

Time-dependent vehicle routing problem, Multi-graph, FIFO property, Disaster relief, Service time

1. Introduction

Natural disasters such as droughts, earthquakes, hurricanes and floods have proven a global challenge in their unpredictable nature and potential scale of impact represented by fatalities and social, environmental and economic costs.

On the other hand, Supply chain and logistics management is important key in humanitarian relief context. Emergency logistics is the process of planning, managing, and controlling the flow of resources to provide relief to people affected by disasters [1]. It relies on a given number of strategically located vehicles over affected area.

Transportation is one of the most crucial processes in the supply chain network. Vehicle routing problem (VRP) is one of the famous problems in transportation optimization. It aims to determine least cost routes from a depot to a wide range of customers. The routes have to satisfy the following set of constraints [2]:

- Each customer is visited exactly once
- All routes start and end at the depot
- Sum of all demands on a route must be less than capacity of a vehicle
- A subset of arcs must be traversed by only one vehicle

The Vehicle Routing Problem (VRP) in disaster situations is applied to design an optimal route for a fleet of vehicles, to service a set of affected areas given a set of constraints such as delivering the commodities in a specifically restricted time. The VRP is used in supply chain management in the physical delivery of goods and services. There are various version of VRP which are modeled based on the nature of the transported goods, the quality of service required and the characteristics of the customers and the vehicles. The vehicle routing problem is shown in Figure 1. As can be seen in the Figure 1 the vehicle's tour is start at the depot and visits all the nodes once. Models in emergency logistics are employing to manage inventory prepositioning and vehicle routing which are considered as two of the most important operations in disaster risk management, separately. The main differences between humanitarian logistics and business logistics include the object of minimization (minimizing time of response or maximizing fairness of distribution) in contrast maximizing profit or minimizing costs and the circumstances in which operations are to be performed (known resources and infrastructure in contrast to uncertainty regarding supplies, available vehicles and the condition of the road network. their

demands at the nodes and then return to the depot again [3].

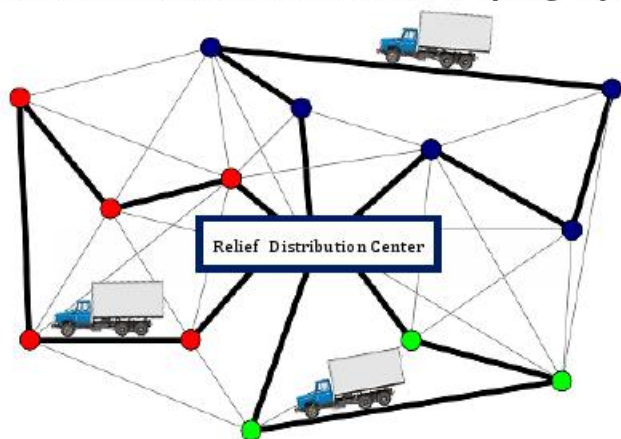


Figure 1. An example solution to a vehicle routing problem.

One of the most important disasters is an earthquake, which often causes huge property damage, human injuries and casualties. Increase of this type of disaster over the last years [4] along with growth of population congestion in areas sensitive to earthquakes, create a growing need for designing emergency response procedures prior to an earthquake disaster. These emergency procedures guide the set of actions taken during the initial phase of this emergency situation, the so-called "Earthquake Response Phase" [5].

Oh and Haghani, analyzed the transportation of different disaster relief commodities such as medical, clothing, food, medicine, machinery and personnel to minimize the loss of life and maximize the efficiency of the rescue operations. The authors modeled a multi-commodity, multi-modal network flow models for generic disaster-relief operations [6,7]. Other commodity logistic planning models are provided by Barbarosoglu et al. [8], Ozdamar et al. [9], Tzeng et al. [10], Sheu [1,11] and Nolz et al. [12]. Barbarosoglu et al. focus on the use of helicopters for aid delivery and rescue missions during natural disasters. Lin et al. propose a multi-item, multi-vehicle, multi-period and multi-objective model for delivery of prioritized items in disaster-relief operations [16]. This model includes two objective functions, which minimize the total unsatisfied demands and the total travel time for all tours and all vehicles.

Time dependent vehicle routing problems have received significantly little attention among researchers. Variations in travel time are common in disaster-affected areas due to, poor roads, security and weather hostile condition. Therefore, the assumption of constant travel times is unrealistic [5]. One of the first approaches using the later interpretation was the approach of Malandraki and Daskin

[13], where the objective is minimization of the total travel time. Nevertheless, time dependent problems might produce sub-optimal solutions if there are high uncertainties in the assumed travel times. The approach presented in this article differs from papers, which do not ensure the "first-in-first-out" property correctly, because the objective is to guarantee that the FIFO property is realized as what done by Ichoua et al. [14]. We will completely explain the problem in Section 2.

Setak et al [15] proposed a new extension of the time-dependent vehicle routing problem with the existence of more than one arc between two nodes. They modeled the problem as the time-dependent vehicle routing problem in multi-graph which provided the FIFO property.

Thus, the possible existence of more than one arc between two specified nodes has not been considered in the Time dependent Relief Vehicle Routing Problem (TDRVRP) in the literature.

The studied problems are limited to the existence of only one edge between two disaster regions or between distribution center and disaster region. Although it is reasonable to assume that there is only one arc with minimum travel time between different points in humanitarian transportation, it is not very acceptable in large disaster situation according to the complexities of humanitarian logistic and traffic restriction in first hours after earthquake. Urban areas in disaster situation usually have a complicated structure, which provides accessibility to different nodes by more than one edge. In this condition, traffic rules for arcs (e.g. determining maximum allowable speed and vehicle traffic constraints) affect arc selection. Choosing suitable edges is important humanitarian transportation due to the reduction of time. The problem is called time-dependent relief vehicle routing problem in multi-graph (TDRVRPM) which allowed for considering more than one arc between two points.

In Section 2, TDRVRPM is described. In Section 3 TDRVRP modeled using mixed integer linear programming. Computational results are presented in section 4. Finally, the results are summarized in conclusions and future works.

2. Problem definition

An important property for time dependent problems is the First In-First Out (FIFO) property proposed by Ichoua et al. [14]. In this paper, the FIFO property was assumed. In TDVRP, FIFO guarantees following assumption when the vehicle "A" overpass the distance from Affected Region (AR) i to region j and a similar vehicle "C" starts to move from AR i to AR j after "A", vehicle "C" will reach AR j later than "A" [14]. The early papers related to TDVRP

have a main shortcoming: they have modeled travel time as a discrete function of time [8]. For instance, Figure 2 indicates this function as it pertains to an arc with the length of 1. So, if vehicle #1 leaves the Relief Distribution Center (RDC) node at time $t_1 = 1$, then it reaches the destination node (AR) at $t_2 = 5$. Nonetheless, when vehicle #2 leaves the RDC at time $t_3 = 2 (> t_1)$, it will reach the destination at $t_4 = 4 (< t_2)$, which means that, despite the fact that vehicle #2 leaves the origin node later than vehicle #1, it reaches the destination node earlier. Thus, the result does not satisfy the FIFO property. Currently, researchers apply continuous travel time functions over the time horizon, instead of discrete travel time functions. In order to do so, travel speed function is applied, as shown in Figure 3. In this paper, a process was provided to transform a travel speed function into a continuous travel time function according to the represented approach by Ichoa et al. [14]

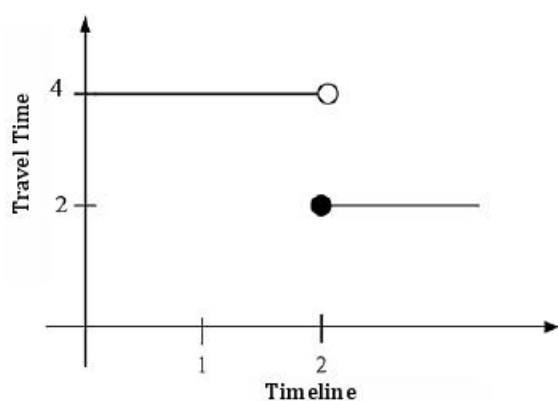


Figure 2. Discrete travel time function [15]

In the aftermath of a large disaster, the routing of vehicles carrying critical supplies can greatly affect the arrival times to those in need. Since it is critical that the deliveries are both fast and fair to those being served, it is not clear that the classic cost-minimizing routing problems properly reflect the priorities relevant in disaster relief.

Thus far, TDVRP papers have been based on this assumption: arc (i,j) is the only shortest distance between two locations i and j . Accordingly, the transportation network is based on a simple graph in which there is only one edge between two specific nodes. In static problems, this assumption is suitable with the consideration of fixed travel time. However, in the real world, particularly in the urban transportation networks, there is more than one edge between two locations, in which their travel time is different according to the daytime. Thus, choosing the edge for traveling depends on the specific time of the day. Multi-graph can be used for this type of problems. In

contrast with the simple graph, multi-graph lets the model to establish more than one edge (or parallel edges) between two nodes. In the multi-graph, the edge is shown by (i,j,m) , in which m shows the m^{th} parallel edge between nodes i and j . Figure 4 demonstrates an example of the multi-graph vs. simple graph. In basic TDVRP, the problem is defined on a simple graph such as Figure 4(a), in which the travel speed changes in each time interval with congestion. However, the network with the lowest travel time is fixed. In contrast, TDVRPM employs a multi-graph-based network, in which the network with the lowest travel time edges is not fixed. In fact, this network usually changes in each time interval. For additional explanations, see Figure 5, which implies a simple multi-graph-based network. It shows the network of edge with the lowest travel time in three time intervals by bold continuous lines. Based on Figure 5(a), in time interval number 1, travel time throughout the edge $(i,j,1)$ is less than $(i,j,2)$. Consequently, this edge is selected for the network that includes the lowest travel time edges. The same explanation is used to determine other edges of the network. In this paper, the minimum travel time of humanitarian aid is aimed to be determined in the transportation networks based on multi-graph.

3. Model formulation

The main purpose of this paper is to minimize the total travel time to serve the affected areas and the number of vehicles in multi-graph transportation network. There are the following assumptions in this model:

1. It is possible to get from a location to another with more than one arc (multi-graph).
2. The shortest edge between two locations is different according to the crisis situation after disaster
3. All vehicles leave the Crisis Management Headquarters or local aid Location (depot) simultaneously.
4. All vehicles return to the depot after finishing the product delivery of relief goods in case of disasters.
5. The demand of aid packages in all case of disaster are given and fixed.
6. Capacity of each vehicle is given and fixed.
7. The first purpose of this model is to minimize travel time routes.

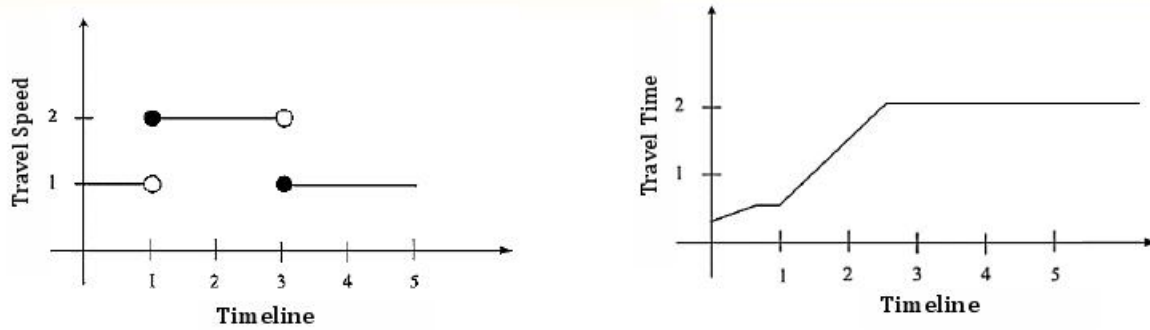


Figure 3. A travel speed function and its travel time function [15]

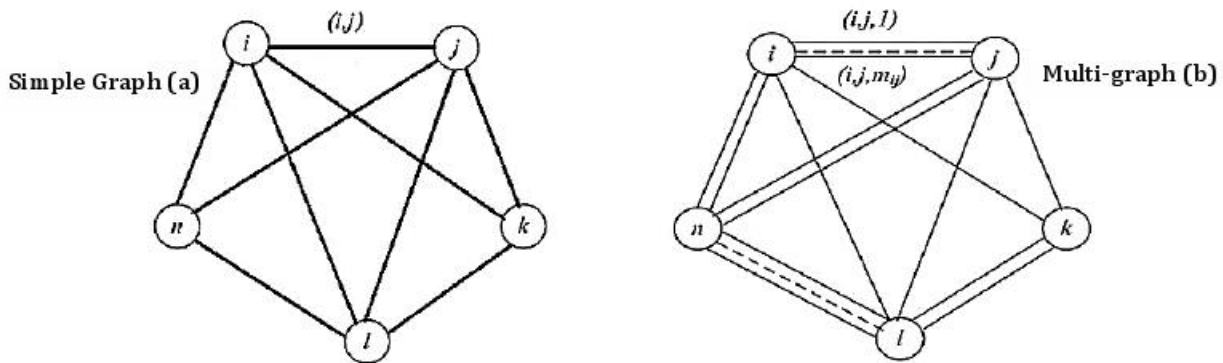


Figure 4. Representation of multi-graph vs. simple graph [15]

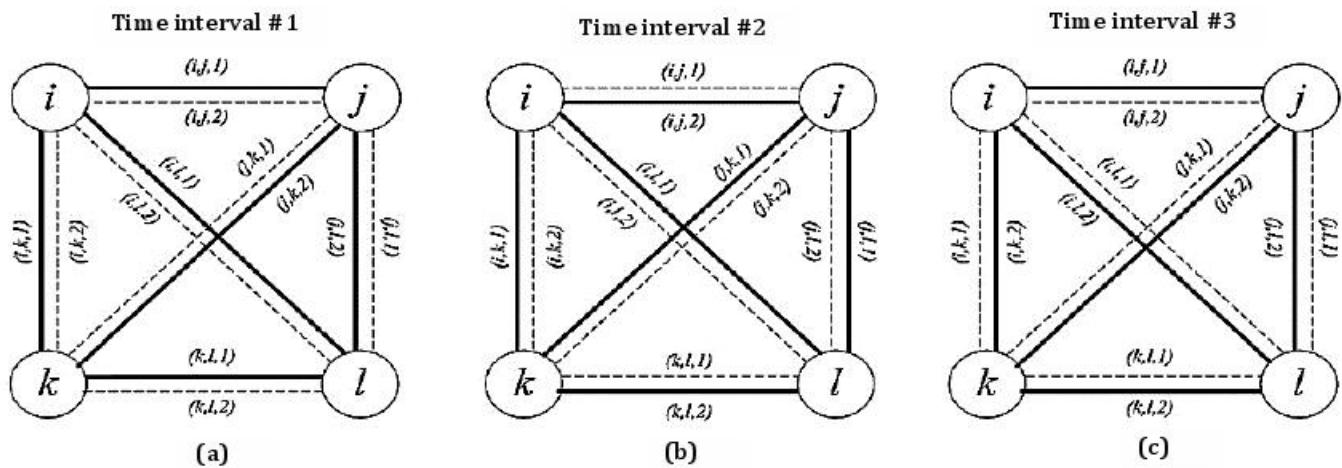


Figure 5. Network of edges with less travel time in multi-graph network transportation [15]

In order to model TDRVRPM problem, notations are described as:

Suppose $G = (V, E)$ is a complete graph, in which V and E are the set of nodes and arcs, respectively. Each arc can be defined by a regular triplex as (i, j, m_{ij}) . In which i , j , and m_{ij} represent Relief distribution center (first node of link), destination node or case of disaster (second node of link), and the m^{th} arc between those two nodes, respectively.

In this model, H and $Tnew_{m_{ij}}^h$ are the number of new time intervals and the head point of new time intervals, respectively. Other notations are as follows:

N : A Set of relief distribution center (depot), case of crisis places and copied from the distribution center

$Ps = \{0, N + 1\}$ Relief distribution center and its Copy

$Ns = \{1, \dots, N\}$ Set of disaster areas

$N = [Ns \cup Ps]$

C_t Transportation cost per unit of travel time of relief operations

C_k Fixed cost for vehicles

E_1, E_2, E_3 Large numbers

k Number of available vehicles

s_i The time required for relieving to the affected people in i th node

Q Capacity of vehicle

M_{ij} The number of arc between affected region i and j ;

q_i The demand of necessary disaster relief commodities relief for location i

e_i Emergency relief time for affected region i

There are two decision variables in this formulation. In addition, the following variables are calculated in the model:

$$x_{ijm}^{hk} = \begin{cases} 1 \\ 0 \end{cases}$$

If one of the vehicles moves through the m^{th} arc from node i to node j in the h^{th} time interval; 0 otherwise;

y_i^k **Departure time of vehicle k from node i**

The formulation of TDVRPM model is presented as follows:

$$\text{Min} \quad C_k \sum_{k \in K} \sum_{m \in M_{ij}} \sum_{h \in H_{m_{ij}}} \sum_{j \in N_s} x_{0jm}^{hk} + C_t \left(\sum_{k \in K} y_{N+1}^k - \sum_{k \in K} y_0^k \right) \quad (1)$$

$$\sum_{i \in (0 \cup N_s)} \sum_{m \in M_{ij}} \sum_{h \in H_{m_{ij}}} \sum_{k \in K} x_{ijm}^{hk} = 1 \quad \forall j \in N_s, \quad i \neq j \quad (2)$$

$$\sum_{j \in (N_s \cup N+1)} \sum_{m \in M_{ij}} \sum_{h \in H_{m_{ij}}} \sum_{k \in K} x_{ijm}^{hk} = 1 \quad \forall i \in N_s, \quad i \neq j \quad (3)$$

$$\sum_{i \in (0 \cup N_s)} \sum_{m \in M_{ij}} \sum_{h \in H_{m_{ij}}} x_{ijm}^{hk} = \sum_{j \in (N_s \cup N+1)} \sum_{m \in M_{ij}} \sum_{h \in H_{m_{ij}}} x_{ijm}^{hk} \quad \forall r \in N_s, \quad \forall k \in K \quad (4)$$

$$y_j^k - y_i^k - E_1 x_{ijm}^{hk} \geq a_{ijm}^h + b_{ijm}^h y_i^k + s_j - E_1 \quad \forall i \in (0 \cup N_s); \quad \forall j \in (N_s \cup N+1); \quad \forall k \in K; \quad \forall m \in M_{ij}; \quad \forall h \in H_{m_{ij}}; \quad i \neq j \quad (5)$$

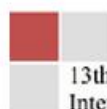
$$y_i^k + E_2 x_{ijm}^{hk} \leq Tnew_{ijm}^h + E_2 \quad \forall i \in (0 \cup N_s); \quad \forall j \in (N_s \cup N+1); \quad \forall k \in K; \quad \forall h \in H_{m_{ij}}; \quad i \neq j; \quad \forall m \in M_{ij} \quad (6)$$

$$y_i^k - Tnew_{ijm}^{h-1} x_{ijm}^{hk} \geq 0 \quad \forall i \in (0 \cup N_s); \quad \forall j \in (N_s \cup N+1); \quad \forall k \in K; \quad \forall h \in H_{m_{ij}}; \quad i \neq j; \quad \forall m \in M_{ij} \quad (7)$$

$$y_j^k \leq E_3 \sum_{i \in N} \sum_{m \in M_{ij}} \sum_{h \in H_{m_{ij}}} x_{ijm}^{hk} \quad \forall k \in K; \quad \forall j \in N \quad (8)$$

$$y_i^k - s_i \leq e_i \sum_{j \in (N_s \cup N+1)} \sum_{m \in M_{ij}} \sum_{h \in H_{m_{ij}}} x_{ijm}^{hk} \quad \forall i \in N_s; \quad \forall k \in K \quad (9)$$

$$\sum_{i \in N_s} q_i \sum_{j \in (N_s \cup N+1)} \sum_{m \in M_{ij}} \sum_{h \in H_{m_{ij}}} x_{ijm}^{hk} \leq Q \quad \forall k \in K \quad (10)$$



$$x_{im}^{hk} = 0 \quad \forall i \in N; \quad \forall m \in M_{ij}; \quad \forall h \in H_{m_j}; \quad \forall k \in K \quad (11)$$

$$x_{i0m}^{hk} = 0 \quad \forall i \in N; \quad \forall m \in M_{ij}; \quad \forall h \in H_{m_j}; \quad \forall k \in K \quad (12)$$

$$x_{(N+1)jm}^{hk} = 0 \quad ; \forall j \in N; \forall m \in M_{ij}; \quad \forall h \in H_{m_j}; \quad \forall k \in K \quad (13)$$

$$x_{ijm}^h = \{0,1\} \quad \begin{array}{l} \forall i, j \in N; \quad \forall m \in M_{ij} \\ \forall h \in H_{m_j}; \quad \forall k \in K \end{array} \quad (14)$$

$$y_i^k \geq 0 \quad \forall i \in N; \quad k \in K \quad (15)$$

The objective function minimizes the total travel time and the number of vehicles, because often available vehicles are rarely sufficient in disaster relief situations. Without minimizing the number of vehicles, the model would probably use more vehicles to minimize the total travel time.

The constraints are defined as follows:

Constraints (2) and (3) show that all the disaster area must be served just once. Constraint (4) guarantees that the routes must be finished in the depots. In fact, a vehicle must leave the node which entered before. Constraint (5) shows the departure time from disaster areas. This constraint also prevents the creation of sub tour. Constraints (6) and (7) determine the appropriate time interval according to the vehicle departure time from the relief distribution center. An inequality auxiliary (8) is provided in order to estimate the time of finishing the travel. Constraints (9) imply that the exit time from each affected area must be less than the related emergency time of that node. Constraints (10) reflect the vehicle capacity limitations. It exerts that the mass of any load being carried on a vehicle must not exceed the maximum load capacity. Constraints (11-13) are used to ensure that the achieved solutions are accurate and practical. Decision variables type and domains are indicated in (14) and (15). In the next section, the problem has been modeled in General Algebraic Modeling System (GAMS).

4. Experimental study

In this section, we produce different examples and the related results are presented. The commercial software GAMS and the MIP solver GAMS/OSL are employed to solve the proposed MIP problem. All tests were executed on a personal computer equipped with a 3.2 GHz Intel Pentium 4 CPU and 1 GB RAM, using the Microsoft Windows 7

operating system. The results are represented in Table 1. In this table, the first column shows the number of parallel edges. Second column indicates the number of available vehicles. The obtained number of required vehicles for each sample problem is represented in column 3. The next three columns are related to CPLEX results.

As can be seen from Table 1, achieved solution is affected by various parameters such as number of parallel edges and number of vehicles. In order to investigate the impact of different parameters on the objective function a brief sensitivity analysis is provided in the next section.

5. Sensitivity analysis

In this section, Twenty six instances were generated to evaluate the proposed model and analyzed the impact of number of parallel edge on the objective function and computational time. As mentioned, we have solved several examples to investigate the effect of six nodes with different number of vehicle. The related input parameters are provided in Table 1. The first column relates to number of parallel edge in six nodes and the second column show the number of vehicles and forth column show the objective function and the Fifth column relates to the number of vehicle. First, we investigate the impact of parallel edge transportation on the objective functions in the proposed model. As can be seen in Figure 6, the values of objective function are continually reduced with increasing the number of parallel edge for different examples. The analysis implies that all arcs in the network are not equally valuable. In fact, due to the variation of travel time between parallel arcs, some of them are more desirable to be utilized in transferring the injured people. In addition, the sensitivity analysis of computational time by changing the number of parallel edge for transportation network is depicted in

Figure 7. It shows that due to the NP hardness of the problem, the computational time increases by increasing the number of vehicles and parallel edge. Based on the results

from Figure 7, there is significant decrease in the objective function when the numbers of parallel edge increase.

Table 1 - Structures of the sample problems and the Results of CPLEX

Number Of parallel Edges	Number Of vehicle	Number Of used Vehicle	Result of example	Time	Result Gap
0	2	2	23.0859	84.7	0
	3	2	23.0859	327.9	0
1	2	2	23.0859	137.2	0
	3	2	23.0859	548.9	0
3	2	2	23.0859	144.1	0
	3	2	23.0859	723.9	0
5	2	2	22.53238	225.6	0
	3	2	22.53238	845.8	0
7	2	2	22.53238	309.9	0
	3	2	22.53238	1361.5	0
9	2	2	22.53238	364.1	0
	3	2	22.53238	1661.5	0
11	2	2	22.53238	436.1	0
	3	3	22.53238	2166.9	0
13	2	2	22.532	629.687	0
	3	2	22.532	2235.31	0
15	2	2	22.532	679.45	0
	3	2	22.532	2951.686	0
17	2	2	22.532	742.911	0
	3	2	22.532	3105.16	0
19	2	2	21.977	920.243	0
	3	2	21.977	5152.446	0
21	2	2	21.977	1162.516	0
	3	2	21.977	5454.434	0
23	2	2	21.771	1255.857	0
	3	2	21.771	7459.98	0

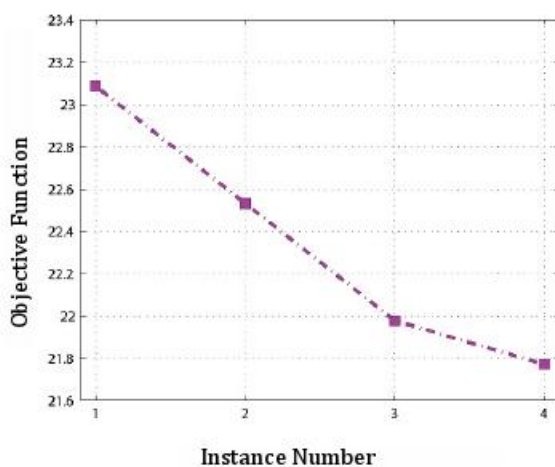


Figure 6. Objective function value for various instances

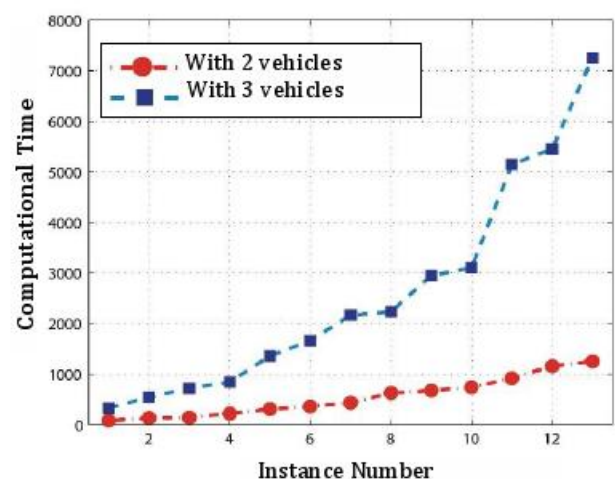


Figure 7. Computational time for different instances with various numbers of parallel edges.



6. Conclusions and future works

In emergencies, humanitarian logistics management is applied to improve the efficiency of scarce resources. In this paper, we studied a time dependent vehicle routing problem in disaster relief efforts which more than one edge exist between nodes. Such problems can be demonstrated in urban areas with different traffic conditions. Therefore, vehicle routing problem is utilized to assign routes for a fleet of vehicles in order to give emergency commodities to affected areas and injured people. Such decisions are significant since prompt delivery can save the lives of many people. We provide a time dependent vehicle routing problem by applying FIFO property in multi-graph network in purpose of minimizing waiting time and number of vehicle. In the following, we implemented the problem in GAMS (with CPLEX solver). Finally, a brief sensitivity analysis is brought to examine the impact of different problem structure on the obtained solution.

Some directions for future works can be incorporating other important features of VRP problems such as a heterogeneous fleet of vehicles and consideration of demands uncertainty in the proposed model. Moreover, provide an efficient meta-heuristic to solve the large-scale problems.

7. References

- [1] Sheu, Jiuh-Biing. "Challenges of emergency logistics management." *Transportation research part E: logistics and transportation review* 43, no. 6 (2007): 655-659.
- [2] Kumar, Suresh Nanda, and Ramasamy Panneerselvam. "A Time-Dependent Vehicle Routing Problem with Time Windows for E-Commerce Supplier Site Pickups Using Genetic Algorithm." *Intelligent Information Management* 7, no. 04 (2015): 181
- [3] Vitoriano, Begonia, M. Teresa Ortuño, Gregorio Tirado, and Javier Montero. "A multi-criteria optimization model for humanitarian aid distribution." *Journal of Global Optimization* 51, no. 2 (2011): 189-208.
- [4] Eshghi, Kourosh, and Richard C. Larson. "Disasters: lessons from the past 105 years." *Disaster Prevention and Management: An International Journal* 17, no. 1 (2008): 62-82
- [5] Najafi, Mehdi, Kourosh Eshghi, and Wout Dullaert. "A multi-objective robust optimization model for logistics planning in the earthquake response phase." *Transportation Research Part E: Logistics and Transportation Review* 49, no. 1 (2013): 217-249.
- [6] Haghani, Ali, and Sei-Chang Oh. "Formulation and solution of a multi-commodity, multi-modal network flow model for disaster relief operations." *Transportation Research Part A: Policy and Practice* 30, no. 3 (1996): 231-250
- [7] Haghani, Sei-Chang Oh Ali. "Testing and evaluation of a multi-commodity multi-modal network flow model for disaster relief management." *Journal of Advanced Transportation* 31, no. 3 (1997): 249-282.
- [8] Barbarosoglu, Gulay, Linet Ozdamar, and Ahmet Cevik. "An interactive approach for hierarchical analysis of helicopter logistics in disaster relief operations." *European Journal of Operational Research* 140, no. 1 (2002): 118-133.
- [9] Ozdamar, Linet, Ediz Ekinci, and Beste Kucukyazici. "Emergency logistics planning in natural disasters." *Annals of operations research* 129, no. 1-4 (2004): 217-245.
- [10] Tzeng, Gwo-Hsiung, Hsin-Jung Cheng, and Tsung Dow Huang. "Multi-objective optimal planning for designing relief delivery systems." *Transportation Research Part E: Logistics and Transportation Review* 43, no. 6 (2007): 673-686.
- [11] Sheu, Jiuh-Biing. "Dynamic relief-demand management for emergency logistics operations under large-scale disasters." *Transportation Research Part E: Logistics and Transportation Review* 46, no. 1 (2010): 1-17.
- [12] Nolz, Pamela C., Frédéric Semet, and Karl F. Doerner. "Risk approaches for delivering disaster relief supplies." *OR spectrum* 33, no. 3 (2011): 543-569.
- [13] Malandraki, Chryssi, and Mark S. Daskin. "Time dependent vehicle routing problems: Formulations, properties and heuristic algorithms." *Transportation science* 26, no. 3 (1992): 185-200.
- [14] Ichoua, Soumia, Michel Gendreau, and Jean-Yves Potvin. "Vehicle dispatching with time-dependent travel times." *European journal of operational research* 144, no. 2 (2003): 379-396.
- [15] Setak, Mostafa, Majid Habibi, Hossein Karimi, and Mostafa Abedzadeh. "A time-dependent vehicle routing problem in multigraph with FIFO property." *Journal of Manufacturing Systems* 35 (2015): 37-45.
- [16] Lin, Yen-Hung, Rajan Batta, Peter A. Rogerson, Alan Blatt, and Marie Flanigan. "A logistics model for emergency supply of critical items in the aftermath of a disaster." *Socio-Economic Planning Sciences* 45, no. 4 (2011): 132-145.

