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Dynamic public transportation system design based on school bus routing problem In a competitive environment

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Abstract

This paper presents the development of Dynamic school bus routing problem to design transport systems. In a real world the using private cars are rival for public transportation systems. So the shortcomings of these systems will lead to an increase in the demand for using private cars. This issue will lead to problems such as increased traffic and urban pollution. Hence, a new dynamic bi-level model is presented for designing such a system. In this model, the public transport system designer as a high-level decision maker is to locate appropriate bus stops and buses' route. Any decisions made at the upper level model will be followed by students' reaction at the lower level.
To solve this problem a hybrid SA-EX-TS approach has been proposed based on a location-allocation-routing (LAR) strategy. The results showed good performance of solution approach proposed in reasonable time.

Keywords:

Dynamic school bus routing problem, bi-level programming, demand outsourcing, meta-heuristic algorithm

Introduction

Nowadays, due to urban development, urban transportation has found great importance. In the big cities of the world, the reluctance of people to use public transport systems is one the main challenges and people prefer use of their private cars and rent private taxis companies instead of using public transportation systems. This reluctance of people to use public transportation system will cause a lot of problems including urban traffic, increase pollution and environmental damage. Therefore, in this paper is provided a dynamic bi-level mathematical model of school bus routing problem (DSBRP) to improve transportation program of students and employee in such systems. The proposed model is based on a leader-follower game theory and public transportation designers play the role of leader and students or employee play the role of follower.

School bus routing problem

School bus routing problem is one of the most widely used branches of Vehicle Routing Problem (VRP). This problem was first introduced by Newton and Thomas [19]. The main objective of this problem is to rout and create an efficient schedule for bus transportation, so that the bus takes the students in pre-marked stations and brings them to their schools. School bus routing problem (SBRP) can be categorized into five sub problems based on the division by Desrosiers, Soumis, Desrocher and Windows [6] including: data collection, location of bus stops, bus routing, and balancing the school start time and timing routs. Routing sub problems can be considered as the most important part of SBRP problem that most of the papers are such as Newton and Thomas [19], Li and Fu [12], Derbel, Jarboui, Hanafi and Chabchaub [5], Dulac, Ferland and Forgues [7], Schittekat, Kinable, Sorensen, Sevaux and Spieksma [23, 24] and Bodin and Berman [2] based on it. Also Rojas, Delgado, Giesen and Munoz [11] review the literature on Transit Network Planning problems and real-time control strategies suitable to bus transport systems in their paper.

Strategies of Location-allocation-routing (LAR) and allocation-routing-location (ARL) are two solving approach that is used in this type of papers. In LAR strategy, at first, a set of designated stations are determined and students are allocated to each station, then routing is performed for each bus station. Desrosiers et al. [6], Dulac et al. [7], Bodin and Berman [2] proposed an innovative approach based on the LAR strategy. In ARL strategy, students are classified based on the capacity of the vehicle. Then, bus stations and bus routes are determined for each category. Chapleau, Ferland

and Rousseau [4] and Bowerman, Hall and Calami [3] used an algorithm based on this strategy. Similarly, Schittekat, Kinable, Sorensen [24] developed a simple mathematical model for selecting an appropriate station between potential ones and routing the buses for a single school problem. Park and Kim [21] in their research classified papers in terms of the number of schools, normality of students and homogeneous or heterogeneous buses [13]. Also, in studies used different objective functions according to the terms of the issue. The used objective functions are: the number of buses [12, 4, 3, 20-1], the total distance or time spent by buses [12, 23, 24, 22, 1], the total time or distance of transporting the students [12], the distance walked by students [4, 3], the maximum length of the route [20] and the time lost [25]. Minimizing the number of buses is the most frequently used objective function in resources. In some papers, multi-objective states are evaluated. For example, Ripplinger [22], Bektas and Elmastas [1] have considered minimizing the number of buses and minimizing the total traveled bus distance.

Ellegood, Campbell and North [8] provides a general strategic analysis using continuous approximation models to assess the conditions under which mixed loading is likely to be beneficial. Also present a case study for a semi-rural Missouri school district to illustrate the application of the models in practice.

Application of bi-level programming routing problems

In the majority of existing studies on routing problem, decision space is examined from the perspective of only one decision-maker. While in the real world problems, there are different stakeholders in the transportation chain that may be exist conflict between them. Given this kind of attitude can be effective in increasing performance programming models. One of the practical approach in modeling such problems is bi-level programming. On these problems there are two level of the decision making. Each level controls part of decision variables in decision space.

In this structure each level has its own objective function. Furthermore, any objective function in each level has its own constraints. While similar constraints may also exist for the entire problem. Fisher and Jaikumar [9] modeled the VRP problem with the two levels of decision making. In this structure, the solution of a level depends on the solution of the other. In fact, it is assumed that the decisions of the second level are made based on the decisions of the first level. Thus, decisions at the first level can be made based on estimation from the results and output of the second level. This type of formulation is called bi-level programming [9]. The general form of bi-level problems can be shown as follows:

$$\min_{x \in X,Y} F(x,y)$$
$$\text{s.t.}\quad \begin{aligned} & G(x,y) \le 0 \\ & \min f(x,y) \\ & \quad \text{s.t.}\ \ g(x,y) \le 0 \end{aligned} \qquad (1)$$

Among papers in the field of vehicle routing which use bi-level programming we can point to Marinakis et al. [19]. Also, Marinakis and Marinaki [16] formulated a bi-level routing location- routing problem. Its upper level is related to strategic decisions such as facilities location. The lower level is related to tactical decisions related to routes which must be visited by customers. Ma and Xu [14], like the Marinakis [15], formulated VRP problem as a bi-level model. With the difference that their problem has been formed in a fuzzy space and some parameters of the problem such as the demand are uncertain.

There is a gap in number of buses in problems. Furthermore, most of the existing researches assumed that the number of buses is a fixed number. In order to fill these gaps, this paper contributes to the literature in the following ways:

- Developing a dynamic bilevel integer programming model to maximize the income and minimize total cost of students.
- Considering the budget to hire buses in each period.
- Considering the possibility of outsourcing for demands.
- Developing a meta-heuristic algorithms to solve the instances.

The rest of the paper is organized as follows: first problem definition and mathematical model are presented. Then the proposed solution methods is explained. Next section handles the computational experiments to show the efficiency of model.

Model Description

In this section we first describe the problem and then we present its mathematical model. The basic idea in model of this paper in locating bus stops in the school bus routing problem is to design a transportation network with the goal of maximizing the profit of transportation firm in a competitive space. This competitive space is modeled as a Stackelberg game such that in this structure a firm that offers transport services (public transport system) play the role of leader and students or employee play the role of follower. In this model, the public transport system designer as a high-level decision maker (ULP) is to locate appropriate bus stops and buses' route. Any decisions made at the upper level model will be followed by students' reaction at the lower level (LLP). The ULP objective function maximizes the profit. This function is dependent on the companies' costs and income and the students seek to minimize their costs in LLP. In fact, in LLP, according to the buses' capacity, the students are allocated to bus stations to minimize the costs. Thus, when there is no justification for the access to this system for a student, the student can select other transportation systems according to outsourcing feature. According to different available budgets in different periods of time presented model is dynamic. Also, depending on the amount available budget in each period the number of hired buses is determined (If a bus is hired in the one period it is also used in later periods). Attract more students equal to earning much income, but it is also need to activate more stations and longer routs. Therefore, it affects the service costs of the firm. According to the following assumptions, an integer mathematical programming model is suggested in this paper:

- Considered only one school, one kind of bus and also one kind of student.
- The constant amount of income per serving each student and an outsourcing constant cost caused by the lack of allocation of each student to a specific station.
- Two-way paths or in other words we have an undirected graph.

Index and Sets

V	Set of potential stations, $i, j \in V$,
T	Set of time periods, $t \in T$,
K	Set of bus transportation, $k \in K$,
S	Set of students, $l \in S$.

Parameters

CB_k	Capacity of k^{th} bus
d_{il}	The distance of l^{th} student from the i^{th} potential station (in the case of lack of access, the Big-M distance is considered)
S_{il}	If l^{th} student has the access to the i^{th} potential station, it will be equal to 1 and otherwise 0
C_{ijt}	The cost of the travel along the arc ij in period t
CI_t	The income obtained from providing service for each student in period t
CW_t	Costs (fines) of students' walk per unit of length in period t
CO_t	Costs of outsourcing or fines for lack of service to each student in period t
$i = 0$	Index for the school
f_{kt}	The present value of the fixed cost of hiring the k^{th} bus in period t
r_{kt}	The present value of the current cost of hiring the k^{th} bus in period t
B_t	The present value of the budget in period t for hire buses

Decision variables

x_{ijkt}	1 if bus k traverses the arc from i to j in period t, 0 otherwise
y_{ikt}	1 if bus k visits stop i in period t, 0 otherwise
z_{ilkt}	1 if student l is picked up by bus k at stop i in period t, 0 otherwise
φ_{kt}	1 if bus k is hired in period t, 0 otherwise
φ'_{kt}	1 if bus k is hired in period t or previous periods, 0 otherwise

The Dynamic bi-level school bus routing problem (DBSBRP)

$$Max_{x,y,\varphi} F(x,y,z,\varphi) =$$

$$\sum_{t\in T} CI_t \times \sum_{i\in V}\sum_{l\in S}\sum_{k=1}^{n} z_{ilkt} - \sum_{t\in T}\sum_{i\in V}\sum_{j\in V} C_{ij} \sum_{k=1}^{n} x_{ijkt} \quad (1)$$

$$-\sum_{t\in T}\sum_{k\in K} f_{kt}\varphi_{kt} - \sum_{t\in T}\sum_{k\in K} r_{kt}\varphi'_{kt}$$

$$\sum_{j\in V} x_{ijkt} = \sum_{j\in V} x_{jikt} = y_{ikt} \quad ;\forall i \in V, t\in T, k\in K \quad (2)$$

$$\sum_{i,j\in Q} x_{ijkt} \le |Q| - 1 \quad ;\forall Q \in V/\{v_0\}, t\in T, k\in K \quad (3)$$

$$\sum_{k=1}^{n} y_{ikt} \le 1 \quad ;\forall i \in V/\{0\} \quad (4)$$

$$\sum_{t\in T}\sum_{k\in K} f_{kt}\varphi_{kt} + \sum_{t\in T}\sum_{k\in K} r_{kt}\varphi'_{kt} \le B_t \quad (5)$$

$$y_{ikt} \le \varphi'_{kt} \quad ;\forall i \in V, t\in T, k\in K \quad (6)$$

$$\sum_{t\in T} \varphi_{kt} \le 1 \quad ;\forall k = 1,2,\ldots,n \quad (7)$$

$$\varphi_{kt} \le \varphi'_{kt} \quad ;\forall k = 1,2,\ldots,n \quad (8)$$

$$\varphi'_{kt} \le \varphi'_{k,t+1} \quad ;\forall t\in T, k = 1,2,\ldots,n \quad (9)$$

$$x_{ijkt} \in \{0,1\} \quad ;\forall i,j \in V, i\ne j, k = 1,\ldots,n \quad (10)$$

$$y_{ikt} \in \{0,1\} \quad ;\forall i \in V, k = 1,\ldots,n \quad (11)$$

$$\varphi_{kt} \in \{0,1\} \quad ;\forall t\in T, k = 1,2,\ldots,n \quad (12)$$

$$\varphi'_{kt} \in \{0,1\} \quad ;\forall t\in T, k = 1,2,\ldots,n \quad (13)$$

$$f(x,y,z,\varphi) =$$

$$Min_z (CW_t \times \sum_{i\in V}\sum_{l\in S} d_{il} \sum_{k=1}^{n} z_{ilkt} + CO_t \times \sum_{l\in S}(1 - \sum_{i\in V}\sum_{k=1}^{n} z_{ilkt})) \quad \forall t\in T \quad (14)$$

$$\sum_{i\in V}\sum_{k=1}^{n} z_{ilkt} \le 1 \quad ;\forall l \in S, t\in T \quad (15)$$

$$\sum_{i\in V}\sum_{l\in S} z_{ilkt} \leq CB_k \quad ;\forall t \in T, k-1,2,...,n \tag{16}$$

$$\sum_{k=1}^{n} z_{ilkt} \leq S_{il} \quad ;\forall i \in V, l \in S, t \in T \tag{17}$$

$$z_{ilkt} \leq y_{ikt} \quad ;\forall i \in V, l \in S, t \in T, k = 1,2,...,n \tag{18}$$

$$z_{ilkt} \in \{0,1\} \quad ;\forall i \in V, l \in S, t \in T, k = 1,2,...,n \tag{19}$$

Expressions 1-13 show upper level of the model and expressions 13-19 indicate low level of the model. The objective of the upper level of the model (expression 1) is maximizing profit of the firm. Actually the ULP objective function is formed by two parts. The first part includes the sum of the incomes obtained from providing service for students and the second part includes a set of costs resulting from the travel of buses in route and the fixed and current cost of hiring buses in different periods. Constraint (2) ensures flow of buses from activated stations. So that if a bus station is considered for a special bus, then once the bus enter into station and once out of it. And if the station is not activated the buses won't be allowed to pass it. In addition, constraints (3) causes that each station is visited at most once. Of course, the school station is an exception. So each station is not active or if activated only is visited by one bus [6]. The constraint (4) is considered to fix the grating problem. Constraint (5) relating to restrictions on the hiring of buses due to budget each period. Constraint (6) guarantee that if the bus passes through station i in period t, should be hired in that period or previous periods. Constraint (8) and (9) guarantee that the hired buses in one period not remove in next periods. Equations 9-12 indicate that the firm uses the binary variables in order to make decision for selecting the bus stations and routing the buses.

The second level objective function (equation 13) seeks to minimize the students' costs. These costs consist of two parts. The first part is related to the students who use the transportation system firm and includes the costs of reaching the nearest active station with capacity. Otherwise, if it does not justify the use of this system for students, student transportation is outsourced. Costs of outsourcing calculated in second part of objective function. Constraint (14) ensures that the number of students allocated to each bus does not exceed its capacity. Constraint (15) show that each student would be allocated a maximum of one station. If a student is allocated to one of the stations ($\sum_{i\in V}\sum_{k\in K} z_{ilkt} = 1$), the cost of it is calculated through the first part of the objective function and otherwise ($\sum_{i\in V}\sum_{k\in K} z_{ilkt} = 0$), it costs is calculated through the second part of the objective function. Constraint (17) show students access to the station. In the other words, this constraint lets each student make decision about available bus stations. Constraint (17) shows the level of students' access to the stations. Constraint (18) ensures that, if the k^{th} bus does not pass the i^{th} station ($y_{ikt} = 0$), then no student will get into the bus in that station. In fact, these decision constraints, harmonize the LLP optimization problem with ULP decisions. Finally, equation (19) indicates that, all lower level decision making variables are binary.

Solution of DBSBRP model

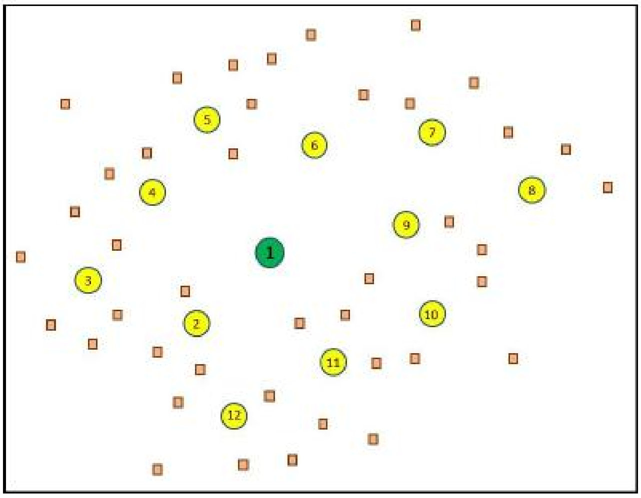
In this paper is proposed a dynamic bi-level problem to locate stations and school bus routing. Generally Bi-level programming problems are considered NP-hard problems [3]. On the other hand, problems solving related to routing problems and the use of integer variables intensifies the complexity of the proposed model. Therefore, to solve the model, we have suggested a three-stage hybrid approach on location-allocation-routing (LAR) strategy. This approach have been described in Table 1.

Table 2- Described steps to problem solving approach DSBRP

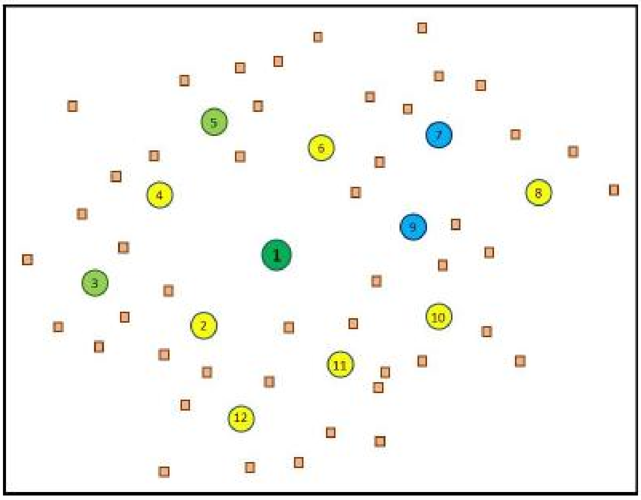
	Decision Type	Decision Variables	Decision level	outputs
Step 1	Location	y_{ikt}	ULP	Determine active bus station related to each bus
Step 2	Allocation	z_{ilkt}	LLP	Predict student's reaction against activated stations and determine number of students who will use the system.
Step 3	Routing	x_{ijkt}	ULP	Bus stations routing assigned to them (solve the Traveling Salesman Problem (TSP) for each bus).

For instance, consider the problem illustrated in Figure 1. This problem has a Depot (the station related to school) and 11 potential places to determine the bus stations. Assume that $\bar{y}_{ikt}$ is a suggested solution from the firm for the location of bus stations in period one ($t = 1$). In this step, assume that stations 3 and 5 are activated for the first bus (k=1) and stations 7, 9 are activated for the second bus (k=2). Then, the reaction of students towards this decision will be estimated by using the values of the y variable as a parameter to the low level. In final step, for activated stations of each bus, we are dealing with a Traveling Salesman problem in this step. This profit (the value of first level objective function) can be considered as a measure for determining the proposed solution fitness of $\overline{y}_{ikt}$.

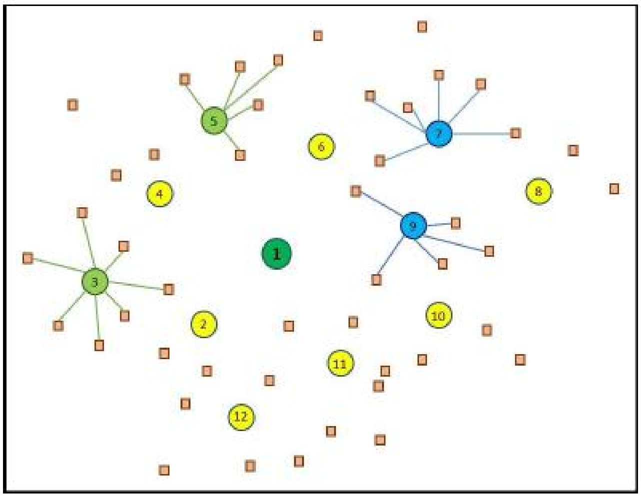
We need to note that in first period due to budget constraints for hiring buses only 2 buses have been hired. Then in the second period due to increase in income the number of hired buses increased to 3 (stations 11, 12 are activated for the third bus (k=3).) and demand of more students have been answered. Also, the red squares indicate the students who have been outsourced.



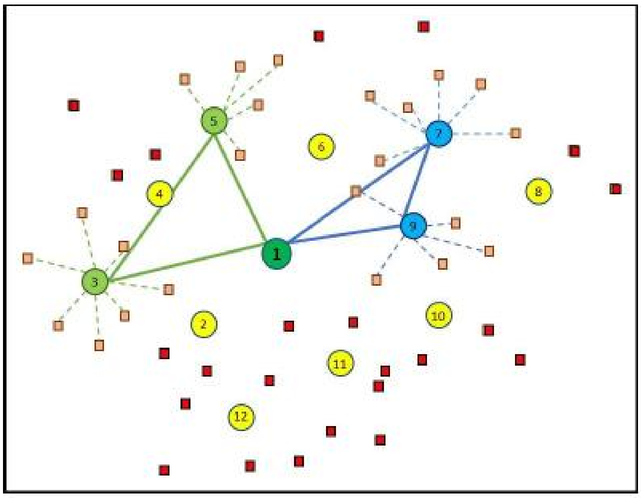
(a) An instance with 12 potential bus stops, 41 students and 2 buses.



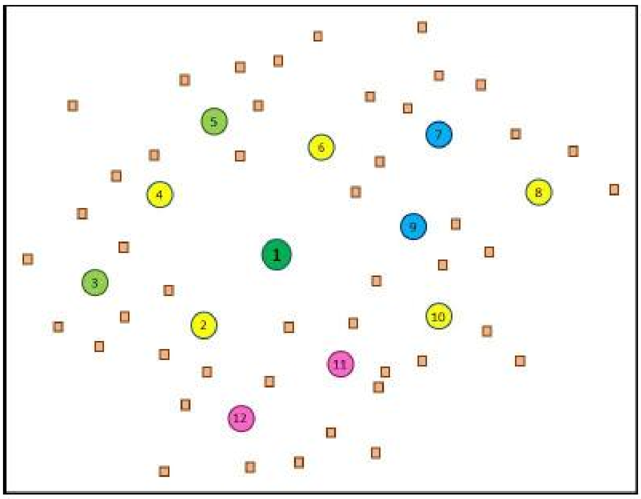
(b) Step 1: A proposed solution ($\overline{y}_{ikt}$) by the firm in period 1.



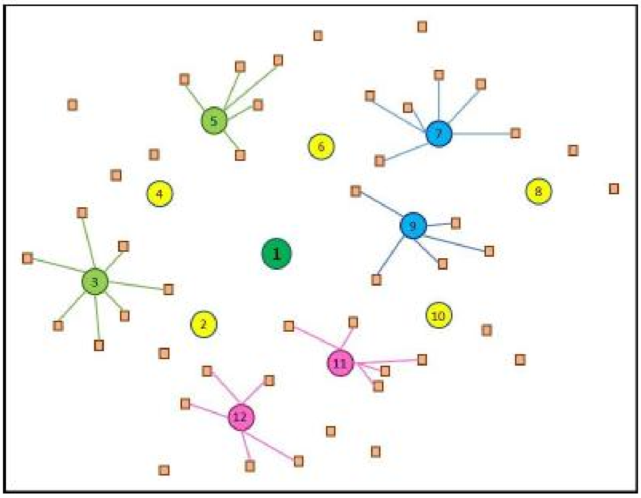
(c) Step 2: Assigning the students to bus stops (z^{*}_{ilkt}) by the students in period 1.



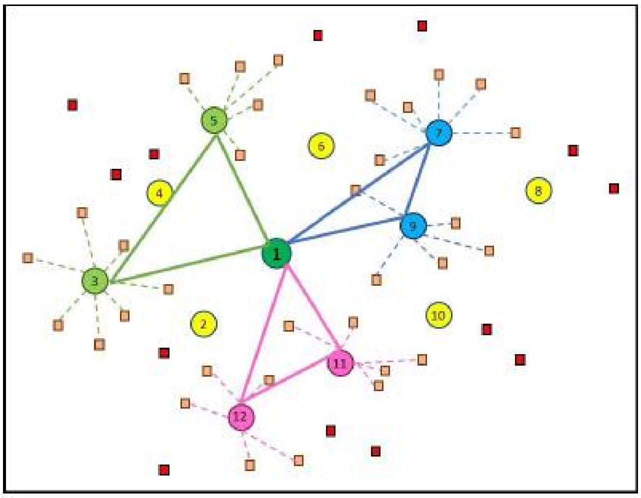
(d) Step 3: Routing each bus (x^{*}_{ijkt}) by the firm



(e) Step 4: A proposed solution ($\overline{y}_{ikt}$) by the firm in period 2.



(f) Step 5: Assigning the students to bus stops (z^*_{ilkt}) by the students in period 2.



(g) Step 6: Routing each bus (x^*_{ijkt}) by the firm in period 2.

Figure 3. The steps of the proposed solving approach through an instance

SA-EX-TS method

In this paper, a methods is presented based on the proposed approach for solving the DBSBRP model. In this method for step 1, we will use simulated annealing because of their suitable performance in similar problems to solve the bus stops selection problem. In step 3, for each bus we are dealing with a TSP. To search the solution's space in the Tabu search algorithm will be used. Table (2) shows the general characteristics of these proposed approaches.

Table 2. Suggested methods to solve the DSBRP model based on LAR strategy.

Object	Decision's type	Decision Variables	SA-EX-TS
Step 1	Location	y_{ikt}	Simulated Annealing
Step 2	Allocation	z_{ilkt}	Exact Solution
Step 3	Routing	x_{ijkt}	Tabu-Search

Simulated Annealing (SA) is a computational stochastic meta-heuristic which is used effectively to search the solutions close to global optimization. The basic idea of the SA was introduced by Metropolis and Rosenbluth [17]. Later, Kirkpatrick, Gelatt and Vecchi [10] used the simulated annealing as an optimizing technique. SA starts the search in the search space from an initial solution, considering this point as the current solution, and searches a neighboring solution. Then the quality of the neighboring solution will be compared with the current solution. If the quality of the neighboring solution is higher than the current solution, the current solution will be replaced with the neighboring one. Otherwise, the neighboring solution will be accepted with an acceptance probability (Boltzmann distribution). This acceptance probability is dependent on the temperature and of quality difference between the neighboring solution and the current one. That is, in higher temperatures, the probability of accepting the bad solutions as the current solutions is higher than in lower temperatures. In fact, escaping from local optimized solutions by accepting worse solutions is one of the main ideas of SA. The SA temperature decreases periodically and based on a cooling schedule scheme. Ultimately, the algorithm will stop when it reaches a predetermined temperature [14].

Computational experiment

In this section, the computational results to evaluate the model is presented. We tested performance of the solution algorithm on a number of instance, which have randomly generated. The proposed hybrid algorithm was coding on software in MATLAB 2012. For the solution of third level of the model, for each response generated in the second level GAMS software was called. Codes on computers with processor Intel (R) Core (TM) i5-3230M @ 2.60 GHz and an internal memory of4/00 GB and Windows 10 operating system was running.

Problem instance generation

Random generation of the problem's parameters was done based on the approach applied in Schittekat et al. [23]. This approach presented in table (3). Then 15 random examples were created for 3,4,5,6 and 10 buses in 4 periods.

Table 3. Generating the instance problems for the proposed model

Parameters	Value
Solution space	$(1500) \times (1500)$
m	$\{3,4,5,6,10\}$

$\lvert k \rvert$	m
$\lvert s \rvert$	i
$\lvert V \rvert$	$round(m \times CB/2)$
$\lvert E \rvert$	$\frac{\lvert V \rvert \times (\lvert V \rvert - 1)}{2}$
CB	20
d_{il}	$\left[(x_l - x_i)^2 + (y_l - y_i)^2\right]^{\frac{1}{2}}$
S_{il}	If $d_{il} \prec 800$, S_{il} equal 1 else S_{il} equal 0
C_{ij}	$\left(20 \times \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{\frac{1}{2}}\right)$
CI	$\{6000, 6250, \ldots, 7000\}$
CW	5
CO	6000

Computational results

After generating 15 random instances in different sizes, the SA-EX-TS method are used to solve samples. The objective function value of the first level, the optimal solutions variables and CPU time, for each of instances, depicted in Table 4.

Table 4. Computational results by SA-EX-TS

Inst 1	Max No of buses	No of student	No of potential location
	3	54	31
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	3	217000	658
2	3	231360	617
3	3	242477	630
4	3	248790	635
Inst 2	Max No of buses	No of student	No of potential location
	3	58	31
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	2	226760	907
2	3	231360	403
3	3	282400	833
4	3	258800	1585
Inst 3	Max No of buses	No of student	No of potential location
	3	54	31
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	3	215630	636
2	3	241630	642
3	3	266140	615
4	3	298000	657
Inst 4	Max No of buses	No of student	No of potential location
	4	70	41
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	3	272260	2968
2	4	289790	3128
3	4	313070	3073
4	4	333110	3210
Inst 5	Max No of buses	No of student	No of potential location
	4	64	41
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	4	311830	1169
2	4	334820	1135
3	4	360830	1132
4	4	385680	1156
Inst 6	Max No of buses	No of student	No of potential location
	4	77	41
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	4	271910	988
2	4	293040	1042
3	4	314210	1079
4	4	339490	1136
Inst 7	Max No of buses	No of student	No of potential location
	5	93	51
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	4	386400	1589
2	5	409950	1585
3	5	434510	1520
4	5	456960	1533
Inst 8	Max No of buses	No of student	No of potential location
	5	5	91
No of	No of	SA-EX-TS	

period	hired buses	z_1^*	CPU time (sec)
1	5	407860	1553
2	5	438610	1465
3	5	470190	1435
4	5	498300	1552
Inst 9	Max No of buses	No of student	No of potential location
	5	87	51
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	5	422760	1397
2	5	450240	1431
3	5	484020	1453
4	5	513840	1483
Inst 10	Max No of buses	No of student	No of potential location
	6	97	60
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	4	426570	2015
2	5	444870	2121
3	5	468090	2113
4	5	511410	2098
Inst 11	Max No of buses	No of student	No of potential location
	6	97	60
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	5	520640	1821
2	5	559130	1834
3	6	570400	1849
4	6	615470	1859
Inst 12	Max No of buses	No of student	No of potential location
	6	102	60
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	5	425730	2251
2	5	452530	2217
3	5	483040	2266
4	5	517890	2209
Inst 13	Max No of buses	No of student	No of potential location
	10	165	100
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	8	885600	3945
2	9	935800	3970
3	9	1006100	3951
4	10	10052800	3917
Inst 14	Max No of buses	No of student	No of potential location
	10	192	100
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	9	901400	3979
2	9	960300	4045
3	10	1004400	4044
4	10	1072800	4053
Inst 15	Max No of buses	No of student	No of potential location
	10	165	100
No of period	No of hired buses	SA-EX-TS	
		z_1^*	CPU time (sec)
1	8	757630	3453
2	8	808490	3463
3	9	848380	3446
4	9	900400	3442

Conclusion

The school bus routing problem plays an important role in urban transportation systems. In this paper, we have presented a development of this problem to design transportation systems, with the outsourcing demand possibility. The mathematical model presented in this paper has formulated such a condition under a dynamic bi-level problem. In this model, the public transport system designer as a high-level decision maker is to locate appropriate bus stops and buses' route. Any decisions made at the upper level model will be followed by students' reaction at the lower level. To solve this problem a hybrid SA-EX-TS approach has been proposed based on a location-allocation-routing (LAR) strategy. The results showed good performance of solution approach proposed in reasonable time.
Since the presented mathematical model is an initial model, it can be developed in future researches. Given multiple schools and students with different characteristics and transportation system development under the optimal strategies, are suggested for upper application of these models. Moreover, other innovative and meta-heuristic methods and comparing their results with the proposed approaches can be presented.

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