



A hybrid approach for shelf space planning considering of stochastic demand and display facing area

S.M.T. Fatemi Ghomi^{a,1} and Sadjad Khalesi^b

^a Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran Tel: +9A-21- 64545381, Fax: 66954569, +9A-21- 64545381, E-mail: <u>fatemi@aut.ac.ir</u>

^b Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran Tel: +9A-2J-44978879, E-mail: <u>sadjadkhalesi@aut.ac.ir</u>

Abstract

Shelf space planning is an important part of retailing management and consists topics such as analysis on the dependence between demand and inventory and also determining the allocation of products to the shelves. Deterministic demand is one of the most common assumptions in these models while most of the time in the real world we are faced with uncertain demand. On the other hand display facing area is an essential variable in these problems because of different physical forms of goods and its impact on space requirements. Hence we develop the shelf space optimization models considering of stochastic demand and display facing area and then use a hybrid approach using the basic form of the bees algorithm and a heuristic algorithm to improve performance of algorithms as a solving method for this type of models. In the end, the sensitivity analysis on parameters and also pricing policies analysis is made.

Keywords:

Shelf space planning, display facing area, stochastic demand, hybrid approach, retailing management

Introduction

Shelf space allocation planning problems often used for consumer products and its impact will be significant when the customer is not looking for a particular brand of a product. Shelf space planning as an important part in logistic decision can consists topics such as analysis on the dependence between demand and inventory and also determining the allocation of products to the shelves. On the other hand we can classify products to three types: unresponsive product or commodities, general use product or staples and occasional purchase products or impulse buys. The first two categories of products are not affected by how the allocation of space and therefore occasional purchase products are the only issues to be considered in shelf space allocation studies. Yang and Chen [1] believe that decision about shelf space management plays a very important role in attracting customers and retail operation managements.

Retailing management can be divided into five subcategories include: category sales planning, assortment planning, shelf space planning, instore logistic planning and shelf layout design. In category sale planning portion we attempt to estimate the expected demand for different groups and also select the groups of products and depth of each of them for midterm planning in store. On the other hand in assortment portion, our main goal will be to analyze the role of substitutionary and complementary products and determining different brands of a product while instore logistic and shelf layout design concentrate on inventory control, replenishment policies and store layout design. Decisions about buying more dependent on various factors such as in-store layout, shelf location, item location and shopping path can be caused changes in customer demand while only 1/3 of shopping decision are pre-planned [2]. Retailers need to ensure that any set of products on the shelves are available in sufficient numbers and so will need to regularly check the shelves, their planogram and their products as well as allocate space available [3]. Due to the competitive environment and limited spaces available and introduction of new products, determination the best assortment, allocation and pricing will be critical for retailers [4].

Management and inventory control and determination the order quantity and re-order point are other factor that play an important role in increasing retailer's productivity and profitability that often known as the replenishment problem. On the other hand is proved that sales of stores is dependent

¹ corresponding author

on the amount of inventory displayed. Therefore, the elastic coefficient for the consideration of customer sensitivity to the product displayed and the impact on the purchases is applied [5].

In general, many factors affect the demand for a product and also the improvement in business often depends on demand management. The pricing policy and revenue management as a tool to control demand are a crucial factor in this field [6]. Multiple product orientations is another important factor that can affect product's demand that in real problem would cause more complexity.

In the rest of this paper, in section 2 we first present the related literature and then in the third section we describe the assumptions and model development. Solution procedure, computational results and sensitivity analysis will be discussed in the fourth section and then in the last section we will summarize our results.

Literature review

In recent years, many researchers study the shelf space optimization with deterministic models (see [7, 8, 9, 1, 5, 10, 11, and 12]).

The second category of problem is uncertain models that can include stochastic demand or robust optimization and fuzzy programming approach (see [3, 13, and 14]). On the other hand, data mining approach and association rules usually are used for shelf space problems and specially to determine assortment and finding relationships between products with respect to their prices [4]. Shelf space allocation in supply chain also include the game theory techniques such as a stackelberg game between retailer and manufacturers especially for consideration retailer shelf space management with trade allowance [15].

Authors in [16] survey customers' shopping path and behavior with consideration a shelf space allocation problem (SSAP) with multi-level shelves and propose an iterative simulation-optimization approach and apply factors such as shelf level utility, attraction of store' zones and demand substitution effects. SSAP belongs to the class of knapsack problems that is known to be NP-hard [3]. Therefore a variety of approaches such as heuristic, meta-heuristic methods, goal programming and also exact methods have been used in this area (see [17, 18, 19, 20, 21, and 22]). Space allocated, order quantity, reorder point, assortment, shelf space design, item location and promotion level are some of most common decision variables considered by different researchers. Authors in [23] introduced a single store multiproduct inventory problem that the amount of inventory of products on shelves will affect sales and show that the relationship between allocated space to a product and demand rate is linear.

Many existing studies typically do not consider parameters such height of the display shelves and display orientations and researchers in [24] try to develop a model with these key aspects of shelf space problem to achieve realistic decisions and be more practical for retailers. In many cases researchers concentrated on integrating the inventory lot sizing, display area and shelf space allocation, and product assortment problem that can increase the model complexity and also usually these models are classified as a mixed integer programming (MIP/MINLP) problems [5].The main parameters that are commonly used in these models are as follows: space elasticity effects, cross-space elasticity effects and positioning effects (vertical/ horizontal). Several researchers prefer to ignore positioning effects to simplify (see [25, 18, and 11]) and this is mainly because of difficulty of estimating positioning effects parameters. However many studies, especially in recent years considering positioning effects in the model were conducted and this causes more actual results and reduces assumptions in the simplified model (see [26, 27, 28, 29, 30, 31, 12]).

Mathematical model

This paper developed the model with reference to the basic idea presented by [3] by adding the following issues that were not considered previously.

•Display facing area

•Embedding the goods on each other just for similar items •Considering the width, height and depth of products and shelves simultaneously

•Ability to select the available products for placement on the shelf, especially when the number of products is high and space constraints are very important.

•Adding some costs such as holding inventory cost and costs associated with inventory and maintenance

Model assumptions are as follows:

•Retailers use direct replenishment policies.

•Backroom storage is not available.

•Shelf space for each class or family of products is limited. •One kind of orientation can be selected for each product •Height and depth of all levels is similar but their width can be different

Table 1 summarizes the notations.

Table 1. Notations of mathematical model

$ \begin{array}{cccc} z_{ij} & & \mbox{Height of item i for} \\ & & \mbox{orientation j} \\ Zs & & \mbox{Height of shelf} \\ Ds & & \mbox{Depth of shelf} \\ d_{ij} & & \mbox{Depth of item i for orientation} \\ & & & \mbox{j} \\ SW_t & & \mbox{Width of level t} \\ w_{ij} & & \mbox{Width of item i for orientation} \\ & & & \mbox{j} \\ C_i & & \mbox{Wholesale price for item i} \\ r_{ij} & & \mbox{Selling price for item i} \\ v_i & & \mbox{Salvage value for item i} \\ i & \mbox{Prenalty cost for item i} \\ I & & \mbox{Interest expense} \\ cp_{ij} & & \mbox{Replenishment cost} \\ D_{ij}^{min} & & \mbox{Minimum demand at one} \\ facing \\ \beta_i & & \mbox{Space elasticity parameter} \\ \delta_{il} & & \mbox{Cross- space elasticity} \\ \end{array} $		
orientation j Zs Height of shelf Ds Depth of shelf d_{ij} Depth of item i for orientation j SW_t $Width$ of level t w_{ij} Width of item i for orientation j C_i Wholesale price for item i r_{ij} Selling price for item i v_i Salvage value for item i s_i Penalty cost for item i I Interest expense cp_{ij} Replenishment cost D_{ij}^{min} Minimum demand at one facing β_i Space elasticity parameter δ_{il} Cross- space elasticity	Z_{ij}	Height of item i for
ZsHeight of shelfDsDepth of shelf d_{ij} Depth of item i for orientationjSWtSWtWidth of level t w_{ij} Width of item i for orientationjCi r_{ij} Selling price for item i v_i Salvage value for item i s_i Penalty cost for item iIInterest expense cp_{ij} Replenishment cost D_{ij}^{min} Minimum demand at one facing β_i Space elasticity parameter δ_{il} Cross- space elasticity	,	orientation j
Ds Depth of shelf d_{ij} Depth of item i for orientation j SW_t SW_t Width of level t w_{ij} Width of item i for orientation j C_i r_{ij} Selling price for item i v_i Salvage value for item i s_i Penalty cost for item i I Interest expense cp_{ij} Replenishment cost D_{ij}^{min} Minimum demand at one facing β_i Space elasticity parameter δ_{il} Cross- space elasticity	Zs	Height of shelf
$\begin{array}{cccc} d_{ij} & & \text{Depth of item i for orientation} \\ & & j \\ SW_t & & \text{Width of level t} \\ w_{ij} & & \text{Width of item i for orientation} \\ & & j \\ C_i & & \text{Wholesale price for item i} \\ r_{ij} & & \text{Selling price for item i} \\ v_i & & \text{Salvage value for item i} \\ s_i & & \text{Penalty cost for item i} \\ I & & & \text{Interest expense} \\ cp_{ij} & & & \text{Replenishment cost} \\ D_{ij}^{min} & & & & \text{Minimum demand at one} \\ facing \\ \beta_i & & & & \text{Space elasticity parameter} \\ \delta_{il} & & & & \text{Cross- space elasticity} \\ \end{array}$	Ds	Depth of shelf
$\begin{array}{cccc} & & & & j \\ SW_t & & Width of level t \\ w_{ij} & Width of item i for orientation \\ & & & j \\ C_i & & Wholesale price for item i \\ r_{ij} & Selling price for item i \\ v_i & Salvage value for item i \\ s_i & Penalty cost for item i \\ I & Interest expense \\ cp_{ij} & Replenishment cost \\ D_{ij}^{min} & Minimum demand at one \\ facing \\ \beta_i & Space elasticity parameter \\ \delta_{il} & Cross- space elasticity \\ \end{array}$	d_{ii}	Depth of item i for orientation
$\begin{array}{llllllllllllllllllllllllllllllllllll$		j
	SW_t	Width of level t
$\begin{array}{ccc} j \\ C_i & Wholesale price for item i \\ r_{ij} & Selling price for item i \\ v_i & Salvage value for item i \\ s_i & Penalty cost for item i \\ I & Interest expense \\ cp_{ij} & Replenishment cost \\ D_{ij}^{min} & Minimum demand at one \\ facing \\ \beta_i & Space elasticity parameter \\ \delta_{il} & Cross- space elasticity \end{array}$	W _{ij}	Width of item i for orientation
$ \begin{array}{ccc} C_i & & \text{Wholesale price for item i} \\ r_{ij} & & \text{Selling price for item i} \\ v_i & & \text{Salvage value for item i} \\ s_i & & \text{Penalty cost for item i} \\ I & & \text{Interest expense} \\ cp_{ij} & & \text{Replenishment cost} \\ D_{ij}^{min} & & \text{Minimum demand at one} \\ facing \\ \beta_i & & \text{Space elasticity parameter} \\ \delta_{il} & & \text{Cross- space elasticity} \end{array} $		j
$\begin{array}{lll} r_{ij} & \text{Selling price for item i} \\ v_i & \text{Salvage value for item i} \\ s_i & \text{Penalty cost for item i} \\ I & \text{Interest expense} \\ cp_{ij} & \text{Replenishment cost} \\ D_{ij}^{min} & \text{Minimum demand at one} \\ facing \\ \beta_i & \text{Space elasticity parameter} \\ \delta_{il} & \text{Cross- space elasticity} \end{array}$	C_i	Wholesale price for item i
$ \begin{array}{cccc} v_i & & \text{Salvage value for item i} \\ s_i & & \text{Penalty cost for item i} \\ I & & \text{Interest expense} \\ cp_{ij} & & \text{Replenishment cost} \\ D_{ij}^{min} & & \text{Minimum demand at one} \\ & & & facing \\ \beta_i & & \text{Space elasticity parameter} \\ \delta_{il} & & \text{Cross- space elasticity} \\ \end{array} $	r_{ij}	Selling price for item i
s_i Penalty cost for item iIInterest expense cp_{ij} Replenishment cost D_{ij}^{min} Minimum demand at one facing β_i Space elasticity parameter Cross- space elasticity	v_i	Salvage value for item i
IInterest expense cp_{ij} Replenishment cost D_{ij}^{min} Minimum demand at one facing β_i Space elasticity parameter Cross- space elasticity	Si	Penalty cost for item i
$\begin{array}{ccc} cp_{ij} & \text{Replenishment cost} \\ D_{ij}^{min} & \text{Minimum demand at one} \\ & facing \\ \beta_i & \text{Space elasticity parameter} \\ \delta_{il} & \text{Cross- space elasticity} \end{array}$	Ι	Interest expense
$ \begin{array}{c} D_{ij}^{min} & \text{Minimum demand at one} \\ & facing \\ \beta_i & \text{Space elasticity parameter} \\ \delta_{il} & \text{Cross- space elasticity} \end{array} $	cp_{ij}	Replenishment cost
$ \begin{array}{c} \beta_{i} \\ \delta_{il} \end{array} \qquad \begin{array}{c} \text{facing} \\ \text{Space elasticity parameter} \\ \text{Cross- space elasticity} \end{array} $	D_{ii}^{min}	Minimum demand at one
$β_i$ Space elasticity parameter δ_{il} Cross- space elasticity	()	facing
δ_{il} Cross- space elasticity	β_i	Space elasticity parameter
	δ_{il}	Cross- space elasticity

	parameter
μ	Price sensitivity parameter
α_t	Scale parameter

Table 2 describes decision variables:

Table 2. Decision variables

Pr _{ijk}	Unit profit for item i with orientation j and
.,	composition of k, h, p
Yijkh	{ 1 if composition of (i
	0 otherwise
e_i	{ 1 if item i is selected
	lo otherwise
j	variable that determine the orientation of product
k	variable that determine the number of facing of
	each item per shelf level
h	Variables related to the number of levels of shelves
	where a product is displayed
р	Variables associated with the highest level of shelf
	that a product will be displayed

Now the shelf space allocation problem with stochastic demand and display facing area can be formulated as follows:

$$Pr_{ijkhp}\left(k_{ij}, h_{ij}, p_{ij}, x_{ij} \middle| x_{ij} = k_{ij} \cdot \left| \frac{Zs}{z_{ij}} \right| \cdot \left| \frac{Ds}{d_{ij}} \right| \cdot h_{ij}\right) = -C_{ij} \cdot x_{ij} + r_{ij} \cdot \int_{0}^{x_{ij}} y \cdot f_{ij}^{*} \cdot dy + r_{ij} \cdot \int_{x_{ij}}^{\infty} x_{ij} \cdot f_{ij}^{*} \cdot dy + v_{ij} \cdot \int_{0}^{x_{ij}} (x_{ij} - y) \cdot f_{ij}^{*} \cdot dy - s_{ij} \cdot \int_{x_{ij}}^{\infty} (y - x_{ij}) \cdot f_{ij}^{*} \cdot dy - \left(\frac{C_{ij} \cdot I}{2}\right) \cdot x_{ij} - cp_{ij} \cdot \int_{0}^{x_{ij}} y \cdot f_{ij}^{*} \cdot dy - cp_{ij} \cdot \int_{x_{ij}}^{\infty} x_{ij} \cdot f_{ij}^{*} \cdot dy$$

$$D_{ij}(\bar{k},\bar{h},p_{ij}) = D_{ij}^{min}.(k_{ij},h_{ij})^{\beta_i}$$
$$.\prod_{l\in N, l\neq i} (k_{lj},h_{lj})^{\delta_{il}}.a_{ij}(k_{ij},h_{ij},p_{ij}).(\frac{r_{avg}}{r_{ij}})^{\mu}$$
(2)

$$a_{ij}(k_{ij}, h_{ij}, p_{ij}) = \frac{\sum_{t \in T} \kappa_{ij}^t(k_{ij}, h_{ij}) \cdot \alpha_t}{k_{ij} \cdot h_{ij}}$$
(3)

$$\begin{array}{l}
\kappa_{ij}(\kappa_{ij}, n_{ij}) = \\
\begin{cases}
k_{ij}; \quad p_{ij} - k_{ij} + 1 \le t \le p_{ij} \\
0; \quad \text{otherwise} \\
\end{array}$$
(4)

$$MAX \qquad W(\bar{\gamma}) = \sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{T} \sum_{p=1}^{T} Pr_{ijkhp}.\gamma_{ijkhp}$$
(5)

Subject to:

$$\sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{T} \sum_{p=1}^{T} SU_{ijkhp}^{t} \cdot \gamma_{ijkhp} \le SW_{t}$$
(6)

$$\sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{T} \sum_{p=1}^{T} \gamma_{ijkhp} \leq e_i$$

$$SU_{ijkhp}^t = \begin{cases} k_{ij} \cdot w_{ij}; & p_{ij} - h_{ij} + 1 \leq t \leq p_{ij} \\ 0 & ; & \text{otherwise} \end{cases}$$

$$(8)$$

$$\gamma_{ijkhp} \in \{0,1\}$$

$$(9)$$

$$e_i \in \{0,1\}\tag{10}$$

In relation (1) we calculate the profit function for each composition of (i, j, k, h, p) that include eight terms. First term shows the total purchasing cost for retailer and second and third term calculate the total income for different quantity of demand. Term (4) and (5) present the salvage value and penalty cost. The last three terms relate to the holding inventory cost and replenishment. As mentioned earlier, demand in this model is stochastic and calculated by relation (2). Relation (3) indicates weighted average value of the scale parameter for item i and also relation (4) is used to calculate the number of facing of item i if it occupies shelf level t. Relation (5) represents the objective function that maximizes the total profit. Constraint (6) ensures that the total width of all products is not greater than the width of different level of shelf. Constraint (7) is used for select the best items to maximize total profit. Constraint (8) states that how to calculate the total width of the display for each product on the shelf. The last constraint also determines the binary decision variables used in the model.

Computational results and sensitivity analysis

This paper uses a hybrid approach in solution procedure. In this approach first the basis is used of bees algorithm and then a heuristic algorithm is added to improve the accuracy and quality of outputs. Figure 1 shows the pseudo code for this hybrid algorithm. It should be noted that the normal distribution is used to express the nature of stochastic demand.

Initializatio	n
Step 1.1	For $i \in N$:
Step1.2	For all combinations of $j_i \in [1:J], k_i \in [1:k], h_i \in [1:T], p_i \in [1:T]$:
Step1.3	Initialize population with random solutions
Step1.4	End for
Step1.5	Evaluate fitness of the population.
Step1.6	End for
Step1.7	Select the best solution with best fitness value (best profit)
Step1.8	For $i \in non$ selected product set :
Step1.9	Select the product with minimum possible facing in shelf
Step1.10	Update selected solution
Step1.11	End for
Main loop	

Step2.1	While (stopping criterion not met)
Step2.2	//Forming new population.
Step2.3	Select sites for neighborhood search.
Step2.4	<i>Recruit bees for selected sites (more bees for best e sites) and evaluate fitness.</i>
Step2.5	Select the fittest bee from each patch.
Step2.6	Assign remaining bees to search randomly
	And evaluate their fitness.
Step2.7	Select the best solution with best fitness value (best profit)
Step2.8	For i
	\in non selected product set :
Step2.9	Select the product with minimum possible facing in shelf
Step2.10	Update selected solution
Step2.11	End for
Step2.12	End While

Figure 1. The pseudo code for hybrid algorithm

In this approach, $m \times n$ matrices is used for the parameter to create the bees as an input of algorithm that figure 2 show an example of it.

Select/not	j	k	h	р
0	٣	٣	١	١
1	٣	٣	٣	٣
0	١	٣	۲	۲
١	٣	٣	٣	٣
•	۲	٣	١	٣
)	۲	۲	٣	٣

Figure 2. Example of a bee

In this section first use random data for our parameter and the problems is solved in different situation and with different sizes that presented in table (5) and then sensitive associated with its output is shown in Figure (3) and Figure (4) according to the objective function and algorithm's running time.



Figure 3. Comparison of objective functions of two algorithm in different size of problems



Figure 4. Run-time analysis with regard to the dimensions of problems

Furthermore, the output of two algorithm is compared with numerical result of GAMS that indicates in table 3.

Table 3. C	omparison	of output	of algorithms	with GAM	IS
------------	-----------	-----------	---------------	----------	----

(I,J,K,H,P)	BA	HA	GAMS (Baron)
(٤,٣,٣,٣,٣)	18910,888	15179	1077.,7

In the second portion of this section, a list of product with real data is taken and solved. Then the output of these two algorithm is compared for this list. The list of product are presented in table (4) and also table (6) and (7) show the numerical results of bees algorithm and hybrid algorithm.

Table 4. List of products

Prod. number	Title of product	Retail price	Wholesale price
١	Hailey Oil	۷۳٥.	720.
	1.5 L		

۲	Behrouz	5700	3012	
	French Sauce			
٣	Behrouz	9970	YY0.	
	Mayonnaise	•		
٤	Shilton Tuna	0.7.	٤	In the e
٥	Mahmoud	19	170	sample
	Теа			discour

٦	Tabarrok	0	891V
	Tomato Paste		

In the end of this section, we analyze different prices of two sample complementary products according to different discount policy or raising price policy considering of objective function value represented in figure (5).

Number	(J,H,P,T)	Objec	Running time in	
of item		Bees algorithm	Hybrid algorithm	hybrid algorithm
i				
٤	(r, r, r, r, r)	130251.6951	171944.826	499.101426
٦	(3,3,3,3)	209624.664	371340.44546	1581.28
٨	(3,3,3,3)	428699.3739	441803.89633	2713.8545
۱.	(3,3,3,3)	546490.1774	546490.1774	10352.174242
15	(3,3,3,3)	747834.3092	948438.6641	11874.069873

Table	5.	Numerical	results	for	random	data
Luvic	. .	1 milli luul	<i>i</i> couis	101	ranaom	uuuu

Table 6. Numerical results of bees algorithm

	Output of	Objective function value				
i	Select/not	j	k	h	р	
١	•	٣	۲	۲	۲	
۲	١	٣	٣	۲	۲	
٣	•	٣	۲	٣	٣	171.1.
٤	١	٣	٣	٣	٣	
٥	١	١	۲	٣	٣	
٦	•	٣	۲	۲	٣	

Table 7. Numerical results of heuristic algorithm

	Output of he	Objective function value				
i	Select/not	j	k	h	р	
١	١	٣	١	۲	۲	
۲	١	٣	٣	۲	۲	
٣	١	٣	١	٣	٣	14019.
٤	١	٣	٣	٣	٣	
٥	١	١	۲	٣	٣	
٦	١	٣	١	۲	٣	



Figure 5. Pricing analysis for two complementary products

Conclusions and future work

This paper developed a mathematical model for shelf space optimization problem considering of stochastic demand and then used a meta-heuristic algorithm and a hybrid algorithm to solve some random problems. Also it investigates a real case to test solution approach with real data. Then it made the sensitive analysis for running time, objective function value and discount or raising price policies. This model considers one possible type of orientation for each item while it is suggested that in future works multiple orientation for each product be considered. Furthermore, utilization of other meta-heuristic and heuristic methods to solve this type of problems and also adding inventory variables, backroom storage and replenishment time for each product can cause the output of the model to be more realistic.

References

[1] M.-H. Yang and W.-C. Chen, "A study on shelf space allocation and management," International Journal of Production Economics, Vol.60-61, pp. 309–317, 1999.

[2] P. M. Reyes and G. V. Frazier, "Goal programming model for grocery shelf space allocation," European Journal of Operational Research, Vol.181, pp. 634 – 644, 2007.

[3] A. Hübner and K. Schaal, "A shelf-space optimization model when demand is stochastic and space-elastic," Omega, in Press.

[4] M. nafary and j. shahrabi, "A temporal data mining approach for shelf-space allocation with consideration of product price," Expert Systems with Applications, Vol.37, pp. 4066–4072, 2010.

[5] M. A. Hariga, A. Al-Ahmari and A.-R. A. Mohamed, "A joint optimisation model for inventory replenishment, product assortment, shelf space and display area allocation decisions," European Journal of Operational Research, Vol.181, pp. 239 – 251, 2007.

[6] O. Baron, O. Berman and D. Perry, "Shelf Space Management When Demand Depends on the Inventory Level," production and operations management, Vol.20, pp. 714–726, 2011.

[7] P. Hansen and H. Heinsbroek, "Product selection and space allocation in supermarkets," European Journal of Operational Research, Vol.3, pp. 474–484, 1979.

[8] M. Corstjens and P. Doyle, "A dynamic model for strategically allocating retail space," The Journal of the Operational Research Society, Vol.34, pp. 943–951, 1983.

[9] A. P. Bultez, P. Naert, E. Gijsbrechts and P. V. Abelle, "Asymmetric cannibalism in retail assortments," Journal of Retailing, Vol.65, No.2, pp. 153–192, 1989.

[10] R. A. Russell and T. L. Urban, "The location and allocation of products and product families on retail shelves," Annals of Operations Research, Vol.179, pp. 131–147, 2010.

[11] A. H. Hübner and H. Kuhn, "Retail category management: State-of-the-art review of quantitative research and software applications in assortment and shelf space management," Omega, Vol.40, pp. 199 – 209, 2012.

[12] H. N. Geismar, M. Dawande, B. P. S. Murthi and C. Sriskandarajah, "Maximizing Revenue Through Two-Dimensional Shelf-Space Allocation," Production and Operations Management, Vol.24, pp. 1148–1163, 2015.

[13] A. Hübner, S. Kühn and H. Kuhn, "An efficient algorithm for capacitated assortment planning with stochastic demand and substitution," European Journal of Operational Research, Vol.250, pp. 505–520, 2016.

[14] M. Lotfi and S. Torabi, "A fuzzy goal programming approach for mid-term assortment planning in supermarkets," European Journal of Operational Research, Vol.213, pp. 430–441, 2011.

[15] Y.-C. Tsao, J.-C. Lu, N. An, F. Al-Khayyal, R. W.Lu and G. Han, "Retailer shelf-space management with trade allowance: A Stackelberg game between retailer and manufacturers," International Journal Production Economics, Vol.148, pp. 133–144, 2014.

[16] E. Ghazavi and M. Lotfi, "Formulation of customers' shopping path in shelf space planning: A simulation-optimization approach," Expert Systems with Applications, Vol.55, pp. 243–254, 2016.

[17] E. E. Anderson and H. N. Amato, "A mathematical model for simultaneously determining the optimal brand-collection and display-area allocation," Operations Research, Vol.22, pp. 13–21, 1974.

[18] F. S. Zufryden, "A dynamic programming approach for product selection and supermarket shelfspace allocation," Journal of the Operational Research Society, Vol.37, pp. 413–422, 1986.

[19] M.-H. Yang, "An efficient algorithm to allocate shelf space," European Journal of Operational Research, Vol.131, pp. 107–118, 2001.

[20] T. L. Urban, "The interdependence of inventory management and retail shelf management," International Journal of Physical Distribution & Logistics Management, Vol.32, pp. 41–58, 2002.

[21] R. Bai and G. Kendall, "A model for fresh produce shelf-space allocation and inventory management with freshness-condition-dependent demand," INFORMS Journal on Computing, Vol.20, pp. 78–85, 2008.

[22] J. Irion, J.-C. Lu, F. a. Al-Khayyal and Y.-C. Tsao, "A hierarchical decomposition approach to retail shelf space management and assortment decisions," Journal of the Operational Research Society, Vol.62, pp. 1861– 1870, 2011.

[23] H. Abbott and U. S. Palekar, "Retail replenishment models with display-space elastic demand," European Journal of Operational Research, Vol.186, pp. 586–607, 2008.

[24] C. C. Murray, D. Talukdar and A. Gosavi, "Joint optimization of product price, display orientation and shelf-space allocation in retail category management," Journal of Retailing, Vol.86, pp. 125 – 136, 2010.

[25] M. Corstjens and P. Doyle, "A model for

optimizing retail space allocations," Management Science, Vol.27, pp. 822–833, 1981.

[26] A. Lim, B. Rodrigues and X. Zhang, "Metaheuristics with Local Search Techniques for Retail Shelf-Space Optimization," Management Science, Vol.50, pp. 117–131, 2004.

[27] E. vanNierop, D. Fok and P. H. Franses, "Interaction between shelf layout and marketing effectiveness and its impact on optimizing shelf arrangements," Marketing Science, Vol.27, pp. 1065– 1082, 2008.

[28] H. Hwang, B. Choi and G. Lee, "A genetic algorithm approach to an integrated problem of shelf space design and item allocation," Computers Industrial Engineering, Vol.56, pp. 809–820, 2009.

[29] S. Raut, S. Swami and M. P. Moholkar, "Heuristic and meta-heuristic approaches for multiperiod shelfspace optimization: the case of motion picture retailing," Journal of the Operational Research Society,pp. 1335– 1348, 2009.

[30] J. M. Hansen, S. Raut and S. Swami, "Retail shelf allocation: A comparative analysis of heuristic and metaheuristic approaches," Journal of Retailing, Vol.86, pp. 94–105, 2010.

[31] H. Gajjar and G. Adil, "Heuristics for retail shelf space allocation problem with linear profit function," International Journal of Retail & Distribution Management, Vol.39, pp. 144–155, 2011a.

[32] H. Gajjar and G. Adil, "A piecewise linearization for retail shelf space allocation problem and a local search heuristic," Annals of Operations Research, Vol.179, pp. 149–167, 2010.

[33] B. Ranaseshan, N. R. Achuthan and R. Collinson, "A retail category management model integrating shelf space and inventory levels," Asia-Pacific Journal of Operational Research, Vol.26, pp. 457–478, 2009.

[34] J. Irion, J.-C. Lu, F. Al-Khayyal and Y.-C. Tsao, "A piecewise linearization framework for retail shelf space management models," European Journal of Operational Research, Vol.222, pp. 122 – 136, 2012.

[35] M. Lotfi, M. Rabbani and S. F. Ghaderi, "A weighted goal programming approach for replenishment planning and space allocation in a supermarket," Journal of the Operational, Vol.62, pp. 1128 – 1137, 2011.

[36] H. Gajjar and G.Adil, "A dynamic programming heuristic for retail shelf space allocation problem," Asia-Pacific Journal of Operational Research, Vol.28, pp. 183–199, 2011b.

[37] X. Drèze, S. J. Hoch and M. E. Purk, "Shelf management and space elasticity," Journal of Retailing, Vol.70, pp. 301 – 326, 1994.

[38] H. Hwang, B. Choi and M.-J. Lee, "A model for

shelf space allocation and inventory control considering location and inventory level e," International Journal of Production Economics, Vol.97, pp. 185 – 195, 2005.

[39] T. L. Urban, "An inventory-theoretic approach to product assortment and shelf-space allocation," Journal of Retailing, Vol.74, pp. 15 – 35, 1998.

[40] J. Preston and A. Mercer, "The influence of product range in the space allocation procedure," European Journal of Operational Research, Vol.47, pp. 339 – 347, 1990.

[41] S. A. Smith and N. Agrawal, "Management of multi-item retail inventory systems with demand substitution," Operations Research, Vol.48, pp. 50–64, 2000.

[42] G. A. Kök and M. L. Fisher, "Demand estimation and assortment optimization under substitution: Methodology and application," Operations Research, Vol.55, pp. 1001–1021, 2007.

[43] E. Yücel, F. Karaesmen, F. S. Salman and M. Türkay, "Optimizing product assortment under customer-driven demand substitution," European Journal of Operational Research, Vol.199, pp. 759–768, 2009.

[44] A. Katsifou, R. W. Seifert and J.-S. Tancrez, "Joint product assortment, inventory and price optimization to attract loyal and non-loyal customers," Omega, Vol.46, pp. 36–50, 2014.

[45] S. v. Zelst, K. v. Donselaar, T. v. Woensel, R. Broekmeulen and J. Fransoo, "Logistics drivers for shelf stacking in grocery retail stores: Potential for efficiency improvement," International Journal of Production Economics, Vol.121, pp. 620–632, 2009.

[46] K. H. v. Donselaar, V. Gaur, T. v. Woensel, R. A. Broekmeulen and J. C. Fransoo, "Ordering behavior in retail stores and implications for automated replenishment," Management Science, Vol.56, pp. 766–784, 2010.

[47] G. Reiner, C. Teller and H. Kotzab, "Analyzing the efficient execution on in-store logistics processes in grocery retailing – the case of dairy products," Production and Operations Management, Vol.22, pp. 924–939, 2012.