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A hybrid approach for shelf space planning considering of stochastic demand and display facing area

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Abstract

Shelf space planning is an important part of retailing management and consists topics such as analysis on the dependence between demand and inventory and also determining the allocation of products to the shelves. Deterministic demand is one of the most common assumptions in these models while most of the time in the real world we are faced with uncertain demand. On the other hand display facing area is an essential variable in these problems because of different physical forms of goods and its impact on space requirements. Hence we develop the shelf space optimization models considering of stochastic demand and display facing area and then use a hybrid approach using the basic form of the bees algorithm and a heuristic algorithm to improve performance of algorithms as a solving method for this type of models. In the end, the sensitivity analysis on parameters and also pricing policies analysis is made.

Keywords:
Shelf space planning, display facing area, stochastic demand, hybrid approach, retailing management

Introduction

Shelf space allocation planning problems often used for consumer products and its impact will be significant when the customer is not looking for a particular brand of a product. Shelf space planning as an important part in logistic decision can consists topics such as analysis on the dependence between demand and inventory and also determining the allocation of products to the shelves. On the other hand we can classify products to three types: unresponsive product or commodities, general use product or staples and occasional purchase products or impulse buys. The first two categories of products are not affected by how the allocation of space and therefore occasional purchase products are the only issues to be considered in shelf space allocation studies. Yang and Chen [1] believe that decision about shelf space management plays a very important role in attracting customers and retail operation managements.

Retailing management can be divided into five subcategories include: category sales planning, assortment planning, shelf space planning, instore logistic planning and shelf layout design. In category sale planning portion we attempt to estimate the expected demand for different groups and also select the groups of products and depth of each of them for midterm planning in store. On the other hand in assortment portion, our main goal will be to analyze the role of substitutionary and complementary products and determining different brands of a product while instore logistic and shelf layout design concentrate on inventory control, replenishment policies and store layout design. Decisions about buying more dependent on various factors such as in-store layout, shelf location, item location and shopping path can be caused changes in customer demand while only 1/3 of shopping decision are pre-planned [2]. Retailers need to ensure that any set of products on the shelves are available in sufficient numbers and so will need to regularly check the shelves, their planogram and their products as well as allocate space available [3]. Due to the competitive environment and limited spaces available and introduction of new products, determination the best assortment, allocation and pricing will be critical for retailers [4].

Management and inventory control and determination the order quantity and re-order point are other factor that play an important role in increasing retailer's productivity and profitability that often known as the replenishment problem. On the other hand is proved that sales of stores is dependent

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on the amount of inventory displayed. Therefore, the elastic coefficient for the consideration of customer sensitivity to the product displayed and the impact on the purchases is applied [5].

In general, many factors affect the demand for a product and also the improvement in business often depends on demand management. The pricing policy and revenue management as a tool to control demand are a crucial factor in this field [6]. Multiple product orientations is another important factor that can affect product's demand that in real problem would cause more complexity.

In the rest of this paper, in section 2 we first present the related literature and then in the third section we describe the assumptions and model development. Solution procedure, computational results and sensitivity analysis will be discussed in the fourth section and then in the last section we will summarize our results.

Literature review

In recent years, many researchers study the shelf space optimization with deterministic models (see [7, 8, 9, 1, 5, 10, 11, and 12]).

The second category of problem is uncertain models that can include stochastic demand or robust optimization and fuzzy programming approach (see [3, 13, and 14]). On the other hand, data mining approach and association rules usually are used for shelf space problems and specially to determine assortment and finding relationships between products with respect to their prices [4]. Shelf space allocation in supply chain also include the game theory techniques such as a stackelberg game between retailer and manufacturers especially for consideration retailer shelf space management with trade allowance [15].

Authors in [16] survey customers' shopping path and behavior with consideration a shelf space allocation problem (SSAP) with multi-level shelves and propose an iterative simulation-optimization approach and apply factors such as shelf level utility, attraction of store' zones and demand substitution effects. SSAP belongs to the class of knapsack problems that is known to be NP-hard [3]. Therefore a variety of approaches such as heuristic, meta-heuristic methods, goal programming and also exact methods have been used in this area (see [17, 18, 19, 20, 21, and 22]). Space allocated, order quantity, reorder point, assortment, shelf space design, item location and promotion level are some of most common decision variables considered by different researchers. Authors in [23] introduced a single store multi-product inventory problem that the amount of inventory of products on shelves will affect sales and show that the relationship between allocated space to a product and demand rate is linear.

Many existing studies typically do not consider parameters such height of the display shelves and display orientations and researchers in [24] try to develop a model with these key aspects of shelf space problem to achieve realistic decisions and be more practical for retailers. In many cases researchers concentrated on integrating the inventory lot sizing, display area and shelf space allocation, and product assortment problem that can increase the model complexity and also usually these models are classified as a mixed integer programming (MIP/MINLP) problems [5].The main parameters that are commonly used in these models are as follows: space elasticity effects, cross-space elasticity effects and positioning effects (vertical/ horizontal). Several researchers prefer to ignore positioning effects to simplify (see [25, 18, and 11]) and this is mainly because of difficulty of estimating positioning effects parameters. However many studies, especially in recent years considering positioning effects in the model were conducted and this causes more actual results and reduces assumptions in the simplified model (see [26, 27, 28, 29, 30, 31, 12]).

Mathematical model

This paper developed the model with reference to the basic idea presented by [3] by adding the following issues that were not considered previously.

•Display facing area

•Embedding the goods on each other just for similar items

•Considering the width, height and depth of products and shelves simultaneously

•Ability to select the available products for placement on the shelf, especially when the number of products is high and space constraints are very important.

•Adding some costs such as holding inventory cost and costs associated with inventory and maintenance

Model assumptions are as follows:

•Retailers use direct replenishment policies.

•Backroom storage is not available.

•Shelf space for each class or family of products is limited.

•One kind of orientation can be selected for each product

•Height and depth of all levels is similar but their width can be different

Table 1 summarizes the notations.

Table 1. Notations of mathematical model

z_{ij}	Height of item i for orientation j
Zs	Height of shelf
Ds	Depth of shelf
d_{ij}	Depth of item i for orientation j
SW_t	Width of level t
w_{ij}	Width of item i for orientation j
C_i	Wholesale price for item i
r_{ij}	Selling price for item i
v_i	Salvage value for item i
s_i	Penalty cost for item i
I	Interest expense
cp_{ij}	Replenishment cost
D_{ij}^{min}	Minimum demand at one facing
β_i	Space elasticity parameter
δ_{il}	Cross- space elasticity

	parameter
μ	Price sensitivity parameter
α_t	Scale parameter

Table 2 describes decision variables:

Table 2. Decision variables

Pr_{ijk}	Unit profit for item i with orientation j and composition of k, h, p
γ_{ijkh}	$\begin{cases} 1 & \textit{if composition of (i} \\ 0 & \textit{otherwise} \end{cases}$
e_i	$\begin{cases} 1 & \textit{if item i is selected} \\ 0 & \textit{otherwise} \end{cases}$
j	variable that determine the orientation of product
k	variable that determine the number of facing of each item per shelf level
h	Variables related to the number of levels of shelves where a product is displayed
p	Variables associated with the highest level of shelf that a product will be displayed

Now the shelf space allocation problem with stochastic demand and display facing area can be formulated as follows:

$$Pr_{ijkhp}\left(k_{ij},h_{ij},p_{ij},x_{ij}\middle|x_{ij}=k_{ij}.\left\lfloor\frac{Zs}{z_{ij}}\right\rfloor.\left\lfloor\frac{Ds}{d_{ij}}\right\rfloor.h_{ij}\right)= -C_{ij}.x_{ij}+r_{ij}.\int_0^{x_{ij}} y.f_{ij}^*.dy+r_{ij}.\int_{x_{ij}}^{\infty} x_{ij}.f_{ij}^*.dy+ v_{ij}.\int_0^{x_{ij}}(x_{ij}-y).f_{ij}^*.dy-s_{ij}.\int_{x_{ij}}^{\infty}(y-x_{ij}).f_{ij}^*.dy -\left(\frac{C_{ij}.I}{2}\right).x_{ij} -cp_{ij}.\int_0^{x_{ij}} y.f_{ij}^*.dy \quad -cp_{ij}.\int_{x_{ij}}^{\infty} x_{ij}.f_{ij}^*.dy \quad (1)$$

$$D_{ij}(\bar{k},\bar{h},p_{ij})=D_{ij}^{min}.(k_{ij}.h_{ij})^{\beta_i} .\prod_{l\epsilon N,l\neq i}(k_{lj}.h_{lj})^{\delta_{il}}.a_{ij}(k_{ij},h_{ij},p_{ij}).(\frac{r_{avg}}{r_{ij}})^{\mu} \quad (2)$$

$$a_{ij}(k_{ij},h_{ij},p_{ij})=\frac{\sum_{t\epsilon T}K_{ij}^t(k_{ij},h_{ij}).\alpha_t}{k_{ij}.h_{ij}} \quad (3)$$

$$K_{ij}^t(k_{ij},h_{ij})= \begin{cases} k_{ij} & ; \quad p_{ij}-\ k_{ij}+1\le t\le\ p_{ij} \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (4)$$

$$MAX \quad W(\bar{\gamma})= \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{T}\sum_{p=1}^{T}Pr_{ijkhp}.\gamma_{ijkhp} \quad (5)$$

Subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{T}\sum_{p=1}^{T}SU_{ijkhp}^t.\gamma_{ijkhp}\le SW_t \quad (6)$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{T}\sum_{p=1}^{T}\gamma_{ijkhp}\le e_i \quad (7)$$

$$SU_{ijkhp}^t=\begin{cases} k_{ij}\ .w_{ij}; & p_{ij}-\ h_{ij}+1\le t\le\ p_{ij} \\ 0 \quad ; & \text{otherwise} \end{cases} \quad (8)$$

$$\gamma_{ijkhp}\in\{0,1\} \quad (9)$$

$$e_i\in\{0,1\} \quad (10)$$

In relation (1) we calculate the profit function for each composition of (i, j, k, h, p) that include eight terms. First term shows the total purchasing cost for retailer and second and third term calculate the total income for different quantity of demand. Term (4) and (5) present the salvage value and penalty cost. The last three terms relate to the holding inventory cost and replenishment. As mentioned earlier, demand in this model is stochastic and calculated by relation (2). Relation (3) indicates weighted average value of the scale parameter for item i and also relation (4) is used to calculate the number of facing of item i if it occupies shelf level t. Relation (5) represents the objective function that maximizes the total profit. Constraint (6) ensures that the total width of all products is not greater than the width of different level of shelf. Constraint (7) is used for select the best items to maximize total profit. Constraint (8) states that how to calculate the total width of the display for each product on the shelf. The last constraint also determines the binary decision variables used in the model.

Computational results and sensitivity analysis

This paper uses a hybrid approach in solution procedure. In this approach first the basis is used of bees algorithm and then a heuristic algorithm is added to improve the accuracy and quality of outputs. Figure 1 shows the pseudo code for this hybrid algorithm. It should be noted that the normal distribution is used to express the nature of stochastic demand.

Initialization

Step1.1	*For* $i \in N$:
Step1.2	*For all combinations of* $j_i \in [1{:}J], k_i \in [1{:}k], h_i \in [1{:}T], p_i \in [1{:}T]$:
Step1.3	*Initialize population with random solutions*
Step1.4	*End for*
Step1.5	*Evaluate fitness of the population.*
Step1.6	*End for*
Step1.7	*Select the best solution with best fitness value (best profit)*
Step1.8	*For* $i \in$ *non selected product set* :
Step1.9	*Select the product with minimum possible facing in shelf*
Step1.10	*Update selected solution*
Step1.11	*End for*

Main loop

Step2.1	*While (stopping criterion not met)*
Step2.2	*//Forming new population.*
Step2.3	*Select sites for neighborhood search.*
Step2.4	*Recruit bees for selected sites (more bees for best e sites) and evaluate fitness.*
Step2.5	*Select the fittest bee from each patch.*
Step2.6	*Assign remaining bees to search randomly*
	And evaluate their fitness.
Step2.7	*Select the best solution with best fitness value (best profit)*
Step2.8	*For $i \in$ non selected product set :*
Step2.9	*Select the product with minimum possible facing in shelf*
Step2.10	*Update selected solution*
Step2.11	*End for*
Step2.12	*End While*

Figure 1.The pseudo code for hybrid algorithm

In this approach, m × n matrices is used for the parameter to create the bees as an input of algorithm that figure 2 show an example of it.

Select/not	j	k	h	p
0	٣	٣	١	١
1	٣	٣	٣	٣
0	١	٣	٢	٢
١	٣	٣	٣	٣
٠	٢	٣	١	٣
١	٢	٢	٣	٣

Figure 2. Example of a bee

In this section first use random data for our parameter and the problems is solved in different situation and with different sizes that presented in table (5) and then sensitive associated with its output is shown in Figure (3) and Figure (4) according to the objective function and algorithm's running time.

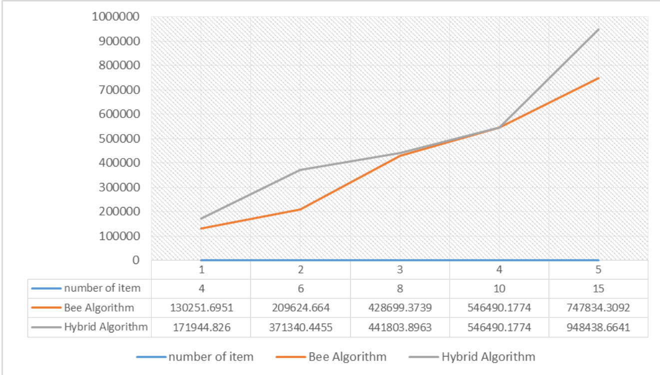

Figure 3. Comparison of objective functions of two algorithm in different size of problems

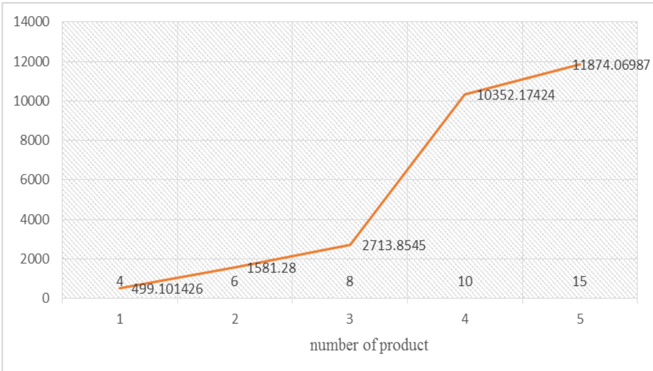

Figure 4. Run-time analysis with regard to the dimensions of problems

Furthermore, the output of two algorithm is compared with numerical result of GAMS that indicates in table 3.

Table 3. Comparison of output of algorithms with GAMS

(I,J,K,H,P)	*BA*	*HA*	*GAMS (Baron)*
(٤,٣,٣,٣,٣)	١٢٩١٥,٣٧٨٤	١٤١٧٩	١٥٢٦٠,٢٠٠

In the second portion of this section, a list of product with real data is taken and solved. Then the output of these two algorithm is compared for this list. The list of product are presented in table (4) and also table (6) and (7) show the numerical results of bees algorithm and hybrid algorithm.

Table 4. List of products

Prod. number	**Title of product**	**Retail price**	**Wholesale price**
١	Hailey Oil 1.5 L	٧٣٥٠	٦٤٥٠

٢	Behrouz French Sauce	٤٣٥٥	٣٥٨٤
٣	Behrouz Mayonnaise	٩٩٢٥٠	٧٧٥٠
٤	Shilton Tuna	٥٠٣٠	٤٠٠٠
٥	Mahmoud Tea	١٩٠٠٠	١٦٥٠٠
٦	Tabarrok Tomato Paste	٥٠٠٠	٢٩١٧

In the end of this section, we analyze different prices of two sample complementary products according to different discount policy or raising price policy considering of objective function value represented in figure (5).

Table 5. Numerical results for random data

Number of item *i*	*(J,H,P,T)*	Objective function		Running time in hybrid algorithm
		Bees algorithm	Hybrid algorithm	
٤	(٣,٣,٣,٣)	130251.6951	171944.826	499.101426
٦	(3,3,3,3)	209624.664	371340.44546	1581.28
٨	(3,3,3,3)	428699.3739	441803.89633	2713.8545
١٠	(3,3,3,3)	546490.1774	546490.1774	10352.174242
15	(3,3,3,3)	747834.3092	948438.6641	11874.069873

Table 6. Numerical results of bees algorithm

Output of bees algorithm						**Objective function value**
i	***Select/not***	***j***	***k***	***h***	***p***	
١	٠	٣	٢	٢	٢	١٢١٠١٠
٢	١	٣	٣	٢	٢	
٣	٠	٣	٢	٣	٣	
٤	١	٣	٣	٣	٣	
٥	١	١	٢	٣	٣	
٦	٠	٣	٢	٢	٣	

Table 7. Numerical results of heuristic algorithm

Output of heuristic algorithm						**Objective function value**
i	***Select/not***	***j***	***k***	***h***	***p***	
١	١	٣	١	٢	٢	۱۴۵۱۹۰
٢	١	٣	٣	٢	٢	
٣	١	٣	١	٣	٣	
٤	١	٣	٣	٣	٣	
٥	١	١	٢	٣	٣	
٦	١	٣	١	٢	٣	

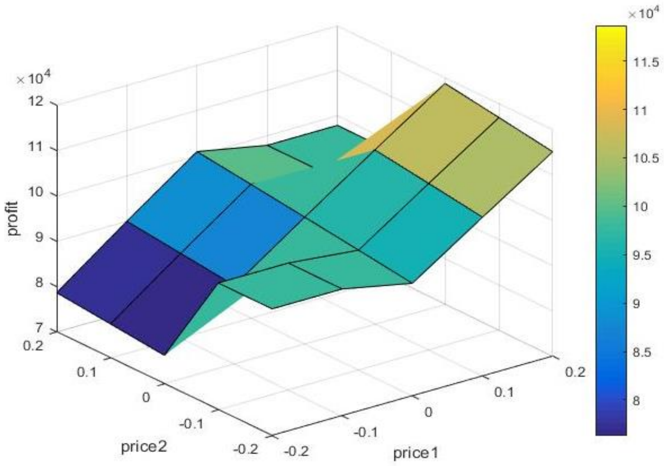

Figure 5. Pricing analysis for two complementary products

Conclusions and future work

This paper developed a mathematical model for shelf space optimization problem considering of stochastic demand and then used a meta-heuristic algorithm and a hybrid algorithm to solve some random problems. Also it investigates a real case to test solution approach with real data. Then it made the sensitive analysis for running time, objective function value and discount or raising price policies. This model considers one possible type of orientation for each item while it is suggested that in future works multiple orientation for each product be considered. Furthermore, utilization of other meta-heuristic and heuristic methods to solve this type of problems and also adding inventory variables, backroom storage and replenishment time for each product can cause the output of the model to be more realistic.

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