Mazandaran University of Science and Technology

Available online at www.iiec2017.com

13th International Conference on Industrial Engineering

(IIEC 2017)

Iran Institute of Industrial Engineering

Effect of a linear price-sensitive demand curve on the multi-echelon supply chain's efficiency

Milad Darzi Ramandi [a], Hamid Mashreghi[b] and Seyed Zeinab Aliahmadi[c]

[a] Department of Industrial Engineering,
Science and Culture University, Tehran, Iran
Tel: +989127750328, E-mail: milad.darzi.ramandi@gmail.com

[b] Corresponding Author: Department of Industrial Engineering,
Babol Noshirvani University of Technology, Shariati Ave., Babol, Iran
Post Code: 47148-71167, Tel: +989112100647, E-mail: Mashreghi.h@nit.ac.ir

[c] Department of Industrial Engineering,
Iran University of Science and Technology, Tehran, IRAN
Tel: +989117734423, E-mail: zeinabaliahmadi9@gmail.com

Abstract

In multi-echelon supply chains, the product made by manufacturer with initial production costs. Then to reach the end consumer, it will pass from different levels of the supply chain. Each member of the chain is in mutual communication with upstream and downstream. Therefore, in order to achieve the best quality and the lowest production cost, a large number of companies has concurrent collaboration with many suppliers. In this paper, we consider a wholesale-price contract where demand of the product is dependent to selling price with linear demand curve. We obtain the amount of optimal profit for each member of two-echelon and three-echelon supply chain and consequently determine the optimal ordering pricing decisions for partners. Finally, it is proved that by increasing the number of echelons under wholesale price contract, the channel's efficiency decreases dramatically.

Keywords:
Supply chain optimization; N-echelon supply chain; Unconstraint optimization; Wholesale price contract; Pricing

1. Introduction

Supply chain partners including upstream and downstream members as suppliers and distributers shape a flow of material and information through the chain. A supply chain consists of a supply side, a production side, and a customer side, which create a "multi-level" environment. Each level has more than one site, therefore it becomes a "multi-site" environment [2].

In an inventory model with ordinary pricing, small changes in the demand curve would lead to large changes at the optimal solutions. In general, by increasing price, demand will decrease. Thus the analysis of supply chain can be developed in under different price-sensitive demands such as linear, exponential, or reverse linear functions [1]. In this paper we focus on linear demand function which is defined as follows:

$$D = a - bp \qquad (1)$$

where a and b are positive constants and p is the selling price of the final product. However the main issue is the impact of demand shape on supply chain efficiency at different levels.

Moreover, the interaction of the supply chain's members must be considered as the basic optimization approach during a multi-level supply chain. Thus we can see that the complexity of the supply chain is not a simple linear structure and often a small change will lead to a chain reaction. For this purpose, a variety of contractual exchanges will be assumed which wholesale price contract is one of the most basic types of contracts to analyze the effect of supply chain members on the chain efficiency [3].

Under contractual assumptions, supply chain coordination with contracts is to eliminate the inefficiencies in the supply chain and align the objectives of supply chain members, which the supplier and buyer reach an agreed in price, order quantity etc. [4]. In this paper, we focus on wholesale price contract, that due to simplicity of its implementation, it is

very popular in chain agreements [4; 5].
Pen et al (2006) consider a supply chain network with two manufacturers and a retailer and showed that each manufacturer can chooses a wholesale price contract or a revenue sharing contract and extracted the results for manufacturer. They also check the impact of demand for the two-level supply chain with one manufacturer and two retailers [6]. Seifert et al (2012) examined a three-level supply chain and showed the impact of the sub-chain coordination with wholesale price contracts or Price-only contract [7]. Chen and Li (2007) analyzed wholesale price contract in a three-stage supply chain. Their model allows the retailer and the distributor that maximize their profits. They showed that when other members of the supply chain have bargaining power, the optimal joint sales is hard to achieve [5]. Iida (2012) studied on reducing participation costs in a decentralized supply chain with one manufacturer and several suppliers. All above studies has been shown that both manufacturers and suppliers are working and investing in order to reduce production costs [8]. Rhee et al. (2010) shows that a n-echelon supply chain can be coordinated if the contractual agreements limited to some specific types of revenue sharing contracts [9]. Yeganehfallah et al (2014) developed different types of revenue sharing contracts including spanning and pair-wise to analyze the possibility of coordination in a telecom supply chain [10].
Therefore, the most studies have focused on analyzing the behavior of two or three-level chain and analyzing the effect of increasing levels of chain. In the case of n-echelon for different demands more research is needed. In this paper we have tried with consider the basic form of supply chain member's communication through wholesale price contracts, investigated changing supply chain efficiency by increasing levels of the supply chain in linear demand.

2. The two-echelon supply chain basic model with linear demand

The components of model and its related parameters are shown in Table 1.

Table 1. Parameters and Indices

Parameters	Variables coefficients	Second subscript	Description
D	Deterministic demand	M	Manufacturer
a , b	Linear demand constants	W	Wholesaler
Q	Retailer's order quantity	R	Retailer
P	Retailer's selling price	C	Integrated supply chain channel
S	Suppliers for n-echelon case	θ	Deterministic profit

Analysis the two-echelon supply chain in two case of integrated and isolated are presented as follows.

2.1. The integrated two-echelon supply chain

There are two levels in this supply chain which in the second level, manufacturer is doing the wholesale and retail operations alone. At first, manufacturer built their goods at cost m, and by adding his profit on it, the goods offers to the customer with price p. Scheme of integrated two echelon supply chain is shown in Figure 1.

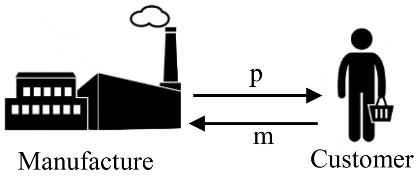

Figure 1. Integrated two-echelon supply chain

The first optimality condition for the retailer's profit function shows that:

$$\theta_{LI} = Q(\mathrm{p-m}) = (\mathrm{a-bp})(\mathrm{p-m}) \rightarrow \text{if } \frac{d\theta_{LI}}{dp} = 0$$

$$p^* = \frac{(\frac{a}{b} + \mathrm{m})}{2} \qquad (2)$$

To ensure that maximum value is obtained, we should check the second optimality condition which indicates that:

$$\frac{d^2\theta_{LI}}{dp^2} = -2b < 0$$

Thus the optimal ordering decision is as follows:

$$Q^* = a - bp^* = a - b\,(\frac{\frac{a}{b} + m}{2}) \rightarrow$$

$$Q_1^* = \frac{(\mathrm{a-bm})}{2} \qquad (3)$$

To obtain optimal chain's profit, p^* and Q^* be put in the profit function:

$$\theta^* = Q^*(\mathrm{p}^* - \mathrm{m}) \rightarrow$$

$$\theta^* = \frac{(\mathrm{a-bm})^2}{4b} \qquad (4)$$

2.2. The isolated two-echelon supply chain

In this case, manufacturer only do the wholesale's operation and the retailer enters into the supply chain as a distribution entity.

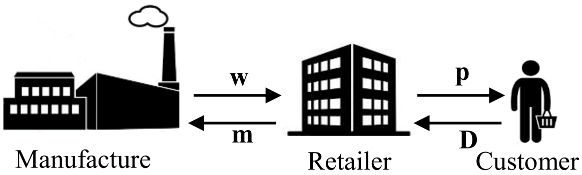

Figure 2. Scheme of isolated two-echelon supply chain

In this supply chain at first, retailer forecast product demand and presents its order to the manufacturer, and manufacturer makes the product with cost of m, and sells its product with price w and finally, with considering retailer's profit on product, this product arrives to consumer with price p.

To calculate the optimal value of w, derived function of the manufacturer's profit from w, as follow:

$$\theta_{LM} = Q_1(\mathrm{w}-\mathrm{m}) \rightarrow \frac{d\,\theta_{LM}}{dw} = 0$$

$$w^* = \frac{(\frac{a}{b}+\mathrm{m})}{2} \qquad (5)$$

It should be noted that in above formulation, we use Q_1^* to calculate the optimal value of w in the function of the manufacturer's profit that been put w instead of m.

The optimal pricing is optimizing by retailer's profit function as follow:

$$p^* = \frac{(\frac{3a}{b}+\mathrm{m})}{4} \qquad (6)$$

And amount of ordering and the expected profit of retailer are then given by:

$$Q_2^* = \frac{(a-bm)}{4} \qquad (7)$$

$$\theta_{LR}^* = Q^*(\mathrm{p}^*-\mathrm{w}) \rightarrow$$

$$\theta_{LR}^* = \frac{(\mathrm{a}-bm)^2}{16b} \qquad (8)$$

In addition economic profit manufacturer is obtained by substituting the optimal values of Q and w, as follows:

$$\theta_{LM}^* = Q^*(\mathrm{w}^*-\mathrm{m}) \rightarrow$$

$$\theta_{LM}^* = \frac{(\mathrm{a}-\mathrm{bm})^2}{8b} \qquad (9)$$

By achieving retailer and manufacturer economic profit, the channel efficiency is obtained as follows:

$$CE_2 = \frac{\theta_{\text{Realchain}}}{\theta_{Idealchain}} = \frac{\theta_{1(\mathrm{p}^*,\mathrm{Q}^*)} + \theta_{2(\mathrm{w}^*,\mathrm{Q}^*)}}{\theta_{(\mathrm{p}^*,\mathrm{Q}^*)}} \rightarrow$$

$$CE_2 = \frac{3}{4} = 0.75 \qquad (10)$$

3. The Three-echelon supply chain modeling

In the three-echelon supply chain, the wholesaler exist as an independent member and W is manufacturer Unit Price to wholesaler, w is wholesale price for retailer and p is customer price. Scheme of Three-echelon supply chain is shown in figure 3.

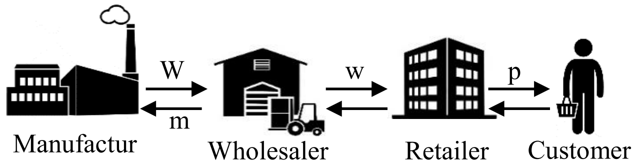

Figure 3. Three-echelon supply chain scheme

In this supply chain manufacturer offers the products to the wholesaler with price W, therefore the manufacturer's profit is as follows:

$$\theta_{LM} = Q_2^*(\mathrm{W}-m) = \frac{a-bm}{4}(\mathrm{W}-\mathrm{m}) \rightarrow$$

Hence by considering the derivative of the function than W can be said:

$$W^* = \frac{(\frac{a}{b}+\mathrm{m})}{2} \qquad (11)$$

Since wholesale received product from manufacturer as price W and sells it to retailer as price w, wholesaler's Profit function is obtained as follows:

$$\theta_{Lw} = Q_1^*(\mathrm{w}-\mathrm{W}) = \frac{(\mathrm{a}-\mathrm{bm})}{2}(\mathrm{w}-\mathrm{W}) \rightarrow \frac{d\,\theta_{Lw}}{dw} = 0$$

$$w^* = \frac{(\frac{3a}{b}+\mathrm{m})}{4} \qquad (12)$$

Similarly, the optimal pricing and amount of ordering are as follows:

$$p^* = \frac{(\frac{7a}{b}+\mathrm{m})}{8} \qquad (13)$$

$$Q_3^* = \frac{(a-bm)}{8} \qquad (14)$$

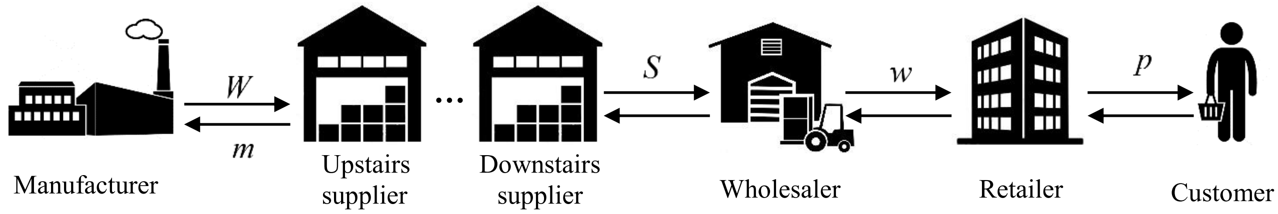

Figure 4. N-echelon supply chain scheme

Hence the economic profit of chain components in order for the retailer, wholesaler and manufacturer obtained as follows:

$$\theta_{LR}^{*}=Q_3^{*}(\mathrm{p}^{*}-\mathrm{w}^{*})=\frac{(\mathrm{a}-\mathrm{bm})^2}{64b} \quad (15)$$

$$\theta_{LW}^{*}=Q_3^{*}(\mathrm{w}^{*}-W^{*})=\frac{(\mathrm{a}-\mathrm{bm})^2}{32b} \quad (16)$$

$$\theta_{LM}^{*}=Q_3^{*}(\mathrm{W}^{*}-m)=\frac{(\mathrm{a}-\mathrm{bm})^2}{16b} \quad (17)$$

Hence the channel efficiency of three-level supply chain calculated as follows:

$$CE_3=\frac{\theta_{\text{Realchain}}}{\theta_{Idealchain}}=\frac{\theta_{1(\mathrm{p}^*,\mathrm{Q}^*)}+\theta_{2(\mathrm{w}^*,\mathrm{Q}^*)}+\theta_{3(\mathrm{W}^*,\mathrm{Q}^*)}}{\theta_{(\mathrm{p}^*,\mathrm{Q}^*)}}\rightarrow$$

$$CE_3=\frac{7}{16}=0.4375 \quad (18)$$

4. The n-echelon supply chain model

The n-echelon model framework shown in Figure. 4. In continue, the optimal pricing, the optimal amount of ordering and the optimal profit functions of the chain's members will be proven, then the optimal n-echelon supply chain channel efficiency is obtained by them.

4.1. Optimal supply chain pricing

Theorem 1 checked optimal decisions of n-echelon supply chain components that is shown below.

Theorem 1. Optimal Pricing of supply chain components in a n-echelon supply chain for $n=K$ from the downstream point (retailer), to the Middle levels (supplier S) and at highest point in chain (manufacturer) will be as follows $(\mathrm{K}\geq 2)$:

$$p_K^{*}=\frac{(2^K-1)\frac{a}{b}+m}{2^K} \quad (19)$$

$$w_K^{*}=\frac{(2^{K-1}-1)\frac{a}{b}+m}{2^{K-1}} \quad (20)$$

$$S_K^{*}=\frac{(2^{K-3}-1)\frac{a}{b}+m}{2^{K-3}} \quad (21)$$

$$W_K^{*}=\frac{(2^{K-2}-1)\frac{a}{b}+m}{2^{K-2}} \quad (22)$$

Proof. By considering n = 2 as first assumption of induction the results of the integrated and isolated two echelon supply chain in the second part, will lead to equation (19) and (20). On the other hand, if $n=K-1$ Then can be achieved equation (21) for a supply chain with $K-1$ echelon, such that the final component manufacturer with middle levels will have communication similar three-echelon supply chain (part III) . Finally, it can be seen by taking $n=K$ that if one other component to be added to chain and it considered in the middle level can developed the results of the equation $n=K-1$ for this case. Thus equations (19) to (22) is attainable to induction. ■

4.2. Optimal n-echelon supply chain ordering

Proposition 1, checked the optimal ordering decision of n-echelon supply chain as follows:

Proposition 1. The economic optimal of order quantity for n-echelon supply chain with $n=K$ will be as follows $(\mathrm{K}\geq 2)$:

$$Q_K^{*}=\frac{(\mathrm{a}-\mathrm{bm})}{2^K} \quad (23)$$

Proof. With $n=2$ as a first assumption of induction, the results of analysis the integrated and isolated two echelon model be extractable. If $Q_{K-1}^{*}=\frac{(\mathrm{a}-\mathrm{bm})}{2^{K-1}}$, Then for $n=K-1$ as the second assumption of inductive can be seen that based on the amounts of optimal pricing obtained from theorem1, equation (23) for $n=K$ is provable■

4.3. Optimal n-echelon supply chain profit

Theorem 2, specifies the profit situation of n-echelon supply chain components that is shown below.

Theorem 2. The amount of n-echelon supply chain components profit for the component k by considering $n = K$ is as follows $(K \geq 2)$:

$$\theta_R^* = \frac{(a-bm)^2}{2^{2K}} \quad (24)$$

$$\theta_w^* = \frac{(a-bm)^2}{2^{2K-1}} \quad (25)$$

$$\theta_S^* = \frac{(a-bm)^2}{2^{2K-2}} \quad (26)$$

$$\theta_M^* = \frac{(a-bm)^2}{2^{2K-3}} \quad (27)$$

$$\theta_k^* = \frac{(a-bm)^2}{2^{2K-(k-1)}} \quad (28)$$

Proof. By taking amount of optimal pricing each component of Theorem 1 and amount of the chain optimal ordering proposition 1, results for the supply chain in different modes $n = K$ is provable. ■

5. Efficiency of n-echelon supply chain

Theorem 3, checked efficiency of n-echelon supply chain that is shown below.

Theorem 3. The amount of supply chain efficiency for $n = K$ is as follows $(K \geq 2)$:

$$CE_K = \frac{2^K - 1}{2^{2(K-1)}} \quad (29)$$

Proof. By using the equation (24) to (28) for a supply chain with $n = K$, total profit of isolated n-echelon supply chain members as follows:

$$\sum_{i=1}^{K} \theta_i^* = [2^0 + 2^1 + ... + 2^{K-1}] \frac{(a-bm)^2}{2^{2K} b}$$

The other hand ideal profit of integration chain is obtained from the equation (4) by $\theta^* = \frac{(a-bm)^2}{4b}$, so we will have:

$$CE_k = \frac{\sum_{i=1}^{K} \theta_i^*}{\frac{(a-bm)^2}{4b}} = \frac{[2^0 + 2^1 + ... + 2^{K-1}] \frac{(a-bm)^2}{2^{2K} b}}{\frac{(a-bm)^2}{2^2 b}} \rightarrow$$

$$CE_K = \frac{2^K - 1}{2^{2(K-1)}}$$

6. Conclusion

The commodity prices for the end customers depends on several factors in reality. On the main influencing factors is the number of the supply chain echelons. Hence investigating these relations is the main issue of n-echelon supply chain modeling. On the other hand, the type of market demand has a significant impact on the chain's efficiency.
In this paper, by analyzing the linear demand function in chain is shown that if a wholesale pricing structure can be used, the supply chain efficiency will be reduced. Regarding the results of the paper if the number of chain levels is more than 5 layers, supply chain efficiency receives around less than 1% in comparison with a coordinated channel by 100% efficiency (Figure. 5).

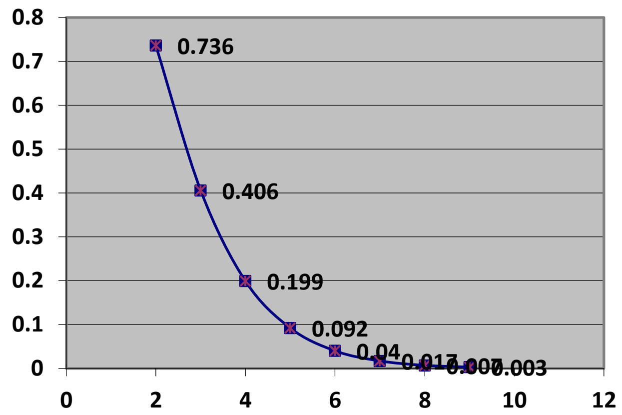

Figure 5. Supply chain efficiency for a n-echelon supply chain

Hence in addition to the amount of price, the performance also will strongly decrease. Therefore, to improve the situation of efficiency, wholesale price mechanism should be changed and profits of supply chain components to be distributed in the better form. Analyzing such enabled contractual mechanisms by revenue sharing or other coordinating contracts will be discussed in future research [11].

7. References

[1] Lau, A. H. L. and Lau H. S., "Effects of a demand-curve's shape on the optimal solutions of a multi-echelon inventory/pricing model," *J. of Operational Research,* Vol.147, No.3 2003, pp.530-548.
[2] Cheng, C. Y., Chen, T. L. and Y. Y. Chen, "An analysis of the structural complexity of supply chain networks," *J. of Applied Mathematical Modelling*, Vol.38, No.9 2014, pp.2328-2344.
[3] Li, X. and Wang, Q., "Coordination mechanisms of supply chain systems," *J. of operational research*, Vol.179, No.1 2007, pp.1-16.
[4] Sluis, S. and De Giovanni P., "The selection of contracts in supply chains: An empirical analysis," *J. of Operations Management,* Vol.41, 2016, pp.1-11

[5] Chen, H., and C.Li, "An analysis of wholesale price contracts in a three-stage supply chain," *The sixth Wuhan international conference on E-business–engineering technology track.* 2007.
[6] Pan, K., Lai K. K., Leung, S. C. H. and Di. Xiao, "Revenue -sharing versus wholesale price mechanisms under different channel power structures," *J. of Operational Research,* Vol.203, No.2 2010, pp.532-538.
[7] Seifert, R. W., Zequeira, R. I. and S.Liao, " A three-echelon supply chain with price-only contracts and sub-supply chain coordination, Int*. ,"J. of Production Economics,* Vol.138, 2012, pp.345–353.
[8] Iida, T., "Coordination of cooperative cost-reduction efforts in a supply chain partnership" *J. of Operational Research,* Vol.222, No.2 2012, pp.180-190.
[9] Van Der Rhee, B., Van Der Veen J. A., Venugopal V. and V. R.Nalla, "A new revenue sharing mechanism for coordinating multi-echelon supply chains" *J. of Operations Research Letters,* Vol.38, No.4 2010, pp.296-301.
[10] Yeganehfallah, A., Mashreghi, H. and M. R.Amin-Naseri , "*Coordinating a Three-Echelon Telecom Supply Chain with Spanning and Pair-Wise Revenue Sharing Contracts*" *J. of* Operations Research 2013 Proceedings, 2014, pp 495-501.
[11] Cachon, G. P., *Supply chain coordination with contracts*, Handbooks in operations research and management science, 2003