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Fraction Defective Chart for High Quality Processes with Adjusted Control Limits to Improve In-control Performance

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Abstract

The np-chart is used to monitor the number of nonconforming items in a sample. The performance of the np-chart in Phase II depends on the accuracy of estimated parameter in Phase I. Despite the fact that taking large sample sizes ensure the performance of estimated parameter charts, it can be impractical for attribute control charts. Recently, the performance of traditional c-chart and np-chart with some adjustments have been studied. In practice, high quality processes with very low rate of nonconformities are often observed. The traditional control charts suffer a serious inaccuracy in control limits specification in this way. Having all this in mind, this paper presents a new np-chart based on simple adjustments derived from Cornish-Fisher expansions to improve such inaccuracy with the application of bootstrap. Based on the conditional in-control average run length (ARL_0), their performances are compared through a simulation study.

Keywords:
np-chart, Bootstrap, Adjusted limits, Cornish-Fisher expansions, ARL_0

Introduction

Control charts are the most popular tool of statistical process control for monitoring a stable process to detect assignable cause(s) when impress the process. For selecting the suitable control chart for monitoring, the type of data being collected must be considered which is mainly classified into variables and attributes. Attribute data, such as discrete and/or categorical data, are monitored using methods that have been conducted by many researchers in literature (see [1] and [2] for comprehensive review). Among them, the traditional p-chart is the standard attribute control chart used to monitor the fraction of nonconforming products on the basis of a Binomial distribution with control limits constructed by the normal approximation [3].

In determining control limits, nonconforming probability for the in-control process must be known. Nevertheless, when this parameter is unknown, firstly it must be estimated. Through m available in-control samples each of size n, the parameter is estimated. This takes place in Phase I of the control chart designing (overview on Phase I methods and analyses can be found in [4]). Following Phase I, monitoring of the process for quickly detection of out-of-control conditions is considered which is generally labelled by Phase II in SPC.

During Phase II, a usual measure of performance evaluation is the average run length (ARL). The in-control ARL, or ARL_0, is the expected number of samples observed before a false alarm, i.e. when there is no shift. Thus, larger value for ARL_0 is desirable. On the other hand, the out-of-control ARL, or ARL_1, is the expected number of samples observed before correctly detection of a shift. Tendency toward rapidly detection of an assignable cause necessitates smaller value for ARL_1. Furthermore, the median of ARL (MDRL), the average of ARL (AARL) and the standard deviation of ARL (SDARL), which is also referred to as practitioner-to-practitioner variability, can be expressed as other performance measures obtained from ARL sampling distribution.

Due to different Phase I data sets, which lead to variation in sampling distribution, the performance of control charts with different estimated parameters, are substantially affected than the situation with known parameter values. On the other hand, since the control limits are constructed using the estimated parameter and a pre-specified false alarm rate (FAR), the accuracy of the estimation play a significant role on determining the performance values [5]. In the recent control chart literature, the impacts of parameter estimation on the performance measures, from a practical as well as a theoretical standpoint, have gain attentions of several researchers to contribute in this area.

Much of such works have focused on variable control charts and little on attributes (for thorough reviews, refer to [6] and [7]).

Relying on recent studies, there are mainly two major points about parameter estimation that must be considered as follows: a) There is no guarantee, even with far larger sizes of Phase I samples, to reach the desired in-control performance and thus the importance of SDARL should not be neglected (as firstly mentioned in [8] by recognizing from [9]), and b) In some cases, the suggested Phase I sample sizes are impractical to satisfactorily decrease the variation in the in-control ARL between practitioners.

The overall conclusion indicates that an alternative method is necessary to overcome the challenges associated with above mentioned reasons. Relating estimation with simulation, Efron [10] introduced the concept of bootstrap as a computer intensive estimation method. Requiring fewer assumptions and giving more accurate results in general together with the fast developments in computing capability, the bootstrap method has turned into an attractive technique to use for wide range of problems. The parametric bootstrap technique, for CUSUM chart in [8] and [11], was applied to determine adjusted control limits based on the data collected to estimate the in-control state. Thus, it guaranteed to achieve the desired in-control conditional performance with a certain probability. Afterwards, extensions of the proposed method for S^2 chart, EWMA chart and *c*-chart were studied in [12], [13] and [14], respectively. Recently, Faraz et al. [15] assessed the *np*-charts with alternative control limits and with estimated parameters as well as applying the bootstrap method to adjust control limits' thresholds. Encouraging to use for being effective, accurate and practical, they suggested to further extension to the other type of attribute control charts such as the Cornish-Fisher corrected p-chart [16]. This correction was performed later in [17] for the *np*-chart which is more convenient to monitor the number of nonconforming items in practice.

In this paper, we investigate the performance of the C-F corrected *np*-charts such that the in-control performance exceeds a desired value with a specified probability. Accordingly, we expect to reduce estimation error when a bootstrap-based methodology is used to adjust the control limits. The rest of the paper is organized as follows: In Section 2, we review the improved *np*-chart using Cornish-Fisher expansion. In the next section, Bootstrap adjusted limits for the improved *np*-charts is described and the procedure is presented in some steps. Section 4 includes some simulation experiments for investigating the sampling distribution of the in-control performance for the improved *np*-chart and, then, comparing the results with bootstrap-based methodology. Finally, conclusions and future research possibilities complete the paper.

Improved np-charts using Cornish-Fisher expansion

A common approach to modeling the number of nonconforming items is to use the Binomial distribution. For a stable process with known in-control nonconforming probability p_0, it is assumed $X_1, X_2, \ldots, X_m$ to be an independent sequence of m initial samples each of size n; that is, $X_i \sim Bin(n, p_0)$ for i=1,…,m. However, the value of p_0 is typically unknown in practice and, thus, it must be estimated from initially sampled data. This parameter is estimated as follows [3]:

$$\overline{p} = \frac{\sum_{i=1}^{m} p_i}{m} = \frac{\sum_{i=1}^{m} X_i}{mn} \qquad (1)$$

The upper and lower control limits of Shewhart *np*-chart, for monitoring the plotted sequence of X_i values over time, are considered as following equations, respectively:

$$\begin{cases} \hat{UCL} = [n\overline{p} + z_{1-\alpha/2}\sqrt{n\overline{p}(1-\overline{p})}\,] \\ \hat{LCL} = \max(0, [n\overline{p} - z_{1-\alpha/2}\sqrt{n\overline{p}(1-\overline{p})}\,]) \end{cases} \qquad (2)$$

where $z_{1-\alpha/2}$ is $1-\alpha/2^{th}$ percentile of the standard normal distribution. Note that when $LCL \leq 0$, the lower control limit holding no meaning, lies on zero and thus, the upper control limit is changed by substituting $z_{1-\alpha}$ instead of $z_{1-\alpha/2}$.

Because of recent high-tech developments, high-quality processes with very low rate of nonconformities are often detected in practice where using the traditional chart leads to high false alarm rate and consequently, unnecessary increasing of inspection costs. For these reasons, alternative methods have been proposed recently [18]. Winterbottom [16] introduced an improved *p*-chart with one correction term, based on the Cornish-Fisher expansion. According to their results the improved *p*-chart: a) shows false alarm risk much closer to the reference risk, and b) allows working with smaller *p* values. The upper control limits of this expansion accommodated for *np*-chart are as follows [17]:

$$\hat{UCL}_1 = n\overline{p} + z_{1-\alpha/2}[n\overline{p}(1-\overline{p})]^{1/2} + \frac{(z_{1-\alpha/2}^2 - 1)(1-2\overline{p})}{6} \qquad (3)$$

where $\hat{UCL}_1$ denote upper control limits with one C-F correction term. When $z_{1-\alpha/2}$ is substituted by $z_{\alpha/2}$, lower control limit can be obtained. Setting $z_{\alpha/2}$,=3 and $z_{1-\alpha/2}$,=-3, the Shewhart *np*-chart can be corrected as follows:

$$\begin{cases} \hat{UCL}_1 = [n\overline{p} + z_{1-\alpha/2}\sqrt{n\overline{p}(1-\overline{p})} + \frac{4(1-2\overline{p})}{3}] \\ \hat{LCL}_1 = \max(0, [n\overline{p} - z_{1-\alpha/2}\sqrt{n\overline{p}(1-\overline{p})} + \frac{4(1-2\overline{p})}{3}]) \end{cases} \qquad (4)$$

In [15] the alternative control limits were also derived for the *np*-chart in order to: 1) adjust the control limits so that ARL_0 is at least the desired value, 2) avoid masking the problem of practitioner-to-practitioner variability using the three-sigma limits, and 3) split α as equally as possible between the two sides of the chart. The equations are as follows:

$$L\hat{C}L = F^{-1}(\alpha/2, n, \overline{p})$$

$$U\hat{C}L = \begin{cases} F^{-1}(1-\alpha/2, n, \overline{p}) & if\ L\hat{C}L \geq 1 \\ F^{-1}(1-\alpha, n, \overline{p}) & if\ L\hat{C}L = 0 \end{cases} \quad (5)$$

where $F^{-1}(\alpha, n, p_0)$ is the inverse cumulative distribution function of $Bin\ (n, p_0)$ at point α.

Despite the fact that the ARL_0 comes to be a random variable in Phase II, the conditional ARL_0 given the estimated nonconforming probability can be obtained as follows:

$$ARL_0|\overline{p} = \frac{1}{\hat{\alpha}|\overline{p}}; \quad \hat{\alpha}|\overline{p} \neq 0 \quad (6)$$

where

$$\hat{\alpha}|\overline{p} = \begin{cases} 1 - F(U\hat{C}L, n, p_0) + F(L\hat{C}L - 1, n, p_0) & if\ L\hat{C}L \geq 1 \\ 1 - F(U\hat{C}L, n, p_0) & if\ L\hat{C}L = 0 \end{cases} \quad (7)$$

where $F(x, n, p_0)$ is the cumulative distribution function of $Bin\ (n, p_0)$ at point x. Note that $L\hat{C}L$ and $U\hat{C}L$ are general terms and their subscripts can be changed according to equation (3). Using a simple example, we compared different np-charts in Table 1.

Table 1 – Comparison of different np-charts for n=50, p_0=0.01, α=0.0027.

np-chart	LCL	UCL	α	ARL_0
Shewhart	0	2	0.0138	72.4
altered in [17]	0	3	0.0016	626.5
C-F with one expan.	0	3.55	0.0016	626.5

When the number of Phase I datasets (m) is not adequate enough, the probability of achievement to the desired performance becomes to a smaller amount. In order to guarantee the desired in-control performance with (1-τ)100%, it is proposed to adjust limits using the bootstrap technique as described in the following section.

Bootstrap adjusted limits for the improved np-charts

In previous studies, it has been revealed that the in-control performance is effected from the Phase I sampling variability obtained by different practitioners. In fact, the average in-control performance value might be close to the target value, while a single in-control value can be vary considerably. The bootstrap approach have been suggested to reduce the effect of sampling variability on the ARL performance with estimated parameters in recent literature.

Similarly, in this section, we apply a bootstrap-based methodology to adjust the limits of the C-F corrected *np*-charts such that the conditional in-control performance meets or exceeds a desired value with a specified probability. The bootstrap algorithm is outlined below:

1. Estimate process parameter from a Phase I dataset, called "training sample", using Equation (1).
2. Draw $y_i^* \sim Bin(mn, \overline{p})$ to estimate $p_i^* = y_i^* / mn$.
3. Calculate the improved *np*-chart limits using the C-F correction for each bootstrap estimates as follows:

$$L\hat{C}L_i^* = F^{-1}(\alpha/2, n, p_i^*)$$

$$U\hat{C}L_i^* = \begin{cases} F^{-1}(1-\alpha/2, n, p_i^*) & if\ L\hat{C}L_i \geq 1 \\ F^{-1}(1-\alpha, n, p_i^*) & if\ L\hat{C}L_i = 0 \end{cases} \quad (8)$$

4. Repeat Steps 2-3 a large number of times, called bootstrap sample size, to obtain $i = 1,..., B$ control limits:
$\{L\hat{C}L_1^*, L\hat{C}L_2^*, ..., L\hat{C}L_B^*\}$
$\{U\hat{C}L_1^*, U\hat{C}L_2^*, ..., U\hat{C}L_B^*\}$
5. Sort the B bootstrap control limits from Step 4 in an increasing order as:
$\{L\hat{C}L_{(1)}^*, L\hat{C}L_{(2)}^*, ..., L\hat{C}L_{(B)}^*\}$
$\{U\hat{C}L_{(1)}^*, U\hat{C}L_{(2)}^*, ..., U\hat{C}L_{(B)}^*\}$
6. Find respectively $L\hat{C}L_\tau^*$ and $U\hat{C}L_\tau^*$ as the τ^{th} and $(1-\tau)^{th}$ percentiles to guarantee the in-control performance with probability (1-τ)100%. For cases with no lower control limit, store only the $(1-\tau)^{th}$ percentile.

Generally, it is expected that adjusted control limits obtained based on the percentiles of bootstrap distribution result in widened control limits to counteract the effect of estimation errors caused by the unknown parameters.

Simulation study

In this section, some simulation experiments were implemented with the purpose of exploring the effect of parameter estimation on the behavior of the performance measure. It is obvious that, when the process parameter is estimated, the calculated control limits and, as a result, the ARL_0 measure turn into functions of $\overline{p}$. Therefore, we expect the performance measure becomes a random variable with the mean and the standard deviation.

In this study, r=10,000 different Phase I samples were simulated consisting of m samples each from $Bin\ (n, p_0)$. Choosing a desired false alarm rate, the control limits with C-F corrections were calculated for each Phase I dataset. Tables 2 and 3, respectively for α=0.0027 and α=0.005, show the 10% and 25% percentiles, the median, the average and the standard deviation of ARL_0 distribution following the computation of the control limits for various cases of the simulation (the case $m=\infty$ relates to the situation with known process parameter in which the performance measure becomes a constant value). Some conclusions about the effects of the number of Phase I datasets, the sample size and the false alarm rate on the amount of guaranteed performance are summarized as following:

- The effect of m: for example, when p_0=0.2, n=100, and α=0.0027, we have $Q_{0.25}$=547 for m=25. That is, with 50 Phase I datasets, the desired ARL_0=370.4 can be guaranteed with probability 75%. Also, $Q_{0.10}$ goes beyond the desired value for the first time when $m\geq75$ which means 75 samples are enough to guarantee $Pr(ARL_0>370.4)$=90%.

Table 2 - The distribution of ARL_0 for improved np-chart with one C-F correction term for different values of n, m, and p_0 when α=0.0027.

n		50					100				
		Lower Quartiles					*Lower Quartiles*				
P_0	m	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$
0.01	25	72.37	626.50	626.50	1407.12	2168.71	291.35	291.35	291.35	957.80	1450.20
	50	626.50	626.50	626.50	906.25	1338.65	291.35	291.35	291.35	862.30	844.01
	75	626.50	626.50	626.50	795.66	1039.68	291.35	291.35	291.35	763.50	746.41
	100	626.50	626.50	626.50	690.51	639.96	291.35	291.35	291.35	747.13	715.77
	125	626.50	626.50	626.50	669.61	520.42	291.35	291.35	291.35	675.15	677.48
	150	626.50	626.50	626.50	650.20	383.81	291.35	291.35	291.35	661.88	669.35
	200	626.50	626.50	626.50	627.75	88.21	291.35	291.35	291.35	622.40	642.93
	∞	626.50	626.50	626.50	626.50	0.00	291.35	291.35	291.35	291.35	0.00
0.02	25	311.55	311.55	311.55	1037.12	1353.16	246.18	246.18	246.18	765.24	1085.86
	50	311.55	311.55	311.55	867.22	826.82	246.18	246.18	246.18	648.36	480.68
	75	311.55	311.55	311.55	769.35	778.29	246.18	246.18	246.18	610.51	410.72
	100	311.55	311.55	311.55	718.31	747.40	246.18	246.18	246.18	620.90	421.87
	125	311.55	311.55	311.55	657.14	704.03	246.18	246.18	246.18	623.02	411.89
	150	311.55	311.55	311.55	620.84	674.40	246.18	246.18	246.18	617.11	411.28
	200	311.55	311.55	311.55	592.72	649.14	246.18	246.18	246.18	603.88	409.69
	∞	311.55	311.55	311.55	311.55	0.00	246.18	246.18	246.18	246.18	0.00
0.05	25	313.64	313.64	313.64	701.05	811.40	233.96	233.96	682.90	616.30	664.36
	50	313.64	313.64	313.64	606.45	489.06	233.96	233.96	682.90	522.23	351.45
	75	313.64	313.64	313.64	561.98	443.64	233.96	233.96	682.90	507.71	266.34
	100	313.64	313.64	313.64	538.47	420.57	233.96	233.96	682.90	502.97	244.67
	125	313.64	313.64	313.64	520.98	407.98	233.96	233.96	682.90	508.53	236.30
	150	313.64	313.64	313.64	499.73	391.39	233.96	233.96	682.90	509.06	222.57
	200	313.64	313.64	313.64	471.07	366.21	233.96	233.96	682.90	512.15	219.27
	∞	313.64	313.64	313.64	313.64	0.00	682.90	682.90	682.90	682.90	0.00
0.10	25	106.90	310.57	310.57	606.41	614.22	218.30	218.30	505.42	551.65	511.19
	50	310.57	310.57	310.57	541.64	371.09	498.72	498.72	885.53	732.86	284.45
	75	310.57	310.57	310.57	502.82	318.64	498.72	498.72	885.53	726.97	245.33
	100	310.57	310.57	310.57	490.41	302.59	498.72	498.72	885.53	715.74	219.90
	125	310.57	310.57	310.57	481.26	296.64	466.73	498.72	885.53	714.40	212.07
	150	310.57	310.57	310.57	455.82	280.31	434.74	498.72	885.53	722.00	204.55
	200	310.57	310.57	310.57	439.41	267.74	434.74	498.72	885.53	717.39	201.57
	∞	310.57	310.57	310.57	310.57	0.00	885.53	885.53	885.53	885.53	0.00
0.20	25	369.84	395.96	888.80	830.92	489.62	293.54	547.22	547.22	549.62	179.03
	50	369.84	395.96	888.80	829.26	397.70	311.75	547.22	547.22	576.49	157.04
	75	369.84	395.96	888.80	798.18	315.23	547.22	547.22	547.22	585.26	141.85
	100	369.84	888.80	888.80	806.72	272.45	547.22	547.22	547.22	595.45	138.49
	125	369.84	888.80	888.80	808.32	232.70	547.22	547.22	547.22	596.94	134.07
	150	369.84	888.80	888.80	812.39	219.95	547.22	547.22	547.22	599.99	132.05
	200	369.84	888.80	888.80	820.70	184.32	547.22	547.22	547.22	607.83	141.12
	∞	888.80	888.80	888.80	888.80	0.00	547.22	547.22	547.22	547.22	0.00

Table 3 – The distribution of ARL_0 for improved *np*-chart with one C-F correction term for different values of *n*, *m*, and p_0 when α=0.005.

n		50					100				
		Lower Quartiles					*Lower Quartiles*				
P_0	*m*	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$
0.01	25	72.37	72.37	626.50	556.44	813.06	54.42	291.35	291.35	484.16	571.23
	50	72.37	626.50	626.50	541.86	387.18	291.35	291.35	291.35	362.04	367.31
	75	72.37	626.50	626.50	533.18	207.39	291.35	291.35	291.35	325.18	251.29
	100	72.37	626.50	626.50	563.99	175.31	291.35	291.35	291.35	303.81	161.74
	125	626.50	626.50	626.50	574.08	162.19	291.35	291.35	291.35	295.08	100.03
	150	626.50	626.50	626.50	591.92	134.05	291.35	291.35	291.35	292.28	61.61
	200	626.50	626.50	626.50	599.12	120.09	291.35	291.35	291.35	292.56	44.79
	∞	626.50	626.50	626.50	626.50	0.00	291.35	291.35	291.35	291.35	0.00
0.02	25	56.31	311.55	311.55	429.64	543.06	64.58	246.18	246.18	371.82	454.93
	50	311.55	311.55	311.55	369.88	380.37	246.18	246.18	246.18	303.64	245.26
	75	311.55	311.55	311.55	326.91	220.72	246.18	246.18	246.18	276.87	179.86
	100	311.55	311.55	311.55	314.94	125.63	246.18	246.18	246.18	268.94	148.45
	125	311.55	311.55	311.55	313.93	104.77	246.18	246.18	246.18	257.02	109.20
	150	311.55	311.55	311.55	312.16	64.74	246.18	246.18	246.18	251.90	79.17
	200	311.55	311.55	311.55	311.60	37.90	246.18	246.18	246.18	249.77	59.20
	∞	311.55	311.55	311.55	311.55	0.00	246.18	246.18	246.18	246.18	0.00
0.05	25	84.84	84.84	313.64	351.01	378.36	87.17	233.96	233.96	299.94	283.46
	50	84.84	313.64	313.64	298.17	201.34	87.17	233.96	233.96	266.72	176.89
	75	84.84	313.64	313.64	292.15	136.75	87.17	233.96	233.96	250.29	129.86
	100	84.84	313.64	313.64	292.91	101.55	233.96	233.96	233.96	242.86	102.95
	125	313.64	313.64	313.64	292.71	76.50	233.96	233.96	233.96	237.68	84.84
	150	313.64	313.64	313.64	297.77	64.26	233.96	233.96	233.96	237.59	74.16
	200	313.64	313.64	313.64	303.78	49.09	233.96	233.96	233.96	234.76	51.62
	∞	313.64	313.64	313.64	313.64	0.00	233.96	233.96	233.96	233.96	0.00
0.10	25	106.90	106.90	310.57	303.76	276.74	203.98	203.98	434.74	397.92	205.10
	50	106.90	310.57	310.57	266.87	160.12	203.98	203.98	434.74	414.31	187.14
	75	106.90	310.57	310.57	262.10	121.80	203.98	363.31	434.74	408.58	162.38
	100	106.90	310.57	310.57	264.23	104.02	203.98	434.74	434.74	407.80	141.06
	125	310.57	310.57	310.57	268.49	90.22	203.98	434.74	434.74	410.09	119.54
	150	310.57	310.57	310.57	272.69	83.73	203.98	434.74	434.74	409.97	110.58
	200	310.57	310.57	310.57	278.93	74.60	203.98	434.74	434.74	410.10	90.95
	∞	310.57	310.57	310.57	310.57	0.00	434.74	434.74	434.74	434.74	0.00
0.20	25	154.96	369.84	369.84	375.28	157.86	157.82	250.93	257.47	291.65	93.39
	50	154.96	369.84	369.84	386.50	143.65	250.93	250.93	257.47	306.89	85.73
	75	369.84	369.84	369.84	394.93	135.42	250.93	250.93	250.93	311.78	84.19
	100	369.84	369.84	369.84	397.26	127.11	250.93	250.93	250.93	314.67	82.52
	125	369.84	369.84	369.84	395.37	120.63	250.93	250.93	250.93	316.51	81.98
	150	369.84	369.84	369.84	396.62	116.34	250.93	250.93	250.93	316.89	81.44
	200	369.84	369.84	369.84	390.81	99.98	250.93	250.93	250.93	314.36	80.19
	∞	369.84	369.84	369.84	369.84	0.00	250.93	250.93	250.93	250.93	0.00

- The effect of n: for the above mentioned example, we have $SDARL_0$=179.03 while for n=50 this value increases to 489.62. That is, for larger sample sizes, the desired ARL_0 can be obtained with smaller standard deviation and, thus, with more precision.
- The effect of α: the results are remarkable for α=0.005. In fact, the desired ARL_0 is guaranteed with probability 75% for m=50 and larger. However, in order to reach desired ARL_0 with probability 90%, at least m=125 samples are required. Nevertheless, there are some exceptions where m=25 samples are enough to have $\Pr(ARL_0>200)$=75% and m=50 samples are enough to have $\Pr(ARL_0>200)$=90%.

As indicated, there are cases where no practical amount of Phase I datasets can guarantee the desired ARL_0 with probability 90% and/or even with probability 75%. Therefore, the control limits should be adjusted. For this reason, similar study was performed for the chart with bootstrap adjusted limits by considering B=500. The gathered results in Tables 4 and 5, respectively for α=0.0027 and α=0.005, indicate that the values of desired ARL_0 are satisfied with probability 90% using the bootstrap adjusted limits, while poor in-control performance are observed for unadjusted limits.

Moreover, incensement in distributional values are significant. For example, when p_0=0.2, n=100, m=200 and α=0.0027, we obtain $Q_{0.9}$=986.32 with bootstrap adjusted limits. Whereas the result with unadjusted limits is 574.22. The comparison of ARL_0 distributions for the improved np-chart using one C-F correction term with adjusted and unadjusted limits is illustrated in Figure 1 for this case.

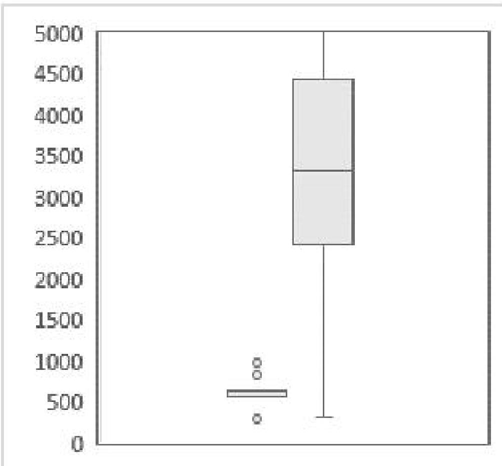

Figure 1 – The box-plot of the distribution for in-control ARL with and without limits adjustment for m=200, n=100, p_0=0.2, and α=0.0027. The reference line is at ARL_0 of 370.

Such results indicate that the bootstrap adjusted limits have better in-control performance. On the other hand, if the amount of Phase I datasets is not available for the np-charts without adjustment, then the control limits should be adjusted. Otherwise, with less chance we expect to have good in-control performance.

Accordingly, once using such limits by practitioners is of primary concern, as when the parameter is unknown and/or practical amount of Phase I datasets is unavailable, it is desirable to estimate the thresholds so that they have a standard guide to achieve the guaranteed performance with probability 90%. The recommended upper and lower thresholds for various values of parameters are shown in Table 6.

Conclusion

In this study, the in-control performance of the improved np-charts using Cornish-Fisher expansions and with estimated process parameters are appraised using some measures. Moreover, the bootstrap-based methodology is applied to adjust the control limits for reducing the effect of sampling variability on the ARL_0 performance and achieving or even exceeding the desired performance value with a specified probability.

This methodology can be extended to other type of control charts. Particularly for the C-F corrected p-chart, Joekesa and Barbosa [19] suggested a rule for selecting the suitable chart as follows:

1. when $np(1\text{-}p)\geq 5$ without correction
2. when $np(1\text{-}p)\geq 0.25$ one term of correction
3. when $np(1\text{-}p)\geq 0.08$ two terms of correction

In this paper, the least value of $np(1\text{-}p)$ was 0.495 and thus the np-chart with one C-F correction term studied. For the time being, we are performing a comprehensive study by considering all above mentioned conditions.

With fast and inexpensive computing developments, the bootstrap has turned into be an attractive technique. Nevertheless, its estimates are subject to bootstrap (statistical) error and a simulation (Monte Carlo) error. The first one, depends on the number of source data and its accuracy, cannot be eliminated using Bootstrap. The second one, due to inadequate randomness, can be reduced by increasing B. Therefore, it was proposed in [12] to run the algorithm for a specified number of times, for example 1000 times, to get the results. In practice, the value of B is left to the experimenter to choose. Note that the problem of selecting the best Bootstrap sample size is also a potential topic which was beyond the scope of this paper (refer to [20] for more information).

The design of control charts from statistical and/or economic aspects have been considered in the literature. For example, a multi-objective economic-statistical design (MOESD) of the improved np-chart using Cornish-Fisher expansions was presented and optimized in [17]. As another future research, such economic-statistical designs can be considered for Bootstrap adjusted limits as well.

Table 4 - The distribution of ARL_0 for improved *np*-chart with one C-F correction term and bootstrap adjusted limits for different values of n, m, and p_0 when α=0.0027.

n		50					100				
		Lower Quartiles					*Lower Quartiles*				
P_0	m	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$
0.01	25	626.50	6863.92	6863.92	27852.6	85886.81	1870.79	1870.79	14067.93	15116.9	31737.79
	50	626.50	6863.92	6863.92	7805.65	13890.08	1870.79	1870.79	1870.79	5347.94	7348.27
	75	626.50	626.50	6863.92	5422.84	4392.98	1870.79	1870.79	1870.79	3043.17	4010.56
	100	626.50	626.50	6863.92	4546.10	3014.43	1870.79	1870.79	1870.79	2192.16	2105.01
	125	626.50	626.50	6863.92	4267.91	3074.91	1870.79	1870.79	1870.79	1926.37	1220.22
	150	626.50	626.50	626.50	3709.03	3118.81	1870.79	1870.79	1870.79	1858.38	811.24
	200	626.50	626.50	626.50	2864.49	2992.07	1870.79	1870.79	1870.79	1799.71	327.46
	∞	626.50	626.50	626.50	626.50	0.00	291.35	291.35	291.35	291.35	0.00
0.02	25	2091.10	2091.10	2091.10	14471.3	36464.69	1073.03	1073.03	5281.61	10187.8	25974.70
	50	2091.10	2091.10	2091.10	4913.81	6184.14	1073.03	1073.03	1073.03	3470.62	3806.59
	75	2091.10	2091.10	2091.10	2839.82	3369.07	1073.03	1073.03	1073.03	2351.40	2078.60
	100	2091.10	2091.10	2091.10	2259.28	1925.43	1073.03	1073.03	1073.03	1734.75	1544.34
	125	2091.10	2091.10	2091.10	2075.24	1123.47	1073.03	1073.03	1073.03	1457.84	1230.19
	150	2091.10	2091.10	2091.10	2001.24	542.47	1073.03	1073.03	1073.03	1247.04	856.16
	200	2091.10	2091.10	2091.10	1982.55	425.94	1073.03	1073.03	1073.03	1107.18	427.26
	∞	311.55	311.55	311.55	311.55	0.00	246.18	246.18	246.18	246.18	0.00
0.05	25	1322.78	1322.78	6306.63	8410.75	17144.00	682.90	2158.55	2158.55	6876.05	19312.10
	50	1322.78	1322.78	1322.78	3150.27	3540.35	682.90	682.90	2158.55	2449.62	2211.98
	75	1322.78	1322.78	1322.78	2127.96	1887.77	682.90	682.90	2158.55	1736.23	1197.64
	100	1322.78	1322.78	1322.78	1618.20	1261.82	682.90	682.90	682.90	1426.01	836.32
	125	1322.78	1322.78	1322.78	1409.60	818.30	682.90	682.90	682.90	1217.33	716.85
	150	1322.78	1322.78	1322.78	1318.51	555.88	682.90	682.90	682.90	1093.26	663.20
	200	1322.78	1322.78	1322.78	1258.35	291.17	682.90	682.90	682.90	903.55	528.09
	∞	313.64	313.64	313.64	313.64	0.00	682.90	682.90	682.90	682.90	0.00
0.10	25	995.40	995.40	3507.29	6479.33	14611.40	1238.28	2954.39	7106.43	7374.86	5853.09
	50	995.40	995.40	995.40	2464.27	2379.94	1198.85	2954.39	2954.39	3918.44	2529.30
	75	995.40	995.40	995.40	1699.36	1235.73	1198.85	1198.85	2954.39	2896.29	1579.85
	100	995.40	995.40	995.40	1355.52	919.78	1198.85	1198.85	2954.39	2460.90	1161.82
	125	995.40	995.40	995.40	1182.34	714.17	1198.85	1198.85	2954.39	2206.42	947.67
	150	995.40	995.40	995.40	1052.82	481.71	1198.85	1198.85	1198.85	2031.54	894.14
	200	995.40	995.40	995.40	979.29	267.98	1198.85	1198.85	1198.85	1749.22	810.62
	∞	310.57	310.57	310.57	310.57	0.00	885.53	885.53	885.53	885.53	0.00
0.20	25	2985.64	2985.64	8587.62	7639.77	6850.81	1323.52	2415.16	3488.32	4198.21	2518.86
	50	1056.29	2985.64	2985.64	3808.13	2453.75	1227.22	1631.12	2415.16	2411.90	993.77
	75	1056.29	1948.02	2985.64	2733.66	1276.00	1227.22	1227.22	1631.12	1842.18	605.73
	100	1056.29	1056.29	2985.64	2323.47	900.96	1227.22	1227.22	1631.12	1567.80	457.95
	125	1056.29	1056.29	1948.02	2107.72	853.90	986.32	1227.22	1227.22	1402.47	357.82
	150	1056.29	1056.29	1948.02	1878.55	797.21	986.32	986.32	1227.22	1293.17	318.88
	200	1056.29	1056.29	1056.29	1568.33	649.44	986.32	986.32	986.32	1151.39	278.56
	∞	888.80	888.80	888.80	888.80	0.00	547.22	547.22	547.22	547.22	0.00

Table 5 – The distribution of ARL_0 for improved np-chart with one C-F correction term and bootstrap adjusted limits for different values of n, m, and p_0 when α=0.005.

n		50					100				
		Lower Quartiles					*Lower Quartiles*				
P_0	m	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$
0.01	25	626.50	626.50	6863.92	8100.08	27025.23	291.35	1870.79	1870.79	4665.57	9552.14
	50	626.50	626.50	626.50	3001.25	3275.83	291.35	1870.79	1870.79	1839.05	2141.72
	75	626.50	626.50	626.50	1957.12	2556.06	291.35	291.35	1870.79	1378.70	933.44
	100	626.50	626.50	626.50	1217.58	1827.47	291.35	291.35	1870.79	1210.90	779.05
	125	626.50	626.50	626.50	940.64	1364.79	291.35	291.35	291.35	1040.95	788.78
	150	626.50	626.50	626.50	759.98	902.73	291.35	291.35	291.35	901.64	769.14
	200	626.50	626.50	626.50	642.72	317.67	291.35	291.35	291.35	700.11	691.82
	∞	626.50	626.50	626.50	626.50	0.00	291.35	291.35	291.35	291.35	0.00
0.02	25	311.55	2091.10	2091.10	4885.64	10199.63	246.18	1073.03	1073.03	3353.38	5636.39
	50	311.55	311.55	2091.10	1836.56	1926.45	246.18	1073.03	1073.03	1360.48	1360.27
	75	311.55	311.55	2091.10	1431.37	1057.68	246.18	1073.03	1073.03	967.81	663.72
	100	311.55	311.55	2091.10	1230.87	889.37	246.18	246.18	1073.03	864.87	433.77
	125	311.55	311.55	311.55	1042.23	875.52	246.18	246.18	1073.03	789.00	418.81
	150	311.55	311.55	311.55	871.04	826.29	246.18	246.18	1073.03	739.16	410.93
	200	311.55	311.55	311.55	641.84	691.93	246.18	246.18	246.18	619.42	411.51
	∞	311.55	311.55	311.55	311.55	0.00	246.18	246.18	246.18	246.18	0.00
0.05	25	313.64	1322.78	1322.78	2862.93	4409.44	682.90	682.90	2158.55	2837.83	7447.39
	50	313.64	313.64	1322.78	1266.27	1174.98	233.96	682.90	682.90	1030.90	841.03
	75	313.64	313.64	1322.78	937.23	569.41	233.96	682.90	682.90	747.67	480.98
	100	313.64	313.64	313.64	803.16	516.21	233.96	682.90	682.90	631.66	325.83
	125	313.64	313.64	313.64	696.82	489.91	233.96	233.96	682.90	578.40	232.76
	150	313.64	313.64	313.64	606.09	457.86	233.96	233.96	682.90	543.93	218.24
	200	313.64	313.64	313.64	479.95	374.43	233.96	233.96	682.90	498.39	220.90
	∞	313.64	313.64	313.64	313.64	0.00	233.96	233.96	233.96	233.96	0.00
0.10	25	310.57	995.40	995.40	2836.95	7717.00	1198.85	1198.85	2767.59	2812.45	2373.21
	50	310.57	310.57	995.40	1024.10	1127.13	498.72	1198.85	1198.85	1439.95	690.07
	75	310.57	310.57	995.40	769.34	463.68	498.72	885.53	1198.85	1075.65	373.00
	100	310.57	310.57	995.40	656.94	351.99	498.72	885.53	885.53	921.74	291.22
	125	310.57	310.57	310.57	567.39	331.74	498.72	885.53	885.53	841.03	247.96
	150	310.57	310.57	310.57	522.06	316.60	434.74	498.72	885.53	774.95	221.42
	200	310.57	310.57	310.57	419.10	250.27	434.74	434.74	885.53	711.74	216.65
	∞	310.57	310.57	310.57	310.57	0.00	434.74	434.74	434.74	434.74	0.00
0.20	25	1056.29	1056.29	2985.64	2540.64	1802.41	637.30	1227.22	1631.12	1691.28	849.06
	50	888.80	888.80	1056.29	1407.92	635.72	614.10	839.29	986.32	1057.60	378.41
	75	888.80	888.80	888.80	1127.15	485.07	547.22	547.22	986.32	859.04	263.02
	100	888.80	888.80	888.80	962.59	358.07	547.22	547.22	628.03	767.82	216.69
	125	395.96	888.80	888.80	888.28	284.64	547.22	547.22	628.03	698.29	198.77
	150	369.84	888.80	888.80	853.17	223.57	547.22	547.22	547.22	641.32	169.53
	200	369.84	888.80	888.80	813.84	199.09	415.66	547.22	547.22	567.67	112.67
	∞	369.84	369.84	369.84	369.84	0.00	250.93	250.93	250.93	250.93	0.00

Table 6 – The bootstrap adjusted limits for the improved np-charts with one C-F correction term to guarantee $Pr(ARL_0>B)=90\%$.

B		200				370.4			
n		50		100		50	100	50	100
P_0	m	LCL_τ	UCL_τ	LCL_τ	UCL_τ	LCL_τ	UCL_τ	LCL_τ	UCL_τ
0.01	25	0.00	4.14	0.00	5.56	0.00	4.56	0.00	6.07
	50	0.00	3.91	0.00	5.28	0.00	4.31	0.00	5.76
	75	0.00	3.80	0.00	5.14	0.00	4.18	0.00	5.59
	100	0.00	3.72	0.00	5.05	0.00	4.09	0.00	5.50
	125	0.00	3.67	0.00	4.99	0.00	4.04	0.00	5.43
	150	0.00	3.63	0.00	4.95	0.00	4.00	0.00	5.38
	200	0.00	3.58	0.00	4.89	0.00	3.94	0.00	5.31
	∞	0.00	3.23	0.00	4.48	0.00	3.56	0.00	4.87
0.02	25	0.00	5.52	0.00	7.80	0.00	5.98	0.00	8.38
	50	0.00	5.22	0.00	7.44	0.00	5.69	0.00	7.99
	75	0.00	5.08	0.00	7.27	0.00	5.53	0.00	7.82
	100	0.00	5.01	0.00	7.17	0.00	5.45	0.00	7.71
	125	0.00	4.95	0.00	7.10	0.00	5.38	0.00	7.63
	150	0.00	4.90	0.00	7.06	0.00	5.33	0.00	7.58
	200	0.00	4.85	0.00	6.98	0.00	5.27	0.00	7.50
	∞	0.00	4.45	0.00	6.51	0.00	4.83	0.00	6.97
0.05	25	0.00	8.61	0.00	13.12	0.00	9.21	0.00	13.87
	50	0.00	8.25	0.00	12.65	0.00	8.82	0.00	13.36
	75	0.00	8.09	0.00	12.43	0.00	8.65	0.00	13.14
	100	0.00	7.99	0.00	12.30	0.00	8.54	0.00	12.99
	125	0.00	7.92	0.00	12.21	0.00	8.46	0.00	12.88
	150	0.00	7.87	0.00	12.15	0.00	8.40	0.00	12.83
	200	0.00	7.80	0.00	12.06	0.00	8.32	0.00	12.72
	∞	0.00	7.31	0.00	11.46	0.00	7.80	0.00	12.07
0.10	25	0.00	12.75	1.61	21.35	0.00	13.46	1.04	22.25
	50	0.00	12.31	1.87	20.78	0.00	12.98	1.42	21.63
	75	0.00	12.12	1.97	20.52	0.00	12.78	1.54	21.35
	100	0.00	12.00	2.04	20.36	0.00	12.65	1.62	21.18
	125	0.00	11.91	2.09	20.27	0.00	12.57	1.66	21.07
	150	0.00	11.86	2.12	20.18	0.00	12.49	1.70	20.99
	200	0.00	11.77	2.18	20.08	0.00	12.40	1.74	20.86
	∞	0.00	11.22	2.50	19.34	0.00	11.80	2.07	20.07
0.20	25	1.87	20.38	7.97	34.25	1.35	21.16	7.27	35.34
	50	2.11	19.90	8.38	33.61	1.67	20.65	7.68	34.62
	75	2.22	19.67	8.57	33.31	1.78	20.40	7.87	34.28
	100	2.29	19.54	8.69	33.14	1.85	20.25	8.00	34.09
	125	2.33	19.44	8.77	33.01	1.90	20.16	8.08	33.96
	150	2.37	19.37	8.83	32.92	1.93	20.07	8.15	33.88
	200	2.42	19.27	8.91	32.79	1.98	19.98	8.23	33.72
	∞	2.75	18.63	9.46	31.92	2.31	19.29	8.80	32.80

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