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Fuzzy Nonparametric Predictive Inference for the Reliability of Parallel Systems

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Soheil Shokri[a], Bahram Sadeghpour-Gildeh[b], Gholam-Reza Mohtashami Borzadaran[c] and Behrouz Fathi Vajargah[b]

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[a] Department of Statistics, International Campus, Ferdowsi University of Mashhad
Mashhad, Iran
E-mail: so_sh270@mail.um.ac.ir

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[b] Department of Statistics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad
Mashhad, Iran
E-mail: sadeghpour@um.ac.ir

[blank line]

[c] Department of Statistics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad
Mashhad, Iran
E-mail: grmohtashami@um.ac.ir

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[d] Department of Statistics, Faculty of Mathematical Sciences, University of Guilan
Rasht, Iran
E-mail: fathi@guilan.ac.ir

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Abstract

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This paper presents fuzzy lower and upper probabilities for the reliability of parallel systems. Attention is restricted to parallel systems with exchangeable components. In this paper we consider the problem of the evaluation of system reliability based on the nonparametric predictive inferential (NPI) approach, in which the defining the parameters of reliability function as crisp values is not possible and parameters of reliability function are described using a triangular fuzzy number. Formula of a fuzzy reliability function and its α-cut set are presented. The fuzzy reliability of structures is defined on the basis of fuzzy number. Furthermore, the fuzzy reliability functions of parallel systems discussed. Finally, some numerical examples are presented to illustrate how to calculate the fuzzy reliability function and its α-cut set. In other words, the aim of this paper is present a new method titled fuzzy non-parametric predictive inference for the reliability of parallel systems.

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Keywords:

Parallel Systems, Lower and Upper Probabilities, Nonparametric Predictive Inference, Fuzzy Number.

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1 Introduction

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Study on the reliability of the engineering design process is an important part of a system in which future performance will be evaluated. Since the future cannot be predicted with certainty be normal in the calculation of reliability, methods are used that allow the modeling of uncertainty[۱۳] . This paper provides a new method for statistical inference about system reliability the basis of limited knowledge resulting from component testing. This method is called Fuzzy Nonparametric Predictive Inference (FNPI). We present FNPI for system reliability, in particular FNPI for parallel systems.The theory of imprecise probabilities [2, 20], Possibility Theory [15], the theory of interval probability [21, 22] and fuzzy reliability theory [5] have been used as a general and promising tool for reliability analysis .[۱۳] Coolen and Utkin [12] provided an insight into imprecise reliability, discussing a variety of issues and reviewing suggested applications of imprecise probabilities in reliability, see [5, 10, 11, 12, 13] for a detailed overview of imprecise reliability and many references. The nonparametric predictive approach is a statistical approach based on few assumptions about probability distributions, with inferences based on data [7]. This method assumes exchangeability of random quantities, both related to observed data and future observations and uncertainty is quantified applying lower and upper probabilities that derived from Coolen [8]. Nonparametric predictive approach that proposed by Coolen [8] has proved to be efficient for measuring the probability of outcomes that cannot be done using precise probabilities. Nonparametric predictive inference (NPI) is a statistical framework which uses few modeling assumptions, with inferences explicitly in terms of future observations[7]. NPI is close in nature to predictive inference for the low structure stochastic case as briefly outlined by Geisser [16], which is in line with many earlier nonparametric test methods where the interpretation of the inferences is in terms of confidence intervals. NPI provides exactly calibrated frequentist inferences [7], and it has strong consistency properties in theory of interval probability [1]. NPI is always in line with inferences based on empirical distributions, which is a charming characteristic when intending at objectivity.

In recent years, many theoretical aspects and a variety of applications of inference based on Hill's assumption *A(n)* for prediction of probabilities, for one (or more) future values, on the basis of *n* prior observations, have been presented, referring to these as 'Nonparametric Predictive Inference' (NPI), see e.g. [1, 2, 6, 7, 8].

This paper aims at studying reliability of parallel systems base on non-parametric predictive inference in a fuzzy environment. In some cases, it may not be possible to define reliability of parallel systems parameters as crisp values. In these cases, these parameters can be represented by linguistic variables. The fuzzy set theory can be applied successfully to cope the vagueness in these linguistic expressions for reliability of parallel systems base on non-parametric predictive inference. In this paper a new method is presented for system reliability. This approach is called Fuzzy Non-parametric Predictive Inference (FNPI). It provides a new method for statistical inference on system reliability on the basis of limited information resulting from component testing. Formula of a fuzzy reliability function and its α-cut set are presented. The equation of a fuzzy reliability function and its α-cut set are determined. The fuzzy reliability of structures is described on the basis of fuzzy number. Finally, some numerical examples are presented to illustrate how to calculate the fuzzy reliability function and its α-cut set. In other words, the aim of this paper is to propose a new method titled fuzzy non-parametric predictive inference for the reliability of parallel systems.

In Section 2 we review briefly the main idea of NPI and Non-parametric Predictive Inference for the reliability of parallel systems. The Fuzzy Non-parametric Predictive Inference for the reliability of parallel systems is presented in Section 3, and finally in section four conclusions and discussion are presented.

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2 Non-parametric Predictive Inference for a Parallel System

Hill [17] proposed the assumption $A_{(n)}$ for prediction about future observations. This assumption was proposed particularly for situations in which there is no strong prior information about the probability distribution for a random quantity of interest. $A_{(n)}$ does not assume anything else, and is a post-data assumption related to exchangeability [7]. Hill [18] discusses $A_{(n)}$ in detail. Inferences based on $A_{(n)}$ are predictive and nonparametric, and can be considered suitable if there is hardly any knowledge about the random quantity of interest, other than the n observations, or if one does not want to apply such knowledge, e.g. to study influences of additional assumptions underlying other statistical methods [7]. $A_{(n)}$ is not sufficient to derive precise probabilities for several events of interest, but it presents optimal bounds for probabilities for all events of interest involving X_{n+1}. These bounds are lower and upper probabilities in the theories of imprecise probability and interval probability, and as such they have strong consistency properties. NPI is a framework of statistical theory and methods that use these $A_{(n)}$ based lower and upper probabilities, and also considers several variations of $A_{(n)}$ which are suitable for different inferences [7]. Augustin and Coolen [1] proved that the lower and upper probabilities obtained based only on the $A_{(n)}$ assumption has strong consistency properties in the theory of interval probability [8]. Coolen [6] used $A_{(n)}$ for NPI in case of Bernoulli data, providing interval probabilities for the number of successes in m future trials, based on the number of successes in n observed trials. This was possible by considering the same representation for such Bernoulli data as was used by Bayes [3], namely as balls on a table [6].

The class of *k*-out-of-*m* systems, also called 'voting systems', was introduced by Birnbaum [4]. These are systems that consist of *m* exchangeable ([9]) components (often the confusing term identical components is used), such that the system functions if and only if at least *k* of its components function. Since the value of *m* is usually larger than the value of *k*, redundancy is generally built into a *k*-out-of-*m* system. Both parallel and series systems are special cases of the *k*-out-of-*m* system. A series system is equivalent to an *m*-out-of-*m* system while a parallel system is equivalent to an *1*-out-of-*m* system [13].

Applications of *k*-out-of-*m* systems can e.g. be found in the areas of target detection, communication, safety monitoring systems, and, particularly, voting systems. The *k*-out-of-*m* systems are a very common type in fault-tolerant systems with redundancy. They have many applications in both industrial and military systems. Fault-tolerant systems include the multi-display system in a cockpit, the multiengine system in an aircraft, and the multi-pump system in a hydraulic control system [13].

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Definition 1 (The $A_{(n)}$ assumption of Hill) [8]

[blank line]

Assume that $X_1, X_2, \dots, X_n, X_{n+1}$ are continuous and exchangeable random quantities. Let the ordered observed values of $x_1, x_2, \dots, x_n$ be denoted by $x_{(1)} < x_{(1)}, \dots < x_{(n)} < \infty$, and let $x_{(0)} = -\infty$ and $x_{(n+1)} = \infty$ for ease of notation. Assume that the possibility of the existence of a nod is zero, and observations specify the real line as $n+1$ intervals in the form of $I_j = \left(x_{(j-1)}, x_{(j)}\right)$ for $j = 1, 2, \dots, n+1$.

For a future observation of X_{n+i} based on n observations, assumption $A_{(n)}$ is written as:

[blank line]

$$P\left(X_{n+i}\in I_j\right)=P\left(X_{n+i}\in\left(x_{(j-1)},x_{(j)}\right)\right)$$

$$=\frac{1}{n+1}\quad for\, i\geq 1,\ j=1,\ldots,n+1 \qquad (1)$$

[blank line]

This assumption implies that the rank of X_{n+i} amongst the observed $x_{(1)}<x_{(2)}<\cdots<x_{(n)}$ has equal probability to be any value in $\{1,2,\ldots,n+1\}$.

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Definition 2 (The predictive interval probabilities) [8]

[blank line]

Assume that $\mathcal{B}$ is the Borel $\sigma-$field on $\mathbb{R}$. For each element $B\in\mathcal{B}$, function sets $\underline{P}(\cdot)$ and $\overline{P}(\cdot)$ for the event $X_{n+1}\in\mathcal{B}$ based on the intervals $I_1,I_2,\ldots,I_{n+1}$ and the assumption $A_{(n)}$ are defined as:

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$$\underline{P}\left(X_{n+1}\in B\right)=\frac{1}{n+1}\left|\left\{j: I_j\subseteq B\right\}\right|, \qquad (2)$$

[blank line]

$$\overline{P}\left(X_{n+1}\in B\right)=\frac{1}{n+1}\left|\left\{j: I_j\bigcap B\neq\varnothing\right\}\right|. \qquad (3)$$

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Theorem 1

[blank line]

Assume a $n+m$ number of Bernoulli's exchangeable experiments whose result can be success or failure. Assume:

[blank line]

$Y_{n+1}^{n+m}\rightarrow$ The random variable of number of successes of m Bernoulli's future ($n+1$ to $+m$) experiments.

$Y_1^n\rightarrow$ The random variable of the number of successes in n Bernoulli's previous (1 to n) experiments.

For the sake of simplicity we define $\binom{s+r_0}{s}=0$, therefore, the upper and lower probabilities of non-parametric predictive inference are

[blank line]

$$\overline{P}\left(Y_{n+1}^{n+m}\in R_t|Y_1^n=s\right)$$

$$=\binom{n+m}{m}^{-1}\sum_{j=1}^{t}\left[\binom{s+r_j}{s}-\binom{s+r_{j-1}}{s}\right]\binom{n-s+m-r_j}{n-s} \qquad (4)$$

[blank line]

And

[blank line]

$$\underline{P}\left(Y_{n+1}^{n+m}\in R_t|Y_1^n=s\right)=1-\overline{P}\left(Y_{n+1}^{n+m}\in R_t^c|Y_1^n=s\right) \qquad (5)$$

[blank line]

Where $R_t=\{r_۱,\ldots,r_t\}$ with $۰\leq r_۱<r_۲<\ldots<r_t\leq m$, $۱\leq t\leq m+۱$ and $R_t^c=\{۰,۱,\ldots,m\}\setminus R_t$.

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Proof. See [6].

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Corollary 1

[blank line]

Considering a *k*-out-of-*m* system, the event $Y_{n+1}^{n+m}\geq k$ is of interest as this corresponds to successful functioning of a *k*-out-of-*m* system, following *n* tests of components that are exchangeable with the *m* components in the system considered. Given data consisting of *s* successes from *n* components tested, the NPI lower and upper probabilities for the event that the *k*-out-of-*m* system functions successfully are also denoted by $\overline{P}\left(S(m:k)|(n,s)\right)$ and $\underline{P}\left(S(m:k)|(n,s)\right)$, respectively. For $k\in\{1,2,\ldots,\mathrm{m}\}$ and $0<s<n$ [13]

[blank line]

$$\overline{P}\left(S(m:k)|(n,s)\right)=\overline{P}\left(Y_{n+1}^{n+m}\geq k|Y_1^n=s\right)$$

$$=\binom{n+m}{m}^{-1}\left[\binom{s+k}{s}\binom{n-s+m-k}{n-s}\right.$$

$$\left.+\sum_{l=k+1}^{m}\binom{s+l-1}{s-1}\binom{n-s+m-l}{n-s}\right] \qquad (6)$$

[blank line]

and

[blank line]

$$\underline{P}\left(S(m:k)|(n,s)\right)$$

$$=\underline{P}\left(Y_{n+1}^{n+m}\geq k|Y_1^n=s\right)$$

$$=1-\overline{P}\left(Y_{n+1}^{n+m}\leq k-1|Y_1^n=s\right)$$

$$=1-\binom{n+m}{m}^{-1}\left[\sum_{l=0}^{k-1}\binom{s+l-1}{s-1}\binom{n-s+m-l}{n-s}\right] \qquad (7)$$

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Corollary 2

[blank line]

For the parallel systems, with *k=1*, NPI upper and lower probabilities can be substantially simplified to give the expressions below, which actually provide insight into the NPI approach for such systems. Representing corresponding lower and upper probabilities for an event A by $(\underline{P},\overline{P})(A)$, the general results above are, for parallel system [13].

[blank line]

$$(\underline{P},\overline{P})(m:1|n,s)$$

$$=\left(1-\prod_{j=1}^{m}\frac{n-s+j}{n+j},\,1-\prod_{j=1}^{m}\frac{n-s-1+j}{n+j}\right)for\,0<s<n \qquad (8)$$

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3 Fuzzy Non-parametric Predictive Inference for the Reliability of *k*-out-of-*m* Systems

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In this Section we consider the problem of the evaluation of system reliability based on the nonparametric predictive inferential (NPI) approach, in which the defining the parameters of reliability function in definite quantities is not possible and parameters of reliability function are described using a triangular fuzzy number.

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3.1 Fuzzy Set Theory

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The theory of sets and fuzzy logic was first proposed by Zadeh (1965). This theory has found wide applications in many fields such as computer, system analysis, electronic and recently in social sciences, economics and industry. Fuzzy logic is a theory for uncertain conditions. This theory can form many of concepts, variables and systems which are imprecise and vague in a mathematical form and provide the way for reasoning, control and decision-making in uncertain conditions. In popular speech if a variable can take a number of terms from the natural language as amounts; we call it a linguistic variable. For the formulation of terms in mathematical expressions, we use fuzzy sets to designate terms. In other words, "if a variable can take terms from the natural language as its amounts, then it is called a linguistic variable in which terms are specified by fuzzy sets domains in which variables have been defined". we recall same concepts of fuzzy set theory used in this article derived from (Zadeh, 1965; Zimmermann, 1991)

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Definition 1

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The set $\tilde{A}$ of R is called a fuzzy number if it satisfies in the following conditions:

$\tilde{A}$ is normal i.e. $\exists x_0 \in R\ ; \tilde{A}(x_0)=1$.

$\tilde{A}$ is convex i.e. for each $x_1, x_2 \in \mathrm{R}$ and each $\lambda \in [0,1]$ we have

$$\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\tilde{A}(x_1), \tilde{A}(x_2))$$

$\tilde{A}$ is the upper semi continuous.

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Definition 2 α- cut of fuzzy set

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The α- cut, A_α, consists of elements whose membership degree in $\tilde{A}$ is not lower than α, i. e.

$$A_\alpha = \left\{ x \in X \ |\tilde{A}(x) \geq \alpha \right\}, \qquad 0 < \alpha \leq 1$$

The α- cut set of a fuzzy number is a closed interval which is shown as $A_\alpha = [A_\alpha^-, A_\alpha^+]$ in which

$$A_\alpha^- = inf \left\{ x \in R\ ; \tilde{A}(x) \geq \alpha \right\}$$

$$A_\alpha^+ = sup \left\{ x \in R\ ; \tilde{A}(x) \geq \alpha \right\}$$

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The most used fuzzy numbers are the trapezoidal and triangular fuzzy numbers. Triangular fuzzy numbers, due to their simple computations, are used more.

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3.2 Fuzzy Number of Success in Tested Components(s)

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The number of success in tested components can be defined by linguistic variables. One of the circumstances that can be assumed is when the number of functioning items is defined as linguistic variables. Fuzzy numbers can be used for showing functioning items. Assume that the number of functioning items can be shown as the following triangular fuzzy number:

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$$\tilde{s} = TFN(s_1, s_2, s_3)$$

[blank line]

and

[blank line]

$$s(\alpha) = (s_1 + (s_2 - s_1)\alpha\ , s_3 + (s_2 - s_3)\alpha)$$

[blank line]

Therefore fuzzy lower non-parametric predictive probability,

[blank line]

$$\underline{P}\left(S(m:1) | (n,s)\right) = \underline{P}\left(Y_{n+1}^{n+m} \geq 1 | Y_1^n = s\right) = 1 - \prod_{j=1}^{m} \frac{n - \tilde{s} + j}{n+j}$$

[blank line]

As a result

[blank line]

$$\underline{P}(\alpha) = \left\{ 1 - \prod_{j=1}^{m} \frac{n-s+j}{n+j} | s \in s(\alpha) \right\} \qquad 0 \leq \alpha \leq 1$$

[blank line]

$$\underline{P}(\alpha) = \left[\underline{P}_l(\alpha), \underline{P}_r(\alpha) \right]$$

[blank line]

In a way that,

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$$\underline{P}_l(\alpha) = min \left\{ 1 - \prod_{j=1}^{m} \frac{n-s+j}{n+j} | s \in s(\alpha) \right\}$$

[blank line]

$$\underline{P}_r(\alpha) = max \left\{ 1 - \prod_{j=1}^{m} \frac{n-s+j}{n+j} | s \in s(\alpha) \right\}$$

[blank line]

If $\tilde{s}$ be the triangular fuzzy number then

[blank line]

$$\underline{P}_l(\alpha)=1-\prod_{j=1}^{m}\frac{n-(s_1+(s_2-s_1)\alpha)+j}{n+j}$$

[blank line]

$$\underline{P}_r(\alpha)=1-\prod_{j=1}^{m}\frac{n-(s_3+(s_2-s_3)\alpha)+j}{n+j}$$

[blank line]

Too fuzzy upper non-parametric predictive probability,

[blank line]

$$\overline{P}(S(m:1)|(n,s))=\overline{P}(Y_{n+1}^{n+m}\ge 1|Y_1^n=s)$$
$$=1-\prod_{j=1}^{m}\frac{n-\tilde{s}-1+j}{n+j}$$

[blank line]

As a result

[blank line]

$$\overline{P}(\alpha)=\left\{1-\prod_{j=1}^{m}\frac{n-s-1+j}{n+j}|s\in s(\alpha)\right\}\quad 0\le\alpha\le 1$$

[blank line]

or

[blank line]

$$\overline{P}(\alpha)=\left[\overline{P}_l(\alpha),\overline{P}_r(\alpha)\right]$$

[blank line]

$$\overline{P}_l(\alpha)=min\left\{1-\prod_{j=1}^{m}\frac{n-s-1+j}{n+j}|s\in s(\alpha)\right\}$$

[blank line]

$$\overline{P}_r(\alpha)=max\left\{1-\prod_{j=1}^{m}\frac{n-s-1+j}{n+j}|s\in s(\alpha)\right\}$$

[blank line]

If $\tilde{s}$ be the triangular fuzzy number then

[blank line]
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$$\overline{P}_l(\alpha)=1-\prod_{j=1}^{m}\frac{n-(s_1+(s_2-s_1)\alpha)-1+j}{n+j}$$

[blank line]

$$\overline{P}_r(\alpha)=1-\prod_{j=1}^{m}\frac{n-(s_3+(s_2-s_3)\alpha)-1+j}{n+j}$$

[blank line]

3.2 Fuzzy Numbers of Tested Components(n)

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One of the other instances that can be assumed is when the number of tested components is defined as linguistic variables. Fuzzy numbers can be used for the depiction of the number of sample elements. Assume that n numbers of tested components are defined as the following triangular numbers:

[blank line]

$$\tilde{n}=\mathrm{TFN}(n_1,\ n_2,\ n_3)$$

[blank line]

And

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$$n(\alpha)=(n_1+(n_2-n_1)\alpha\ ,n_3+(n_2-n_3)\alpha)$$

[blank line]

So fuzzy lower non-parametric predictive probability

[blank line]

$$\underline{P}(S(m:1)|(n,s))=\underline{P}(Y_{n+1}^{n+m}\ge 1|Y_1^n=s)$$
$$=1-\prod_{j=1}^{m}\frac{\tilde{n}-\tilde{s}+j}{\tilde{n}+j}$$

[blank line]

As a result

[blank line]

$$\underline{P}(\alpha)=\left\{1-\prod_{j=1}^{m}\frac{n-s+j}{n+j}|s\in s(\alpha),n\in n(\alpha)\right\}\quad 0\le\alpha\le 1$$

[blank line]

or

[blank line]

$$\underline{P}(\alpha)=\left[\underline{P}_l(\alpha),\underline{P}_r(\alpha)\right]$$

[blank line]

$$\underline{P}_l(\alpha)=min\left\{1-\prod_{j=1}^{m}\frac{n-s+j}{n+j}|s\in s(\alpha),n\in n(\alpha)\right\}$$

[blank line]

$$\underline{P}_r(\alpha)=max\left\{1-\prod_{j=1}^{m}\frac{n-s+j}{n+j}|s\in s(\alpha),n\in n(\alpha)\right\}$$

[blank line]

If $\tilde{s}$ and $\tilde{n}$ be the triangular fuzzy numbers then

[blank line]

$$\underline{P}_l(\alpha)=1-\prod_{j=1}^{m}\frac{(n_1+(n_2-n_1)\alpha)-(s_1+(s_2-s_1)\alpha)+j}{(n_1+(n_2-n_1)\alpha)+j}$$

[blank line]

$$\underline{P}_r(\alpha)=1-\prod_{j=1}^{m}\frac{(n_1+(n_2-n_1)\alpha)-(s_3+(s_2-s_3)\alpha)+j}{(n_3+(n_2-n_3)\alpha)+j}$$

[blank line]

Too fuzzy upper non-parametric predictive probability,

[blank line]

$$\overline{P}(S(m:1)|(n,s))=\overline{P}(Y_{n+1}^{n+m}\ge 1|Y_1^n=s)$$
$$=1-\prod_{j=1}^{m}\frac{\tilde{n}-\tilde{s}-1+j}{\tilde{n}+j}$$

[blank line]

$$\overline{P}(\alpha)=\left\{1-\prod_{j=1}^{m}\frac{n-s-1+j}{n+j}|s\in s(\alpha),n\in n(\alpha)\right\}$$

$0\le\alpha\le1$

[blank line]
or
[blank line]

$$\overline{P}(\alpha)=\left[\overline{P}_l(\alpha),\overline{P}_r(\alpha)\right]$$

[blank line]

$$\overline{P}_l(\alpha)=min\left\{1-\prod_{j=1}^{m}\frac{n-s-1+j}{n+j}|s\in s(\alpha),n\in n(\alpha)\right\}$$

[blank line]

$$\overline{P}_r(\alpha)=max\left\{1-\prod_{j=1}^{m}\frac{n-s-1+j}{n+j}|s\in s(\alpha),n\in n(\alpha)\right\}$$

[blank line]
If $\tilde{s}$ and $\tilde{n}$ be the triangular fuzzy numbers then

[blank line]

$$\overline{P}_l(\alpha)=1-\prod_{j=1}^{m}\frac{(n_1+(n_2-n_1)\alpha)-(s_1+(s_2-s_1)\alpha)-1+j}{(n_1+(n_2-n_1)\alpha)+j}$$

[blank line]

$$\overline{P}_r(\alpha)=1-\prod_{j=1}^{m}\frac{(n_1+(n_2-n_1)\alpha)-(s_3+(s_2-s_3)\alpha)-1+j}{(n_1+(n_2-n_1)\alpha)+j}$$

[blank line]

3.3 Numerical Examples

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Example 1

[blank line]
Consider a parallel system with 5 exchangeable components (so m=5), and the only information available is the result of a test of 4 components, also exchangeable with the 5 to be used in the system. Assume that the numbers of successes in the tests are expressed as “Approximately 2”. Triangular fuzzy numbers are more suitable to convert this definition into a fuzzy number. The number of successes in the tests to be converted to a triangular fuzzy number as $\tilde{s}=\text{TFN}(1,2,3)$. The FNPI lower and upper probabilities for successful functioning of the system are
[blank line]

$$\tilde{s}=\text{TFN}(1,2,3)$$

[blank line]

$$s(\alpha)=(1+\alpha\ ,3-\alpha)$$

[blank line]

$$\underline{P}(\alpha)=\left\{\prod_{j=1}^{5}\frac{4-s+j}{4+j}|s\in s(\alpha)\right\}\qquad \circ\le\alpha\le1$$

[blank line]
or
[blank line]

$$\underline{P}(\alpha)=\left[\underline{P}_l(\alpha),\underline{P}_r(\alpha)\right]$$

[blank line]

$$\underline{P}_l(\alpha)=min\left\{1-\prod_{j=1}^{5}\frac{4-s+j}{4+j}|s\in s(\alpha)\right\}$$

[blank line]

$$\underline{P}_r(\alpha)=max\left\{1-\prod_{j=1}^{5}\frac{4-s+j}{4+j}|s\in s(\alpha)\right\}$$

[blank line]

$$\overline{P}(\alpha)=\left\{1-\prod_{j=1}^{5}\frac{4-s-1+j}{4+j}|s\in s(\alpha)\right\}\quad 0\le\alpha\le1$$

[blank line]
or

$$\overline{P}(\alpha)=\left[\overline{P}_l(\alpha),\overline{P}_r(\alpha)\right]$$

[blank line]

$$\overline{P}_l(\alpha)=min\left\{1-\prod_{j=1}^{5}\frac{4-s-1+j}{4+j}|s\in s(\alpha)\right\}$$

[blank line]

$$\overline{P}_r(\alpha)=max\left\{1-\prod_{j=1}^{5}\frac{4-s-1+j}{4+j}|s\in s(\alpha)\right\}$$

[blank line]
Table (1) and (2) shows α-cuts related to $\underline{\tilde{P}}$ fuzzy lower non-parametric predictive probability and $\overline{\tilde{P}}$ fuzzy upper non-parametric predictive probability and Figures (1) and (2) show diagrams corresponding membership function.
[blank line]

Table 1- α-cuts related to $\underline{\tilde{P}}$ fuzzy lower non-parametric predictive probability.

α	$\underline{P}_l(\alpha)$	$\underline{P}_r(\alpha)$	α	$\underline{P}_l(\alpha)$	$\underline{P}_r(\alpha)$
0	0.5556	0.9524	0.55	0.7341	0.9010
0.05	0.5749	0.9488	0.60	0.7470	0.8948
0.10	0.5935	0.9451	0.65	0.7593	0.8882
0.15	0.6115	0.9411	0.70	0.7712	0.8814
0.20	0.6289	0.9370	0.75	0.7827	0.8743
0.25	0.6456	0.9326	0.80	0.7937	0.8668
0.30	0.6618	0.9279	0.85	0.8042	0.8590
0.35	0.6774	0.9231	0.90	0.8143	0.8508
0.40	0.6924	0.9179	0.95	0.8240	0.8422
0.45	0.7068	0.9126	1	0.8333	0.8333

0.50	0.7207	0.9069	----	----	----

[blank line]

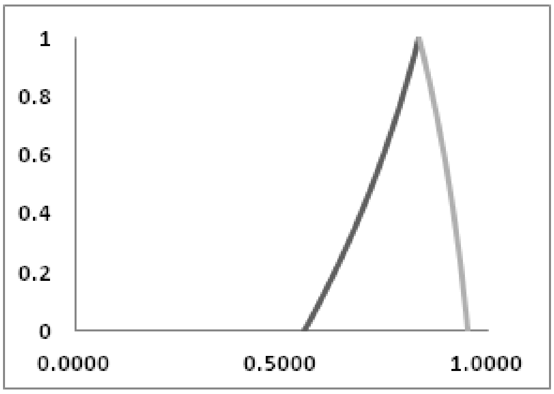

Figure 1- the diagram of membership function of lower non-parametric predictive probability.

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Table 2- α- cuts related to $\overline{P}$ fuzzy upper non-parametric predictive probability.

α	$\overline{P}_l(\alpha)$	$\overline{P}_r(\alpha)$	α	$\overline{P}_l(\alpha)$	$\overline{P}_r(\alpha)$
0	0.8333	0.9921	0.55	0.9126	0.9766
0.05	0.8422	0.9911	0.60	0.9179	0.9745
0.10	0.8508	0.9901	0.65	0.9231	0.9723
0.15	0.8590	0.9890	0.70	0.9279	0.9699
0.20	0.8668	0.9878	0.75	0.9326	0.9674
0.25	0.8743	0.9865	0.80	0.9370	0.9647
0.30	0.8814	0.9851	0.85	0.9411	0.9619
0.35	0.8882	0.9836	0.90	0.9451	0.9589
0.40	0.8948	0.9820	0.95	0.9488	0.9557
0.45	0.9010	0.9803	1	0.9524	0.9524
0.50	0.9069	0.9785	----	----	----

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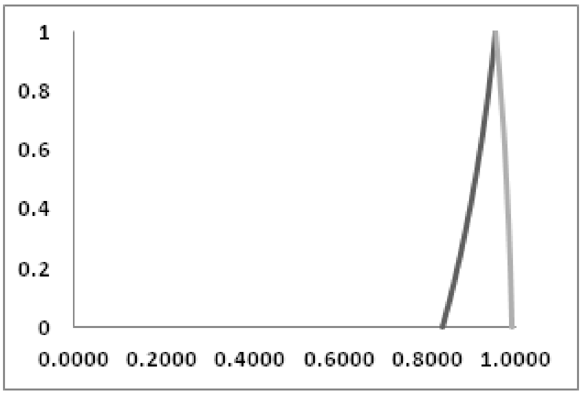

Figure 2- the diagram of membership function of upper non-parametric predictive probability

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Example 2

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Consider a parallel system with 5 exchangeable components (so m=5), and the only information available is the result of a test of "Approximately 4" components, also exchangeable with the 5 to be used in the system. Assume that the numbers of successes in the tests are expressed as "Approximately 2". Triangular fuzzy numbers are more suitable to convert this definition into a fuzzy number. The number of components to be converted to a triangular fuzzy number as $\tilde{n}=\mathrm{TFN}(3,4,5)$ and The number of successes in the tests to be converted to a triangular fuzzy number as $\tilde{s}=\mathrm{TFN}(1,2,3)$. The FNPI lower and upper probabilities for successful functioning of the system are

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$$\tilde{n}=\mathrm{TFN}(3,4,5)$$

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$$\tilde{s}=\mathrm{TFN}(1,2,3)$$

[blank line]

$$n(\alpha)=(3+\alpha\ ,5-\alpha)$$

[blank line]

$$s(\alpha)=(1+\alpha\ ,3-\alpha)$$

[blank line]

$$\underline{P}(\alpha)=\left\{1-\prod_{j=1}^{5}\frac{n-s+j}{n+j}\,|\,s\in s(\alpha),n\in n(\alpha)\right\}\quad 0\le\alpha\le1$$

[blank line]

or

[blank line]

$$\underline{P}(\alpha)=\left[\underline{P}_l(\alpha),\underline{P}_r(\alpha)\right]$$

[blank line]

$$\underline{P}_l(\alpha)=min\left\{1-\prod_{j=1}^{5}\frac{n-s+j}{n+j}\,|\,s\in s(\alpha),n\in n(\alpha)\right\}$$

[blank line]

$$\underline{P}_r(\alpha)=max\left\{1-\prod_{j=1}^{5}\frac{n-s+j}{n+j}\,|\,s\in s(\alpha),n\in n(\alpha)\right\}$$

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Too fuzzy upper non-parametric predictive probability,

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$$\overline{P}(\alpha)=\left\{1-\prod_{j=1}^{5}\frac{n-s-1+j}{n+j}\,|\,s\in s(\alpha),n\in n(\alpha)\right\}\quad 0\le\alpha\le1$$

[blank line]

or

[blank line]

$$\overline{P}(\alpha)=\left[\overline{P}_l(\alpha),\overline{P}_r(\alpha)\right]$$

[blank line]

$$\overline{P}_l(\alpha)=min\left\{1-\prod_{j=1}^{5}\frac{n-s-1+j}{n+j}|s\in s(\alpha),n\in n(\alpha)\right\}$$

[blank line]

$$\overline{P}_r(\alpha)=max\left\{1-\prod_{j=1}^{5}\frac{n-s-1+j}{n+j}|s\in s(\alpha),n\in n(\alpha)\right\}$$

[blank line]

Table (3) and (4) shows □-cuts related to fuzzy lower $\underline{\tilde{P}}$ non-parametric predictive probability and $\overline{P}$ fuzzy upper non-parametric predictive probability and Figures (1) and (2) show diagrams corresponding membership function.

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Table 3: α- cuts related to $\underline{\tilde{P}}$ fuzzy lower non-parametric predictive probability.

α	$\underline{P}_l(\alpha)$	$\underline{P}_r(\alpha)$	α	$\underline{P}_l(\alpha)$	$\underline{P}_r(\alpha)$
0	0.6250	0.9167	0.55	0.7640	0.8795
0.05	0.6411	0.9139	0.60	0.7732	0.8752
0.10	0.6565	0.9111	0.65	0.7820	0.8707
0.15	0.6710	0.9081	0.70	0.7904	0.8661
0.20	0.6848	0.9050	0.75	0.7984	0.8612
0.25	0.6979	0.9018	0.80	0.8061	0.8561
0.30	0.7103	0.8985	0.85	0.8134	0.8508
0.35	0.7222	0.8950	0.90	0.8203	0.8452
0.40	0.7334	0.8913	0.95	0.8270	0.8394
0.45	0.7441	0.8876	1	0.8333	0.8333
0.50	0.7543	0.8836	----	----	----

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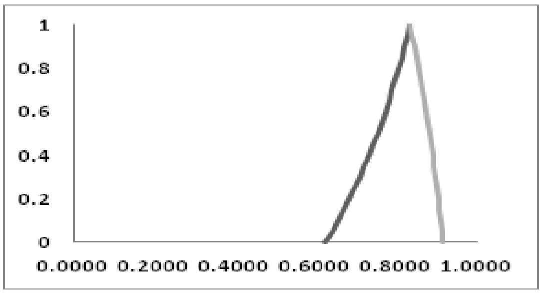

Figure (3) the diagram of membership function of lower non-parametric predictive probability

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Table 4: α− cuts related to $\overline{P}$ fuzzy upper non-parametric predictive probability.

α	$\overline{P}_l(\alpha)$	$\overline{P}_r(\alpha)$	α	$\overline{P}_l(\alpha)$	$\overline{P}_r(\alpha)$
0	0.8929	0.9762	0.55	0.9326	0.9656
0.05	0.8975	0.9754	0.60	0.9352	0.9643
0.10	0.9018	0.9746	0.65	0.9377	0.9631
0.15	0.9060	0.9737	0.70	0.9401	0.9617
0.20	0.9099	0.9729	0.75	0.9424	0.9603
0.25	0.9137	0.9719	0.80	0.9446	0.9589
0.30	0.9172	0.9710	0.85	0.9467	0.9574
0.35	0.9206	0.9700	0.90	0.9487	0.9558
0.40	0.9238	0.9690	0.95	0.9506	0.9541
0.45	0.9269	0.9679	1	0.9524	0.9524
0.50	0.9298	0.9667	----	----	----

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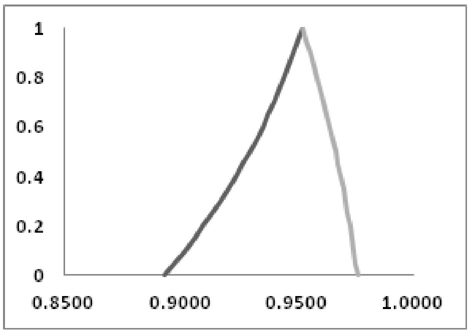

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Figure (4) the diagram of membership function of upper non-parametric predictive probability

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4 Conclusions

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Despite of the usefulness of reliability of parallel systems base of nonparametric predictive approach, it has a main difficulty in defining its parameters as crisp values. Sometimes it is easier to define these parameters by using linguistic variables. For these cases, the fuzzy set theory is the most suitable tool to analyze reliability of parallel systems base of nonparametric predictive approach. The obtained results show that the fuzzy definitions of parameters provide more flexibility and more usability. In this article the non- parametric predictive probability has been analyzed for reliability of parallel systems with fuzzy parameters. We have shown that when the definition of lower and upper predictive probability parameters is not possible as crisp values, and when defining the parameters of number of success in tested components and number of tested components as crisp values is not possible, these parameters can be expressed in linguistic terms, and the fuzzy set theory can be used successfully to overcome ambiguity in such expressions in the form of non-parametric predictive reliability of parallel systems. We also calculate the fuzzy reliability function and its α-cut set.

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References

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[1] Augustin, T. and Coolen, F.P.A. (2004). “Nonparametric predictive inference and interval probability”. *Journal of Statistical Planning and Inference*, 124, 251-272.

[2] Augustin, T., Coolen, F.P.A., Cooman, G. D. and Troffaes, M.C.M., editors , *Introduction to Imprecise Probabilities*. Wiley Series in Probability and Statistics, Wiley,2014.

[3] Bayes, T. (1763). “An essay towards solving a problem in the doctrine of chances”. *Philosophical Transactions of*

the Royal Society London, 53, 370-418; 54, 296-325.
[4] Birnbaum, Z. W., Esary, J. D. and Saunders, S. (1961). “Multi-component systems and structures and their reliability”. *Technometrics*, 3, 5577.
[5] Cai, K-Y., *Introduction to Fuzzy Reliability*, Kluwer Academic Publishers, Boston, 1996.
[6] Coolen, F.P.A. (1998). “Low structure imprecise predictive inference for Bayes' problem”. *Statistics & Probability Letters*, 36, 349-357.
[7] Coolen, F. P. A. (2011). “Nonparametric Predictive Inference”. *In International encyclopedia of statistical science*, ed. M. Lovric, 968–970. Berlin, Springer.
[8] Coolen, F.P.A, *Nonparametric Predictive Inference*. Wiley Series in Probability and Statistics, Wiley, 2015.
[9] De Finetti, B., *Theory of Probability*, 2 vols. Wiley, London, 1974.
[10] Coolen, F.P.A., Coolen-Schrijner, P., Yan, K.J. (2002). “Nonparametric predictive inference in reliability”. *Reliability Engineering and System Safety* 78, 185-193
[11] Coolen, F.P.A. and Coolen-Schrijner, P. (2008). “Nonparametric predictive inference for k-out-of-m systems”. *In Advances in mathematical modeling for reliability*, (Eds T. Bedford, et al.) pp. 185–192 (IOS Press, Amsterdam).
[12] Coolen F.P.A., Utkin L.V. (2011). “Imprecise reliability”. In: *International Encyclopedia of Statistical Science*, M. Lovric (Ed.). Springer, 649-650.
[13] Coolen-Schrijner, P., Coolen, F.P.A. and MacPhee, I.M. (2008). “Nonparametric predictive inference for systems reliability with redundancy allocation”. *Journal of Risk and Reliability* 222, 463-476. 3
[14] De Finetti, B., *Theory of Probability*, 2 vols. Wiley, London, 1974.
[15] Dubois, D. and Prade, H., Possibility Theory: *An Approach to Computerized Processing of Uncertainty*, Plenum Press, New York, 1988.
[16] Geisser, S., *Predictive Inference: An Introduction*. Chapman & Hall, London,1993.
[17] Hill, B.M. (1968). “Posterior distribution of percentiles: Bayes' theorem for sampling from a population”. *Journal of the American Statistical Association*, 63, 677-691.
[18] Hill, B.M. (1988). “De Finetti's theorem, induction, and A(n) or Bayesian nonparametric predictive inference (with discussion)”. In J.M. Bernardo, et al. (Eds.), *Bayesian Statistics 3*, 211-241. Oxford University Press.
[19] Tian, Z., Zuo, M. J. and Yam, R. C. (2009). “Multi-state k-out-of-n systems and their performance evaluation”. *IIE Transactions*, 41, 32-44.
[20] Walley, P., *Statistical Reasoning with Imprecise Probabilities*. Chapman & Hall, London, 1991.
[21] Weichselberger, K. (1995). “Axiomatic foundations of the theory of interval-probability”. In: Mammitzsch, V., SchneeweiU, H. (Eds.), *Proceedings of the Second GauU Symposion*, Section B. De Gruyter, Berlin, pp. 47–64.
[22] Weichselberger, K. (2000). “The theory of interval-probability as a unifying concept for uncertainty”. *Int. J. Approx. Reason*. 24, 149–170.
[23] Zadeh, L.A. (1965*). “*Fuzzy sets*”. Information and Control 8, 338–353*
[24] Zimmermann, H.J., Fuzzy set theory and its applications. Kluwer Academic Publishers, Dordrecht, 1991.