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Bi-objective location routing problem with M/M/c queue system: Application in waste collection problem

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Abstract

In this model, economic objective (opening cost of depots and treatment facility and transportation cost regard to loading time and waiting time) and societal objective function; that is, negative effect, of treatment facilities which are close to towns, for waste collection problem is considered under uncertainty noticing congestion in the treatment facilities. Each treatment facilities are modeled as an M/M/c queuing system. Then locate depots and treatment facilities and design the routes starting from depots to serve customers. Also modified weighted-sum approach and maximize minimum approach is used to validate the proposed model with GAMS.

Keywords: Bi-objective, location routing problem, queue system, multi objective optimization**,** waste collection problem

Introduction

The governments and business organizations have recently close attention to green. The environmental, ecological and social effects are considered for designing logistics policies, in addition to the conventional economic costs. The reverse logistics in product recovery management and the routing of waste collection are some environmental issues related to routing that) specified by [1, 2].Thus Waste collection problem is one of the most important applications of vehicle routing problems in real world includes collect, reuse, dispose and recycling activities.[3] offered some research gaps that link the VRP with Green Logistics issues. Recently, A model that collection waste from customers' location with assumptions that the fleet of vehicles is heterogeneous and vehicles have separated compartments proposed by[4].

Karaoglan, Altiparmak [5] express that location of depots and the distribution routings with efficiency, reliability, and flexibility are of vital importance for managers. If the routes are ignored while locating the depots, the costs of distribution systems might be immoderate [6].The LRP is a combination of facility location problem (FLP) and vehicle routing problem (VRP)[7] .The LRP is an NP-hard problem because both problems classified to NP-hard problem.[8].Beltrami and Bodin [9] proposed vehicle routing models using in waste collection. A model for the hazardous waste location routing problem with minimization of total cost and transportation risk is proposed by[10].

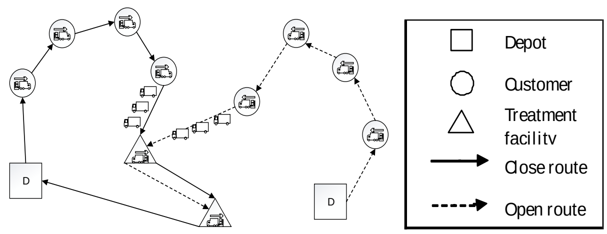
Rabbani, Farrokhi-Asl [11] considered location routing problem and waste collection problem simultaneously. The VRP with stochastic travel time in which there is waiting time is significantly more difficult than the case with no waiting time. [12]

As one of the operational performance metrics is service time level in the transportation system, a mismatch between the arrival rate of flows and the processing rate of recycling may cause congestion in the treatment facility nodes.

Neglecting this situation may result in a queue and long waiting times for flows which has a direct effect on the delivery service performance. [13, 14]In this paper, is considered the waste collection problem and location depot and treatment facilities simultaneously, also the waiting time in treatment facilities and queue system is noticed ,because creating queue in treatment facilities increases the variable cost of transportation and the Greenhouse Gas emissions. This model has two objective functions, first objective minimize the transportation and location cost and second objective attend to environmental issues.in the other word, maximize the distance between treatment facilities and customer's nodes. The rest of the paper is arranged as follows: The formulation of the model is presented in Section 2. The presented solution methodology for the proposed model is provided in Sections 3. Test problems with the developed model and the computational results and discussion are investigated in section 4. Conclusions are presented in Section 5.

Problem description

In this paper, expected demands and locations of customers are deterministic and known. Also, the potential locations for depots and treatment facilities are given (i.e., the set of depots and treatment facilities must be chosen from these potential locations). Two types of fleet are considered in this paper (Internal and external fleets). Vehicle belonging to internal fleet should come back to depot which vehicles exit from, but external fleets are free after unloading the wastes in last treatment facility. Therefore, the routes are mixed open and close. Vehicles are homogeneous and have maximum capacity constraint. We also consider maximum allowable time for servicing customers and maximum allowable route length, simultaneously. Due to the limited capacity of the treatment facilities, the entering flow unit should wait in a queue to receive services (e.g., unloading). Since the amount of flow unit between nodes is stochastic, calculating the exact process time at the treatment facilities is challenging. Since the process time has been considered to be the sum of processing time and waiting time, calculating the waiting time with stochastic flow units requires developing a queuing model for the system. Several papers have justified using queuing approach by both empirical data and simulation [15-17].in order to represent the waiting time in practice. Justification results proved that the queuing models can be adequately used for calculating the waiting time of the flow units. Under specific assumptions, the analysis of the queue formed by vehicle waiting for receiving services is applicable for their unloading and getting stuck in traffic. We use the peak hour's analysis, assuming that during the peak hours the average arrival rate and the service rate are both constant and therefore the arrivals of flow units to treatment facilities nodes follow a Poisson distribution during peak hours. This allows us to model the queue formed by vehicles as an M/M/c queuing system. Vehicle starts its route from depot and collects wastes from customers' locations. After finishing collection of wastes, vehicle moves to treatment facilities. Note that vehicles are multi-compartment and have specific parts to each type of waste. Each waste type should be treated in treatment facility that has compatible technology to treat this type of waste. There is fixed cost associated with opening a treatment facilities and depots at each potential location. Distribution cost associated with routes including the fixed cost of vehicles for external ones and variable cost that is proportional to the total time traveled by the vehicle. We want to determine location of depots and treatment centers and vehicle routes in this problem. The objectives function includes minimization of total cost and maximization of distance between opened treatment facilities and customers. The example of this problem is shown in Figure (I).



Problem assumptions

1. The demand of each customer and its location are deterministic and known
2. Each customer is served by only one vehicle
3. Internal vehicles should return to the depot, but the external ones should not return to depot
4. Capacity of vehicles and depots are limited
5. Vehicles are homogeneous and vehicles are constrained in time and distance
6. There are some types of waste
7. Each type of waste should be treated in compatible treatment technology
8. Only external vehicles have fixed cost for starting their routes
9. Vehicles are multi compartment and have limited capacity for each type of waste.
10. All unloading waste at treatment facility are modeled by M/M/c queuing system.
11. Vehicles arrive at treatment facility nodes with an overall arrival rate λ_j . The arrival follows a Poisson process
12. Unloading waste at treatment facility nodes has no limits on their queuing capacities.

Notations

Sets:	
D	Set of potential depots
C	Set of customers
F	Set of potential treatment facilities
W	Set of wastes type and corresponding treatment technology
K	Set of indexes for vehicles
S	Fleets type (internal or external)
N	Set of depots and customers nodes
P	Set of customers and potential treatment facilities
A	Set of all nodes

Parameters:	
t_{ij}	Loading time per unit of waste *w* in customer node *c* by means of vehicle *k* of fleet *s*
f_1	Fixed cost of external fleet
VC_k	Variable cost of vehicle *k* per unit of time
q_{iw}	Demand of customer i for treatment of waste type w
cap_{kw}	Maximum vehicle k capacity for load waste type w
L	Maximum allowable route length
T	Maximum allowable time to servicing customers in each route
Ω_d	Maximum capacity of depot d
π_{iw}	Fixed cost of opening treatment facility with technology *w* in potential location *f*
π'_d	Fixed cost of opening depot in potential location *d*
VR_{ij}	speed level between node i and j
M	Big value

Decision variables:	
x_{ijsk}	1 if vehicle k of fleet s travels directly from node i to node j; otherwise=0.
y_{iw}	1 if treatment facility with technology w is opened in potential location i; otherwise=0
z_{isk}	1 if vehicle k of fleet s is allocated to customer i; otherwise=0.
O_i	1 If depot is opened in potential location i; otherwise=0.
U_{iskw}	Continuous variable that represents the load of compartment w vehicle k of fleet s just after leaving node i
T_{ij}	Continuous variable that represents the total time between node i and node j
Wq_j	Total queue waiting time at node *j*

Ws_j	Total service time at node *j*
λ_j	Arrival rate of flow unit to node *j*
μ_j	Service rate of node *j*
c_j	Numbers of service providers at node *j* i
Lq_j	Length of queue at node *j*
d_{ij}	Distance between node *i* and node *j*

M/M/c queue system

In this subsection, an M/M/c queuing system is developed for the location routing problem. In other words, Loading waste at customer nodes and unloading waste at treatment facility are modeled by M/M/c queuing system. Alternatively, according to [18] Ws_j , total service time at node *j* is calculated as Eq. (2) which j is customer or treatment facility node, and $1/\mu_j$ is processing time.

$$\lambda_j = \sum_c \sum_w q_{iw} y_{jw} \quad \forall j \in f\ s \in S, k \in K \tag{1}$$

$$\mathrm{Ws}_j = \mathrm{Wq}_j + \frac{1}{\mu_j} \tag{2}$$

$$\mathrm{Wq}_j = \frac{Lq_j}{\lambda_j} \tag{3}$$

$$\mathrm{P}_{0i} = \left[\frac{(a_j)^{c_j}}{c_j!\,(1-\rho_j)} + \sum_{v=0}^{c_j-1} \frac{(a_i)^v}{v!} + 1 \right]^{-1} \tag{4}$$

$$Lq_j = \frac{(a_j)^{c_j}(\rho_j)}{c_j!\,(1-\rho_j)^2} \mathrm{P}_{0j} \tag{5}$$

$$a_{j=} \frac{\lambda_j}{\mu_j} \tag{6}$$

$$\rho_j = \frac{\lambda_j}{c_j \mu_j} \tag{7}$$

Mathematical models

$$\begin{aligned} \min \sum_{k\in K}\sum_{j\in C}\sum_{i\in D} f_1 x_{ij1k} &+ \sum_{s\in S}\sum_{k\in K}\sum_{j\in f}\sum_{i\in p} VC_k \left((d_{ij}/VR_{ij}) + Ws_j\right)x_{ijsk} \\ &+ \sum_{s\in S}\sum_{k\in K}\sum_{j\in c}\sum_{i\in n} VC_k \left((d_{ij}/VR_{ij}) + t_i q_{iw}\right)x_{ijsk} \\ &+ \sum_{k\in K}\sum_{j\in D}\sum_{i\in F} VC_k t_{ij} x_{ij0k} + \sum_{i\in F}\sum_{w\in W} \pi_i y_{iw} + \sum_{i\in D} \pi'_i O_i \end{aligned} \tag{8}$$

$$Max\ Min_{\substack{i \in C \\ j \in F \\ w \in W}} \left(d_{ij} y_{jw}\right) \qquad (9)$$

S.t:

$$\sum_{s \in S} \sum_{k \in K} \sum_{i \in N} x_{ijsk} = 1 \qquad \forall j \in C \qquad (10)$$

$$\sum_{i \in D} \sum_{j \in D} x_{ijsk} = 0 \quad \forall s \in S, k \in K \qquad (11)$$

$$\sum_{i \in N} \sum_{j \in C} (d_{ij}/VR_{ij}) x_{ijsk} + \sum_{w} \sum_{i \in C} t_i q_{iw} z_{isk} \le T \qquad \forall s \in S, k \in K \qquad (12)$$

$$\sum_{i \in N} x_{ijsk} = z_{jsk} \qquad \forall j \in C,\ s \in S, k \in K \qquad (13)$$

$$\sum_{s \in S} \sum_{k \in K} \sum_{j \in C} x_{ijsk} \le \Omega_i o_i \quad \forall i \in D \qquad (14)$$

$$U_{iskw} = 0 \qquad \forall i \in D, s \in S, k \in K \qquad (15)$$

$$U_{iskw} + q_{jw} - M\left(1 - x_{ijsk}\right) \le U_{jskw}\ \forall s \in S, k \in K, w \in W, i, j \in C \qquad (16)$$

$$q_{iw} \le \sum_{s \in S} \sum_{k \in K} U_{iskw} \le \sum_{s \in S} \sum_{k \in K} \sum_{j \in N} x_{jisk} cap_{kw} \qquad \forall w \in W, i \in C \qquad (17)$$

$$\sum_{i \in A} \sum_{j \in A} d_{ij}\, x_{ijsk} \le L \qquad \forall s \in S, k \in K \qquad (18)$$

$$\sum_{i \in C} q_{iw} z_{isk} \le\ cap_{kw} \qquad \forall s \in S, k \in K, w \in W \qquad (19)$$

$$\sum_{s \in S} \sum_{k \in K} \sum_{j \in F} \sum_{i \in D} x_{ijsk} = 0 \qquad (20)$$

$$\sum_{s \in S} \sum_{k \in K} \sum_{j \in D} \sum_{i \in C} x_{ijsk} = 0 \qquad (21)$$

$$\sum_{s \in S} \sum_{k \in K} \sum_{j \in C} \sum_{i \in F} x_{ijsk} = 0 \qquad (22)$$

$$\sum_{s \in S} \sum_{k \in K} \sum_{i \in C} \sum_{j \in D} x_{ijsk} = \sum_{s \in S} \sum_{k \in K} \sum_{p \in P} x_{pfsk} \qquad \forall f \in F \qquad (23)$$

$$\sum_{i \in F} y_{iw} = 1 \quad \forall w \in W \qquad (24)$$

$$\sum_{w \in W} y_{iw} \le 1 \quad \forall i \in F \qquad (25)$$

$$x_{ijsk} = \{0,1\} \quad \forall i, j \in A, s \in S, k \in K \qquad (26)$$

$$y_{iw} = \{0,1\} \quad \forall i \in F, w \in W \qquad (27)$$

$$O_i = \{0,1\} \quad \forall i \in D \qquad (28)$$

$$U_{iskw} \ge 0 \quad \forall i \in N, s \in S, k \in K, w \in W \qquad (29)$$

Objective function (8) calculates economic cost. In objective function (9) makes far treatment facilities from customers' location. Equation (10) assures that each customer is assigned to only one route. Equation (11) prevents travelling between depots. Equations (12) guarantee that serving the customers in each route is less than time limitation. Equation (13) shows the relationship between two types of decision variables. Equation (14) determines numbers of vehicles that depart from each depot should not trespass from depot's capacity. Equations (15)-(17) are lifted Miller–Tucker–Zemlin (MTZ) sub tour elimination constraints[19]. In equation (16), parameter M is set equal to summation of all customers' demand. Equation (18) determine that each route's length do not trespass from maximum allowable route length. Equation (19) consider the capacity of vehicles in each route. Equation (20) prohibits moving from depots to treatment facilities directly Equation (21) prohibits moving from customers to depots before crossing treatment facilities. Equation (22) prohibits moving from treatment facilities to customers. Equation (23) satisfy that all vehicles that depart from depots must visit all treatment facilities established in potential locations. Equation (24) represent that only one treatment facility for each type of waste must be opened. Equation (25) guarantee that treatment facilities do not overlap with each other. Equations (26)- (29) shows the ranges of the variables

methodology

Multi objective Optimization

Three objectives are considered in this paper that these objectives are in conflict with each other. therefore, it is difficult to find the best values which optimize all objective functions. The weighted-sum method and maximize minimum method are well-known methods for multi objective optimization. These methods make multi objective functions convert into a single objective function.

To solve the multi objective problem with k objectives, the weighted-sum method can be written as follows:

$$minimize\ f = \sum_{j=1}^{k} w_j \left(\frac{f_j(x)}{f_j^*}\right)$$

$$Subject\ to:$$

$$x \in X$$

$$\sum_{j=1}^{k} w_j = 1$$

$w_j \geq 0 \;,\; j = 1, \dots \dots k$

Where $f_j(x)$the objective is function for the jth design objective, $f(x)$ is the design objectives vector and X is the feasible design space, x is an n-dimensional vector of design variables, and f_j^* is optimal value of jth design objective while the optimization problem is solved by of jth design objective

The maximize minimum method can be written as follows:

$maximize \;\; y$

$Subject\ to:$

$$\left(\frac{f_j(x)}{f_j^*}\right) \geq y$$

$x \in X$

Where $f_j(x)$the objective is function for the jth design objective, X is the feasible design space, x is an n-dimensional vector of design variables, and f_j^* is optimal value of jth design objective while the optimization problem is solved by of jth design objective.

Computational Results and Discussion

To validate the reliability of the proposed model fifteen test problem are presented that solve by two different multi objective method in GAMS, the initial data for parameters is shown in Table 1 and the computational results of weighted-sum method and maximize minimum method is shown in Table 3. As Fig. 1, Fig. 2 show both of the result have Normal distribution, therefor t-test problem is used to comparative these result in MINITAB Fig3.

Table 1. Sources of random generation of the parameters.

Service rate	Poison(1000)
Demands of customers	Poison(100)
Coordinate of nodes	U(0,100)
Opening cost of depots	U(100,1000)
Opening cost of facilities	U(100,1000)
Fixed cost of external vehicles	U(10,100)
Collection time in nodes	U(1,10)
Capacity of vehicles	U(100,500)
Maximum allowable distance	10000
Maximum allowable travelling time	10000

Table 1

Problem	Customers	Waste Type	Potential locations for depot	Potential locations for treatment facilities
1	5	4	5	5
2	5	4	6	5
3	6	4	6	6
4	6	4	5	5
5	7	4	6	5
6	8	4	6	6
7	9	4	5	5
8	10	4	9	6
9	10	4	10	10
10	11	4	5	5
11	11	4	10	10
12	12	4	5	5
13	12	4	10	10
14	13	4	5	5
15	14	4	5	5

Table 2

Test problem	Z_T maximize minimum	
	Z12	Z23
1	181063.17	81.33
2	198043.74	79.05
3	242319.31	68.39
4	268837.4	57.7
5	315126.02	65.1
6	381863.93	54.95
7	415801.37	55.85
8	456311.01	52.62
9	501084.59	61.29
10	695809.44	48.73
11	910231.26	39.18
12	1350174.53	43.87
13	1391008.14	44.09
14	1702962.36	47.25
15	190561.92	53.14

Table 3

Test problem	Z_T weighted-sum	
	Z1	Z2
1	151166.25	101.51
2	168078.34	84.11
3	182597.27	69.62
4	198045.61	73.21
5	239134.69	71.49
6	297801.15	63.02
7	323489.8	59.83
8	359841.32	70.13
9	470834.41	57.64
10	663436.07	50.31
11	839744.45	42.39
12	1196489.23	40.26
13	1273991.09	66.21
14	1641367.71	49.03
15	1738990.25	37.02

Table 4

Table 5. scale multi objective data to one objective data

Test problem	Z_T weighted-sum	Z_T maximize minimum
1	0.81	0.78
2	0.766	0.774
3	0.709	0.63
4	0.683	0.618
5	0.642	0.612
6	0.614	0.593
7	0.609	0.545
8	0.582	0.527
9	0.556	0.513
10	0.534	0.501
11	0.519	0.493
12	0.492	0.47
13	0.452	0.363
14	0.413	0.346
15	0.379	0.342

Table 5

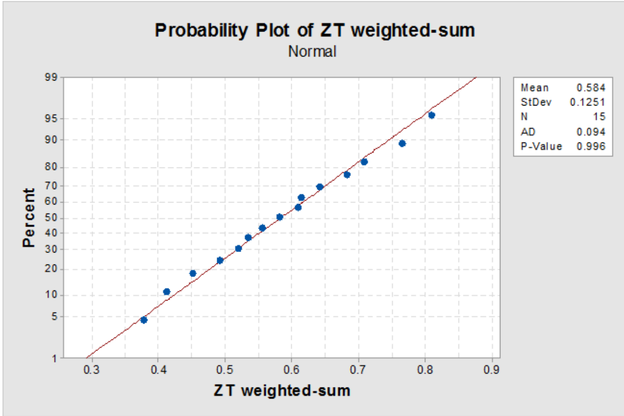


Figure 1

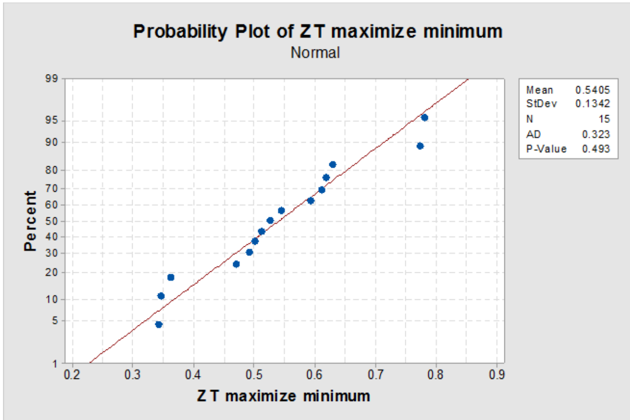


Figure 2

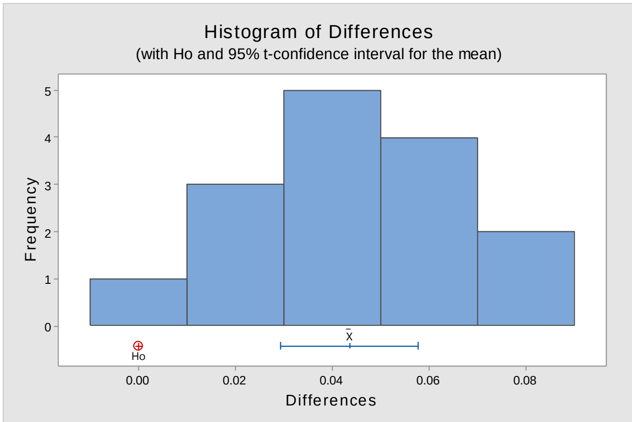


Figure 3

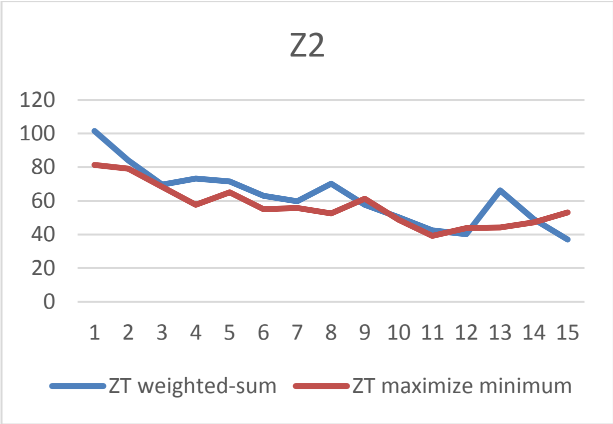


Figure 4

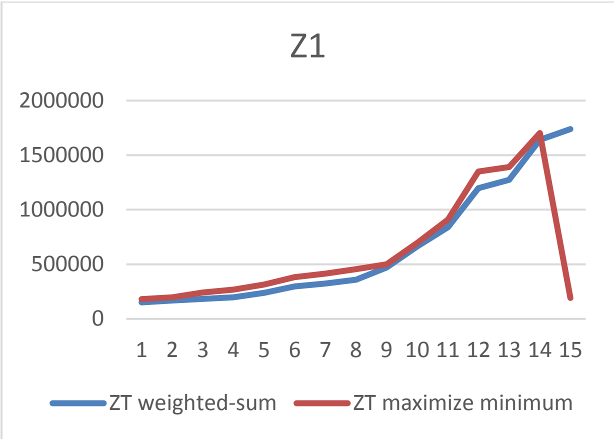


Figure 5

Conclusion

In this study, two objectives are considered that minimize transportation cost and maximize the minimum distance between each customer and treatment facilities. The waste are collection from each customers then are recycled in treatment facilities. The arrival rate of flows and the processing rate of recycling may cause a queue and long waiting times in the treatment facility nodes, so M/M/C queue is considered. Also weighted-sum method and maximize minimum method for optimization two different and conflicting objective functions to find the optimal solutions, AZ is shown in Figure 6 there is no difference between the weighted- sum method and Maximize minimum method .

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